

Variables-in-the-Choices Problems Answers

1. **(B)**. Since Josephine reads b books per week and each book has an average of 100,000 words, she reads $100,000b$ words per week. However, the question asks for words per *day*, so divide this quantity by 7.

Alternatively, you could try picking numbers. If $b = 2$, for instance, then Josephine would read 2 books per week and thus 200,000 words per week. Divide by 7 to get 28,571.42... words per day. Plug 2 into each answer choice in place of b , and pick the answer that gives you 28,571.42... Only **(B)** works.

2. **(D)**. Since width is twice length, write $W = 2L$. However, you want your answer in terms of w , so solve for L :

$$L = 1/2 W \text{ or } \frac{w}{2}$$

Since area is $L \times W$ and $L = \frac{w}{2}$:

$$A = \frac{w}{2} \times W$$

$$A = \frac{w^2}{2}, \text{ or choice (D).}$$

Alternatively, pick values. If width were 4, length would be 2. The area would therefore be $4 \times 2 = 8$. Plug in 4 for w to see which answer choice yields 8. Only **(D)** works.

3. **(E)**. This problem requires you to know that profit equals revenue minus cost. You could memorize the formula Profit = Revenue - Cost (or Profit = Revenue - Expenses), or you could just think about it logically — of course a business has to pay its expenses out of the money it makes: the rest is profit.

The revenue for all 100 shirts was \$50, and the cost to purchase all 100 shirts was \$ x . Therefore:

$$\text{Total profit} = 50 - x$$

The question does not ask for the total profit, but for the profit per shirt. The store sold 100 shirts, so divide the total profit by 100 to get the profit per shirt:

$$\text{Profit per shirt} = \frac{50 - x}{100}$$

None of the answer choices match this number, so you need to simplify the fraction. Split the numerator into two separate fractions:

$$\frac{50-x}{100} = \frac{50}{100} - \frac{x}{100} = 0.5 - \frac{x}{100}$$

4. **(A)**. Let s = the height of the shorter tree. Let t = the height of the taller tree.

If the combined height of the trees is 60 feet, then:

$$s + t = 60$$

You also know that the height of the taller tree is x times the height of the shorter tree:

$$t = xs$$

You need to solve for the height of the shorter tree, so substitute (xs) for t in the first equation:

$$s + (xs) = 60$$

You need to isolate s , so factor s out of the left side of the equation:

$$s(1 + x) = 60$$

$$s = \frac{60}{1 + x}$$

5. **(B)**. First, you need to express all three women's ages in terms of L . If Louise is three times as old as Mary, then Mary's age is $L/3$.

You also know that Mary is twice as old as Natalie. If Mary's age is $L/3$, then Natalie's age is $1/2$ of that, or $L/6$.

Now you can plug those values into the average formula. The average of the three ages is:

$$\text{average} = \frac{L + \frac{L}{3} + \frac{L}{6}}{3}$$

To get rid of the fractions in the numerator, multiply the entire fractions by $6/6$:

$$\frac{6}{6} \times \left(\frac{L + \frac{L}{3} + \frac{L}{6}}{3} \right) = \frac{6L + 2L + L}{18} = \frac{9L}{18} = \frac{L}{2}$$

6. **(A)**. Let T = Toshi's age $(T + x)$ = Toshi's age in x years

Let K = Kosuke's age $(K + x)$ = Kosuke's age in x years

If you know that Toshi is four times as old as Kosuke, then you know that:

$$T = 4K$$

To translate the second sentence correctly, remember that you need to use $(T + x)$ and $(K + x)$ to represent their ages:

$$(T + x) = 3(K + x)$$

You need to solve for Kosuke's age in terms of x , so replace T with $(4K)$ in the second equation:

$$(4K) + x = 3K + 3x$$

$$K + x = 3x$$

$$K = 2x$$

7. **(E)**. If the cost of the shirt is increased 30%, then the new price of the shirt is 130% of the original price. If the original price was k , then the new price is $1.3k$.

Remember that it is this new price that is increased by 50%. You need to multiply $1.3k$ by 1.5 (150%) to get the final price of the shirt:

$$1.3k \times 1.5 = 1.95k$$

8. **(D)**. Carlos's race times can be expressed as x , $x + 5$, and $x - 2$. (Remember, SLOWER race times are LARGER numbers, so "five seconds slower" means *plus* 5, not *minus* 5!) Average the race times:

$$\frac{x + (x + 5) + (x - 2)}{3} = \frac{3x + 3}{3} = x + 1$$

His average time is $x + 1$ *seconds*. But you need *minutes*. Since there are 60 seconds in a minute, divide by 60 to get

$\frac{x + 1}{60}$, or choice (D).

Alternatively, pick values. If x were 60 seconds, for example, Carlos's race times would be 60, 65, and 58. His average

time would be 61 seconds, or 1 minute and 1 second, or $1\frac{1}{60}$ minutes, or $\frac{61}{60}$ minutes. Plug in $x = 60$ to see which

value yields $\frac{61}{60}$. Only (D) works.

9. **(C)**. For every hour Andrew's business is open, he sells \$78 worth of clothes but pays \$10 to his assistant. Thus, he is making \$68 an hour after paying the assistant. He also must pay \$150 for the whole day.

So, the formula for his daily profit, using Revenue - Expenses = Profit and h for hours he is open:

$$68h - 150$$

Written as a function of profit in terms of hours, this is $P(h) = 68h - 150$, or choice (C).

Be careful that you are reading the answer choices as *functions*. P is not a variable that is being multiplied by h ! P is the *name* of the function, and h is the variable on which the output of the function depends.

Note that (D) is a very good trap — this formula represents what the profit would be if Andrew only had to pay the assistant \$10 *total*. However, he pays the assistant \$10 *per hour*.

Alternatively, you could pick numbers. If Andrew were open for an 8-hour day (here, you are testing out $h = 8$), he would make \$68 an hour (\$78 of sales minus \$10 to the assistant), or \$544 total. Subtract the \$150 rental fee to get \$394.

Then, plug 8 into the answer choices in place of h to see which answer yields 394. Only (C) works.

10. (C). Since a , b , c , and d are consecutive and d is largest, you can express c as $d - 1$, b as $d - 2$, and a as $d - 3$. Therefore, the average is:

$$\frac{(d-3) + (d-2) + (d-1) + d}{4} = \frac{4d-6}{4} = d - \frac{6}{4} \text{ or } d - \frac{3}{2}, \text{ which matches choice (C).}$$

Alternatively, plug in numbers. Say a , b , c , and d are simply 1, 2, 3, and 4 (generally, you want to avoid picking the numbers 0 and 1, lest *several* of the choices appear to be correct and you have to start over, but since only d appears in the choices, it's no problem that a is 1 in this example).

Thus, the average would be 2.5. Plug in 4 for d to see which choice yields an answer of 2.5. Only (C) works.

11. (A). If a cheese costs c cents per ounce, it costs $16c$ cents per pound. To convert from cents to dollars, divide by 100:

$$\frac{16c}{100} = \frac{4c}{25}, \text{ or choice (A).}$$

Alternatively, pick numbers. If $c = 50$, a cheese that costs 50 cents per ounce would cost 800 cents, or \$8, per pound. Plug in $c = 50$ and select the answer that gives the answer 8. Only (A) works.

12. (D). To figure out what *fraction* of the mix is fruit, put the amount of fruit over the total amount of the mix:

$$\frac{x}{6+x} \text{ To convert a fraction to a percent, simply multiply by 100: } \frac{x}{6+x}(100) = \frac{100x}{6+x}, \text{ or answer choice (D).}$$

Alternatively, pick smart numbers. For instance, say $x = 4$. In that case, the total amount of mix would be 10 ounces, 4

of which would be dried fruit. Since $4/10 = 40\%$, the answer to the question for your example would be 40%. Now, plug $x = 4$ into each answer choice to see which yields 40%. Only choice (D) works: $\frac{100(4)}{6 + (4)} = \frac{400}{10} = 40$. This will work for any number you choose for x , provided that you correctly calculate what percent of the mix would be dried fruit in your particular example.

13. (C). To increase a number by 1.5%, first convert 1.5% to a decimal by dividing by 100 to get 0.015.

Do NOT multiply the original number by 0.015 — this approach would be very inefficient, because multiplying by 0.015 would give you only the increase, not the new amount (you would then have to add the increase back to the original amount, a process so time-wasting and inefficient that it would not likely appear in a formula in the answer choices).

Instead, multiply by 1.015. Multiplying by 1 keeps the original number the same; multiplying by 1.015 gets you the original number plus 1.5% more.

Finally, if you want to multiply by 1.015 twice per year, you will need to do it $2y$ times. This $2y$ goes in the exponent spot, to give you $s(1.015)^{2y}$, or choice (C).

14. (C). A formula to find the cost of a call under Plan A, using x as the number of minutes:

$$\text{Cost} = 1.25 + 0.15(x - 1)$$

Note that you need to use $x - 1$ because the caller does *not* pay \$0.15 for every single minute — the first minute was already paid for by the \$1.25 charge.

A formula to find the cost of a call under Plan B, using x as the number of minutes:

$$\text{Cost} = 0.90 + 0.20x$$

Note that here you do *not* use $x - 1$ because the connection fee does not “buy” the first minute — you still have to pay \$0.20 for every minute.

To find the length of a call that would cost the same under either plan, set the two formulas equal to one another:

$$1.25 + 0.15(x - 1) = 0.90 + 0.20x$$

This is choice (C). Note that you are not required to solve this equation, but you might be required to solve a similar equation in a different problem on this topic:

$$1.25 + 0.15x - 0.15 = 0.90 + 0.20x$$

$$1.1 + 0.15x = 0.90 + 0.20x$$

$$0.20 = 0.05x$$

$$20 = 5x$$

$$4 = x$$

A 4-minute call would cost the same under either plan. To test this, calculate the cost of a 4-minute call under both plans: it's \$1.70 either way.

15. (D). The mix costs c cents per ounce. Since you want your final answer in dollars, convert right now:

$$c \text{ cents per ounce} = \frac{c}{100} \text{ dollars per ounce}$$

The supplier then purchases p pounds of mix. You cannot simply multiply p by $\frac{c}{100}$, because p is in pounds and $\frac{c}{100}$ is in dollars per OUNCE. You must convert again. Since there are 16 ounces in a pound, it makes sense that a pound would cost 16 times more than an ounce:

$$\frac{c}{100} \text{ dollars per ounce} = \frac{16c}{100} \text{ dollars per pound}$$

Reduce to get $\frac{4c}{25}$ dollars per pound.

Multiply by p , the number of pounds, to get what the supplier paid: $\frac{4cp}{25}$ dollars.

Now, the supplier is going to sell the mix for three times what he or she paid. (Don't worry that the problem says three times the "price per ounce" — whether you measure in ounces or pounds, this stuff just got three times more expensive.)

$$\frac{4cp}{25} \times 3 = \frac{12cp}{25}, \text{ or answer choice (D).}$$

Note: Make sure you were calculating for revenue, not profit! You were not asked to subtract expenses (what the supplier paid) from the money he or she will be making from selling the mix.

An alternative solution is to plug in smart numbers. An easy number to pick when working with cents is 50 (or 25 — whatever is easy to think about and convert to dollars). Write a value on your paper along with what the value means in words:

$$c = 50 \quad \text{mix costs } 50\text{¢ per ounce}$$

Now, common sense (and the fact that 16 ounces = 1 pound) will easily allow you to convert:

$$50\text{¢ per ounce} = \$8.00 \text{ per pound}$$

The supplier bought p pounds. Pick any number you want. For example:

$p = 2$ bought 2 pounds, so spent \$16

Notice that no one asked you for this \$16 figure, but when calculating with smart numbers, it's best to write down obvious next steps in the reasoning process.

Finally, the supplier is going to sell the mix for three times what he or she paid, so the supplier will sell for \$48.

Plug in $c = 50$ and $p = 2$ to see which answer choice generates 48. Only (D) works.

16. (A). Since the figure has six sides, use the formula $(n - 2)(180)$, where n is the number of sides, to figure out that the sum of the angles inside the figure = $(6 - 2)(180) = 720$.

The angle supplementary to x can be labeled on your paper as $180 - x$ (since two angles that make up a straight line must sum to 180). Thus:

$$\begin{aligned} a + b + c + d + e + 180 - x &= 720 \\ a + b + c + d + e - x &= 540 \end{aligned}$$

You are asked to solve for x . Since x is being subtracted from the left side, it would be easiest to add x to both sides, and get everything else on the opposite side.

$$\begin{aligned} a + b + c + d + e - x &= 540 \\ a + b + c + d + e &= 540 + x \\ a + b + c + d + e - 540 &= x \end{aligned}$$

$$e = \frac{1}{2}a$$

Since $d = 2c$ and $e = \frac{1}{2}a$ and the answers are in terms of a , b , and c , you need to make the d and e drop out of $a + b + c + d + e - 540 = x$.

$$e = \frac{1}{2}a$$

Fortunately $d = 2c$ and $e = \frac{1}{2}a$ are already solved for d and e , the variables you want to drop out. Substitute:

$$\begin{aligned} a + b + c + 2c + \frac{1}{2}a - 540 &= x \\ \frac{3}{2}a + b + 3c - 540 &= x \end{aligned}$$

This is a match with answer choice (A).

Alternatively, pick numbers. To do this, use the formula $(n - 2)(180)$, where n is the number of sides, to figure out that the sum of the angles inside the figure = $(6 - 2)(180) = 720$. Then, pick values for a , b , c , d , and e , so that $d = 2c$ and

$$e = \frac{1}{2}a$$

$$a = 100$$

$$b = 110$$

$$c = 120$$

$$d = 240 \text{ (this is twice the value picked for } c)$$

$$e = 50 \text{ (this is } 1/2 \text{ the value picked for } a)$$

Subtract all of these values from 720 to get that the unlabeled angle, for this example, is equal to 100. This makes x equal to $180 - 100 = 80$.

Now plug $a = 100$, $b = 110$, and $c = 120$ into the answers to see which formula yields a value of 80. (A) is the correct answer.

17. (A). One algebraic solution involves defining all three terms in terms of a . Since the terms are consecutive odd integers, they are 2 apart from each other, as such:

$$a$$

$$b = a + 2$$

$$c = a + 4$$

Then, a is halved to become m , b is doubled to become n , and c is tripled to become p , so:

$$\frac{1}{2}a = m$$

$$2b = n$$

$$2(a + 2) = n$$

$$2a + 4 = n$$

$$3c = p$$

$$3(a + 4) = p$$

$$3a + 12 = p$$

Since $k = mnp$, multiply the values for m , n , and p :

$$k = \left(\frac{1}{2}a\right)(2a+4)(3a+12)$$

$$k = \left(\frac{1}{2}a\right)(6a^2 + 24a + 12a + 48)$$

$$k = \left(\frac{1}{2}a\right)(6a^2 + 36a + 48)$$

$$k = 3a^3 + 18a^2 + 24a$$

This is a match with answer choice (A).

A “smart numbers” solution would be to pick three consecutive odd integers for a , b , and c . When picking numbers for a Variables in the Choices problem, avoid picking 0, 1, or any of the numbers in the problem (this can sometimes cause more than one answer to appear to be correct, thus necessitating starting over with another set of numbers). So:

$$\begin{aligned}a &= 3 \\b &= 5 \\c &= 71\end{aligned}$$

Then, a is halved to become m , b is doubled to become n , and c is tripled to become p , so:

$$\begin{aligned}1.5 &= m \\10 &= n \\21 &= p\end{aligned}$$

Since $k = mnp$, multiply the values for m , n , and p :

$$\begin{aligned}k &= (1.5)(10)(21) \\k &= 315\end{aligned}$$

Now, plug $a = 3$ (the value originally selected) into the answer choices to see which choice equals 315. Only (A) works.

Because the correct answer is simply a mathematical way of writing the situation described in the problem, this will work for any value you pick for a , provided that a , b , and c are consecutive odd integers and you calculate k correctly.

18. (C). The phrase “ m pencils cost the same as n pens” can be written as an equation, using x for the cost per pencil and y for the cost per pen:

$$mx = ny$$

Keep in mind here that m stands for the NUMBER of pencils and n for the NUMBER of pens (not the cost). Now, since pencils cost 20 cents or \$0.2 (the answer needs to be in dollars, so convert to dollars now), substitute in for x :

$$0.2m = ny$$

Solve for y to get the cost of 1 pen:

$$y = \frac{0.2m}{n}$$

Since y is the cost of 1 pen and $y = \frac{0.2m}{n}$, multiply by 10 to get the cost of 10 pens:

$$10y = 10\left(\frac{0.2m}{n}\right)$$

$$10y = \frac{2m}{n}$$

$$\frac{2m}{n}$$

Thus, the answer is $\frac{2m}{n}$, or (C).

Alternatively, plug in smart numbers. Since pencils cost 20 cents, maybe pens cost 40 cents (you can arbitrarily pick this number). You are told “ m pencils cost the same as n pens”—pick a number for one of these variables, and then determine what the other variable would be for the example you’ve chosen. For instance, if $m = 10$, then 10 pencils would cost \$2.00. Since 5 pens can be bought for \$2.00, n would be 5. Now, answer the final question as a number: the cost of 10 pens in this example is \$4.00, so the final answer is 4. Plug in $m = 10$ and $n = 5$ to all of the answer choices to see which yields an answer of 4. Only (C) works. For any working system you choose in which “ m pencils cost the same as n pens,” choice (C) will work.

19. (E). One way to do this problem is simply to construct a formula. Randi’s pay is equal to \$2,000 plus commission:

$$P = 2000 + \dots$$

You are only asked to construct a formula for months during which she sells more than 10 forklifts, so you know that she will definitely be receiving 5% commission on 10 forklifts that each cost s . Since the revenue from the forklifts would then be $10s$, Randi’s commission would be $0.05(10s)$, or $0.5s$.

$$P = 2,000 + 0.5s + \dots$$

Now, you must add the commission for the forklifts she sells above the first 10. Since these first 10 forklifts are already accounted for, you can denote the forklifts at this commission level by writing $f - 10$. Since each forklift still costs s , the revenue from these forklifts would be $s(f - 10)$. Since Randi receives 10% of this as commission, the amount she receives would be $0.10s(f - 10)$.

$$P = 2,000 + 0.5s + 0.10s(f - 10)$$

It is possible to simplify further by distributing $0.10s(f - 10)$, but before doing more work, check the answers — you have an exact match already, answer choice (E).

Alternatively, plug in numbers. Say forklifts cost \$100 (so, $s = 100$). Randi makes \$5 each for the first ten she sells, so \$50 total. Then she makes \$10 each for any additional forklifts. Pick a value for f (make sure the value is more than 10, since the question asks for a formula for months in which Randi sells more than 10 forklifts). So, in a month in which she sells, for example, 13 forklifts (so, $f = 13$), she would make $\$2,000 + \$50 + 3(\$10) = \$2,080$.

In this example:

$$s = 100$$

$$f = 13$$

Plug in these values for s and f to see which choice yields \$2,080. Only choice (E) works:

$$P = 2,000 + 0.5(100) + 0.10(100)(13 - 10)$$

$$P = 2,000 + 50 + 10(3)$$

$$P = 2,080$$

20. (A). This question can be solved either with smart numbers or algebra. First, consider plugging in smart numbers.

Set $l = 2$, so $w = 4$. The perimeter will be $2l + 2w = 2(2) + 2(4) = 12$. The answer is the area, which is $wl = (2)(4) = 8$ based on these numbers. Now plug $p = 12$ into the choices to see which choice equals 8:

(A) $144/18 = 8$

(B) $144/36 = 4$

(C) $12/9 = 4/3$

(D) $144/9 = 16$

(E) $12/6 = 2$

The correct answer is (A).

Though smart numbers are easier and faster here, you could also solve with algebra. If $w = 2l$:

$$a = l \times w = l \times 2l = 2l^2$$

$$p = 2l + 2w = 2l + 4l = 6l$$

Solve the second equation for l :

$$l = p/6$$

And plug back into the first equation:

$$2\left(\frac{p}{6}\right)^2 = 2\left(\frac{p^2}{36}\right) = \frac{2p^2}{36} = \frac{p^2}{18}$$

This method is much more difficult than plugging in numbers, but you can still get to the correct answer, (A).