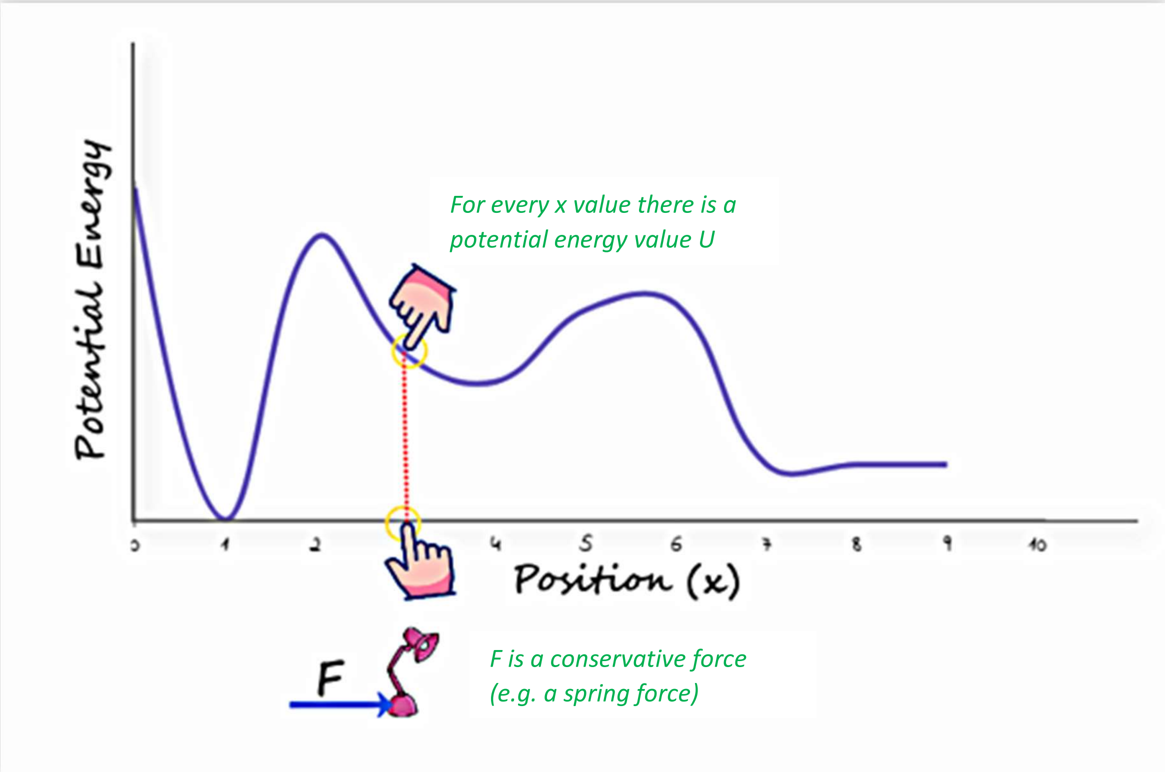
Potential Energy Curves

Key Idea

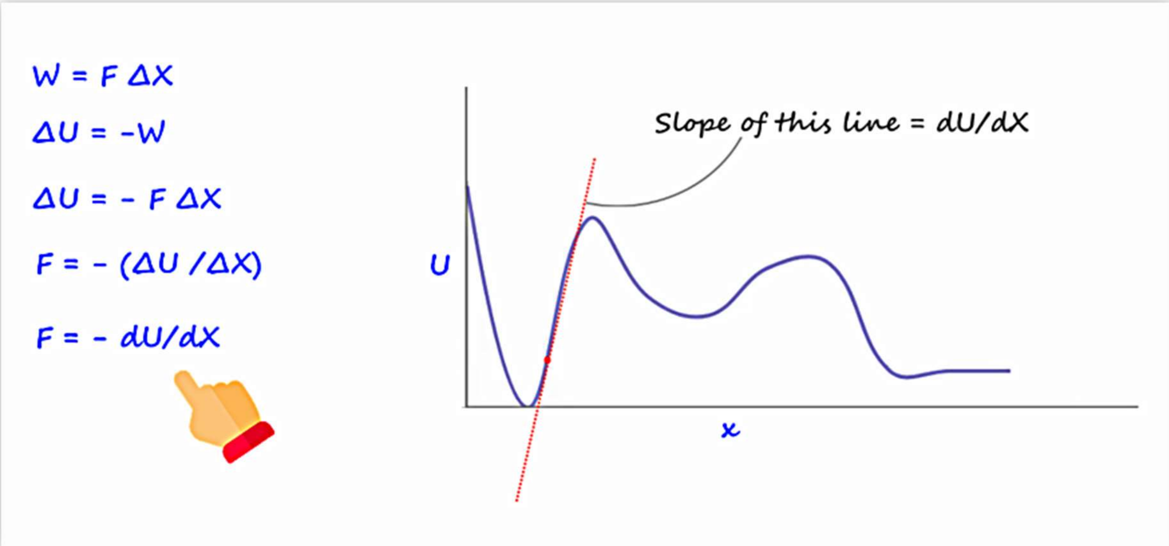
Potential energy curves visualize the relationship between a particle's potential energy (U) and its position (x) under the influence of a conservative force. A graph that shows the change in potential energy U with x is called a potential energy graph or curve



Equation Connecting Force, Potential Energy and Position

$F = -dU/dx$	Force equation	(1)

This formula implies that the force at any position x is equal to the negative of the rate of change of potential energy with respect to that position. *This rate of change is the slope of the curve at that position x*



Validating the equation (F = -dU/dx)

(a) Elastic Potential Energy and Hooke's Law

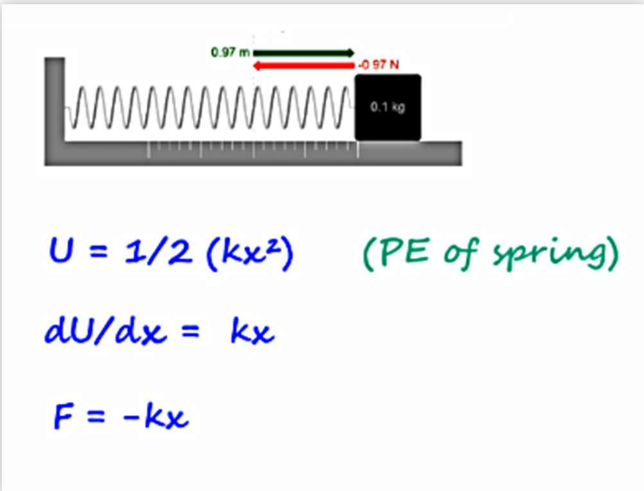
For a spring, the potential energy

$U = 1/2\ kx^2$.

Differentiating with respect to x gives $dU/dx = kx$, which when substituted equation (1) yields

$F = -kx$.

This result confirms Hooke's law

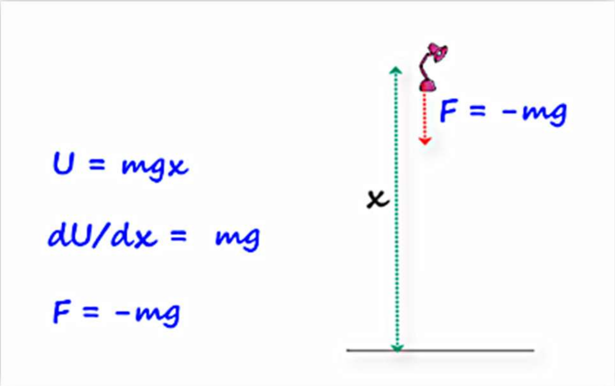


(b) Gravitational Potential Energy

Consider the gravitational potential energy U = mgx.

Differentiating yields dU/dx = mg. Leading to F = -mg when applied to the force equation.

This example reinforces the relationship between force and potential energy.

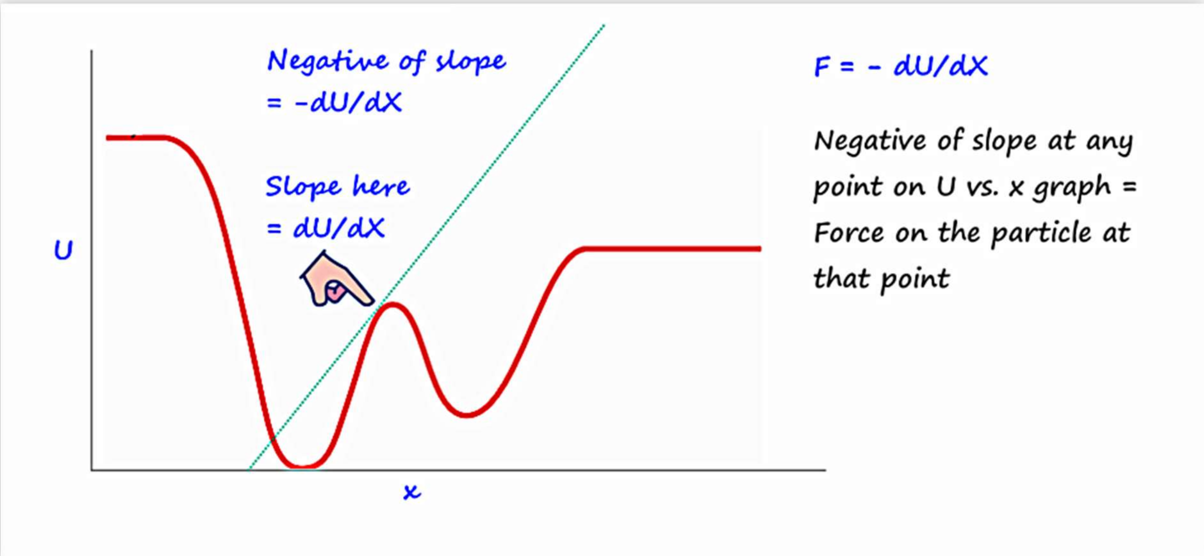


Analysing Potential Energy Graphs

The slope of the graph between potential energy U and position x is dU/dx.

F = *Minus* of Slope at the point x under consideration, or

$$F = -dU/dx$$

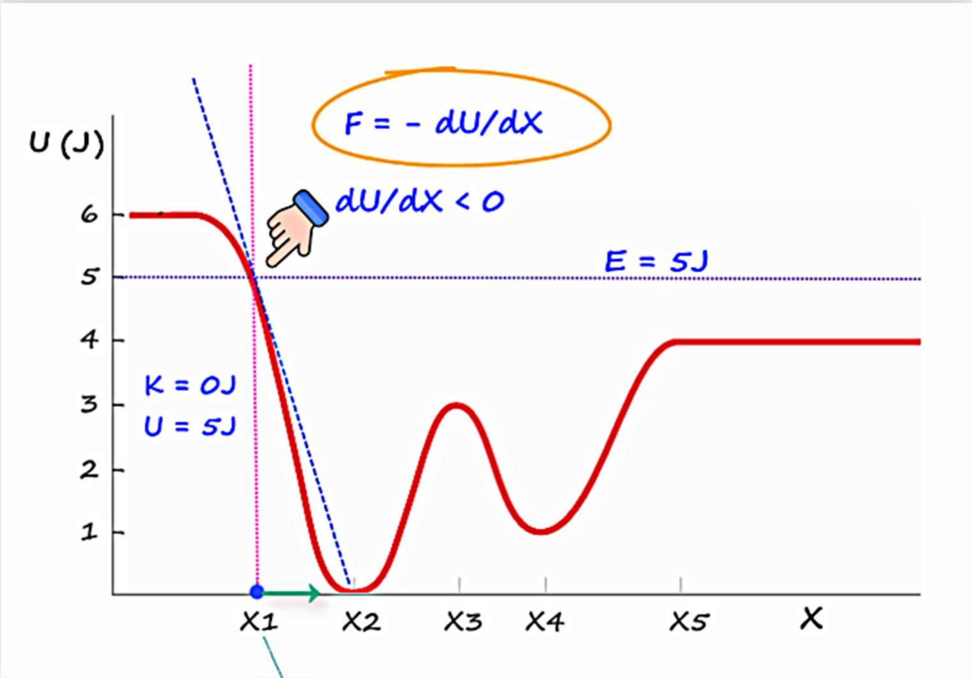


Turning Points

A turning point is a point where a particle changes its direction of motion. Following are characteristics of turning points-

- Kinetic energy of particle becomes zero. Therefore, velocity drops to zero.
- A force exists on the particle but in a direction opposite the initial direction of motion.
- Mechanical Energy E = U since K = 0 (E = U + K)

In the below U vs. x curve, x1 is a turning point



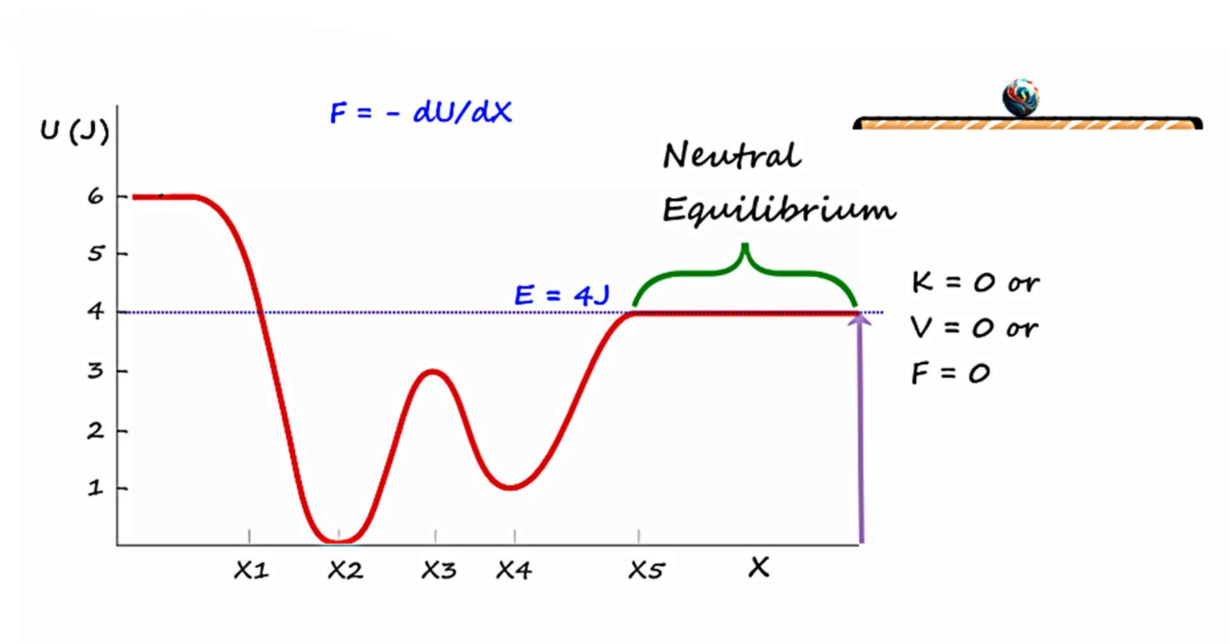
Since the slope dU/dx at x1 < 0, F becomes +ve.

(Remember F is negative of slope)

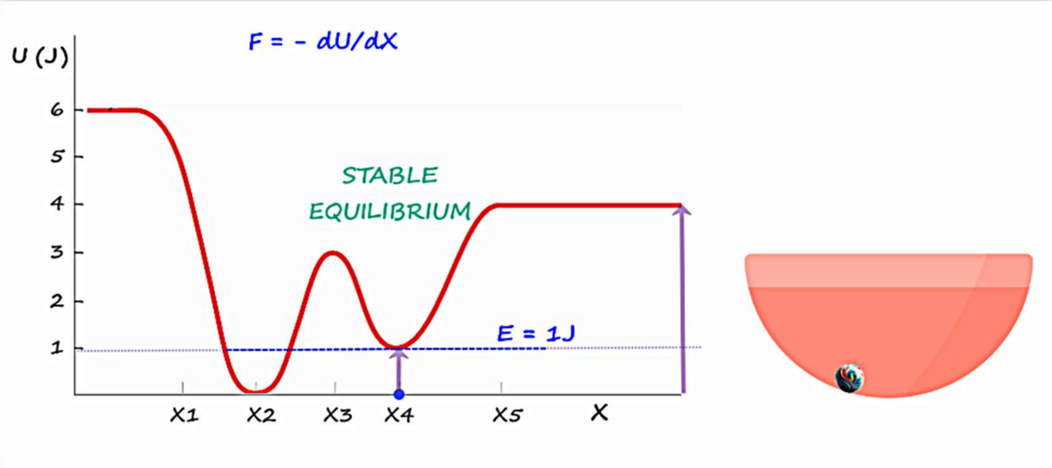
Equilibrium Points

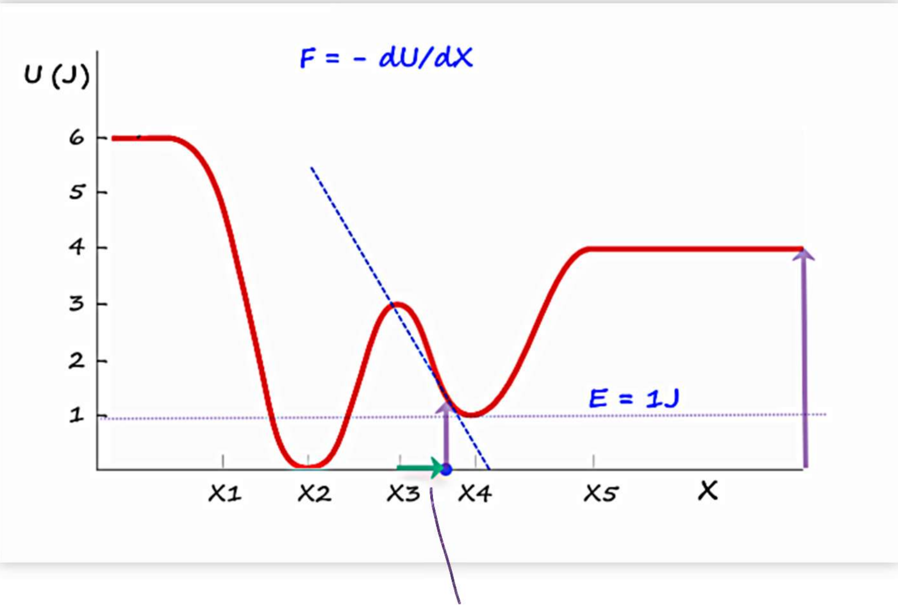
There are three types of equilibrium points

1. Neutral Equilibrium
    - Slope at this point is zero
    - Therefore, force on the particle is zero
    - Therefore KE = 0 and velocity = 0
    - Particle will remain stationary
    - If you push it left or right, it will assume the new position. Notice along the flat line in the diagram below, even if you change position x, the slope remains zero that is force is also zero
    - Example – marble put on a flat table



2. Stable Equilibrium (SE):
   - Slope at the point of SE is zero (x4 below)
   - Therefore, force on the particle is zero
   - KE = 0 and therefore velocity = 0
   - A slight displacement of the particle results in a force that pushes the particle back towards the equilibrium point.
   - Example: A marble at the bottom of a bowl.

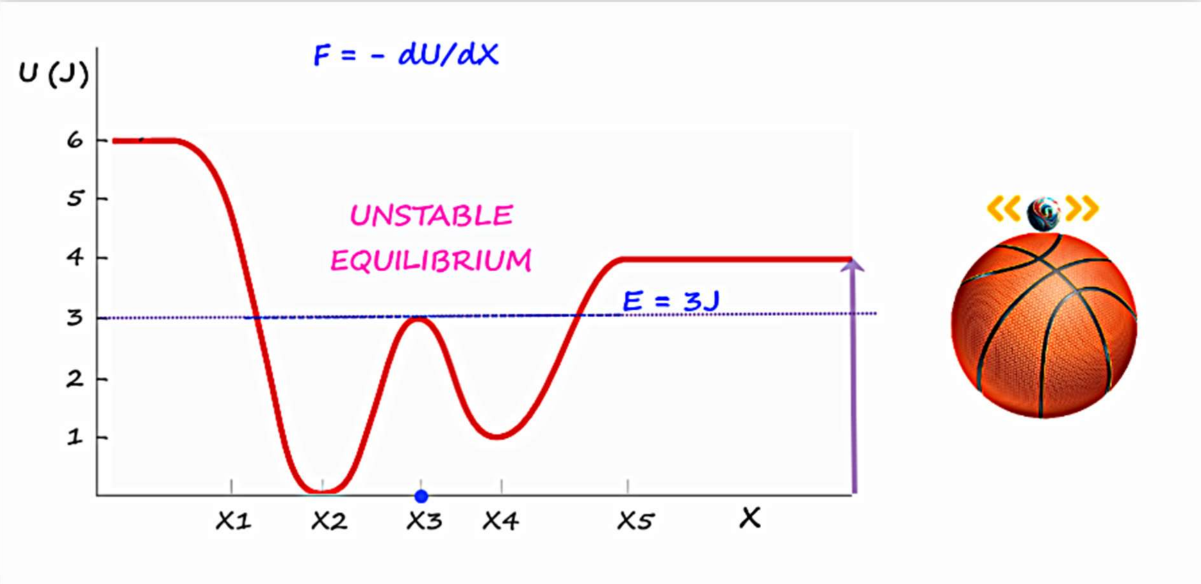


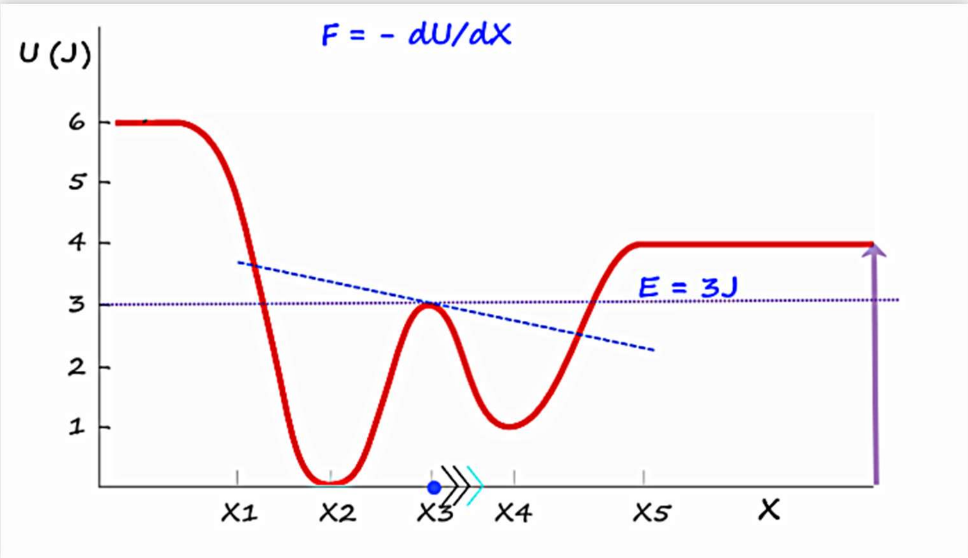


If particle is moved to the left of X4, the slope changes and therefore a force is introduced in the direction of X4. This force restores the position back to x4

3. Unstable Equilibrium (UE)

- Slope at the point of UE is zero (like x3)
- Therefore, force on the particle is zero
- KE = 0 and therefore velocity = 0
- A slight displacement of the particle results in a force that pushes the particle further down in that direction
- Example: A marble at the top of a ball





If particle is pushed a little to the right from X3, the slope changes which means the force has been introduced and it will get pushed further in that direction