H

National
Qualifications
2015

2015 Mathematics

New Higher Paper 2

Finalised Marking Instructions

General Comments

These marking instructions are for use with the 2015 Higher Mathematics Examination.

For each question the marking instructions are in two sections, namely **Illustrative Scheme** and **Generic Scheme**. The **Illustrative Scheme** covers methods which are commonly seen throughout the marking. The **Generic Scheme** indicates the rationale for which each mark is awarded. In general, markers should use the **Illustrative Scheme** and only use the **Generic Scheme** where a candidate has used a method not covered in the **Illustrative Scheme**.

All markers should apply the following general marking principles throughout their marking:

1 Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than deducted for what is wrong.

2 One mark is available for each •. There are **no** half marks.

3 Working subsequent to an error **must be followed through**, with possible full marks for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.

4 As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Throughout this paper, unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.

5 In general, as a consequence of an error perceived to be trivial, casual or insignificant, e.g. $6\times 6=12$, candidates lose the opportunity of gaining a mark. But note the second example in comment 7.

6 Where a transcription error (paper to script or within script) occurs, the candidate should be penalised, e.g.

This is a transcription error and so the mark is not awarded.

Eased as no longer a solution of a quadratic equation.

$x^2+5x+7=9x+4$ ✓

$x-4x+3=0$ ✗

$x=1$ ✓2

Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt.

$x^2+5x+7=9x+4$ ✓

$x-4x+3=0$ ✓

$(x-3)(x-1)=0$

$x=1$ or 3 ✓

7 Vertical/horizontal marking

Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example: Point of intersection of line with curve

Illustrative Scheme: $\bullet^5$ $x = 2, x = -4$

$\bullet^6$ $y = 5, y = -7$

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$y = 5$
$\bullet^6$	$x = -4$	$y = -7$

Markers should choose whichever method benefits the candidate, but **not** a combination of both.

8 In final answers, numerical values should be simplified as far as possible, unless specifically mentioned in the detailed marking instructions.

Examples:

$\frac{15}{12}$ should be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$

$\frac{43}{1}$ should be simplified to 43

$\frac{15}{0\cdot3}$ should be simplified to 50

$\frac{4/5}{3}$ should be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8

The square root of perfect squares up to and including 100 must be known.

9 Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

10 Unless specifically mentioned in the marking instructions, the following should not be penalised:

- Working subsequent to a **correct** answer;
- Correct working in the wrong part of a question;
- Legitimate variations in numerical answers, eg angles in degrees rounded to nearest degree;
- Omission of units;
- Bad form (bad form only becomes bad form if subsequent working is correct), e.g.

  $(x^3 + 2x^2 + 3x + 2)(2x + 1)$

  written as

  $(x^3 + 2x^2 + 3x + 2) \times 2x + 1$

  $2x^4 + 4x^3 + 6x^2 + 4x + x^3 + 2x^2 + 3x + 2$

  $2x^4 + 5x^3 + 8x^2 + 7x + 2$ gains full credit;
- Repeated error within a question, but not between questions.

11 In any 'Show that . . .' question, where the candidate has to arrive at a required result, the last mark of that part is not available as a follow through from a previous error unless specifically stated otherwise in the detailed marking instructions.

12 All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate's response. Marks may still be available later in the question so reference must be made continually to the marking instructions.
All working must be checked: the appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.

13 If you are in serious doubt whether a mark should or should not be awarded, consult your Team Leader (TL).

14 Scored out working which **has not been replaced** should be marked where still legible. However, if the scored out working **has been replaced**, only the work which has not been scored out should be marked.

15 Where a candidate has made multiple attempts using the same strategy, mark all attempts and award the lowest mark.
Where a candidate has tried different strategies, apply the above ruling to attempts within each strategy and then award the highest resultant mark. For example:

Strategy 1 attempt 1 is worth 3 marks	Strategy 2 attempt 1 is worth 1 mark
Strategy 1 attempt 2 is worth 4 marks	Strategy 2 attempt 2 is worth 5 marks
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

16 In cases of difficulty, covered neither in detail nor in principle in these instructions, markers should contact their TL in the first instance.

Paper 2

Question	Generic Scheme	Illustrative Scheme	Max Mark
1(a)			
	•[1] calculate gradient of AB	•[1] $m_{AB} = -3$	
	•[2] use property of perpendicular lines	•[2] $m_{alt} = \frac{1}{3}$	
	•[3] substitute into general equation of a line	•[3] $y - 3 = \frac{1}{3}(x - 13)$	
	•[4] demonstrate result	•[4] $\ldots \Rightarrow x - 3y = 4$	**4**

Notes:

1. •[3] is only available as a consequence of trying to find and use a perpendicular gradient.
2. •[4] is only available if there is/are appropriate intermediate lines of working between •[3] and •[4].
3. The ONLY acceptable variations for the final equation for the line in •[4] are $4 = x - 3y,\ -3y + x = 4,\ 4 = -3y + x$.

Commonly Observed Responses:

Candidate A		**Candidate B**
$m_{AB} = \frac{-1-(-5)}{-5-7} = \frac{4}{-12} = -\frac{1}{3}$	•[1] ✗	For •[4]
$m_{alt} = 3$	•[2] $\boxed{✓1}$	$y - 3 = \frac{1}{3}x - \frac{13}{3}$
$y - 3 = 3(x - 13)$	•[3] $\boxed{✓1}$	$y = \frac{1}{3}x - \frac{4}{3}$
	•[4] ✗	$3y = x - 4$ - not acceptable
•[4] is not available		$3y - x = -4$ - not acceptable
		$x - 3y = 4$ ✓

Question	Generic Scheme	Illustrative Scheme	Max Mark
1(b)			
	•[5] calculate midpoint of AC •[6] calculate gradient of median •[7] determine equation of median	•[5] $M_{AC}=(4,5)$ •[6] $m_{BM}=2$ •[7] $y=2x-3$	**3**

Notes:

4. •[6] and •[7] are not available to candidates who do not use a midpoint.
5. •[7] is only available as a consequence of using a non-perpendicular gradient and a midpoint.
6. Candidates who find either the median through A or the median through C or a side of the triangle gain 1 mark out of 3.
7. At •[7] accept $y-(-5)=2(x-(-1))$, $y-5=2(x-4)$, $y-2x+3=0$ or any other rearrangement of the equation.

Commonly Observed Responses:

Median through A	**Median through C**
$M_{BC}=(6,-1)$ $m_{AM}=\frac{-8}{11}$ $y+1=\frac{-8}{11}(x-6)$ or $y-7=\frac{-8}{11}(x+5)$ **Award 1/3**	$M_{AB}=(-3,1)$ $m_{CM}=\frac{1}{8}$ $y-3=\frac{1}{8}(x-13)$ or $y-1=\frac{1}{8}(x+3)$ **Award 1/3**

Question	Generic Scheme	Illustrative Scheme	Max Mark
1(c)			
	•[8] calculate x or y coordinate •[9] calculate remaining coordinate of the point of intersection	•[8] $x=1$ or $y=-1$ •[9] $y=-1$ or $x=1$	**2**

Notes:

8. If the candidate's 'median' is either a vertical or horizontal line then award 1 out of 2 if both coordinates are correct, otherwise award 0.

Commonly Observed Responses:

For candidates who find the altitude through B in part (b)	**Candidate A**
$x=-\frac{1}{5}$ •[8] $\boxed{\checkmark 1}$ $y=-\frac{7}{5}$ •[9] $\boxed{\checkmark 1}$	(b) $y-5=2(x-4)$ •[7] ✓ $y=2x-13$ - error (c) $x-3y=4$ $y=2x-13$ •[8] × Leading to $x=7$ and $y=1$ •[9] $\boxed{\checkmark 1}$

Question	Generic Scheme	Illustrative Scheme	Max Mark
2 (a)			
	•[1] interpret notation	•[1] $f((1+x)(3-x)+2)$ stated or implied by •[2]	
	•[2] state a correct expression	•[2] $10+(1+x)(3-x)+2$ stated or implied by •[3]	**2**

Notes:

1. •[1] is not available for $g\left(f\left(x\right)\right)=g(10+x)$ but •[2] may be awarded for $(1+10+x)(3-(10+x))+2$.

Commonly Observed Responses:

Candidate A

(a) $f(g(x))='g(f(x))'$

$=(1+10+x)(3-(10+x))+2$ •[1] × •[2] ✓1

(b) $=-75-18x-x^2$ or $-x^2-18x-75$ •[3] ✓1

$=-(x^2+18x$ •[4] ✓1

$=-(x+9)^2$ •[5] ✓1

$=-(x+9)^2+6$

(c) $-(x+9)^2+6=0$ •[6] ✓1

$x=-9+\sqrt{6}$ or $-9-\sqrt{6}$ •[7] ✓1

Candidate B

$f(g(x))$ •[1] ^

$=10((1+x)-(3-x))+2$ •[2] ×

Candidate C

$f(g(x))$ •[1] ^

$=10((1+x)(3-x)+2)$ •[2] ×

Question	Generic Scheme	Illustrative Scheme	Max Mark
2 (b)			
	•[3] write $f(g(x))$ in quadratic form	•[3] $15+2x-x^2$ or $-x^2+2x+15$	
	Method 1	**Method 1**	
	•[4] identify common factor	•[4] $-1(x^2-2x$ stated or implied by •[5]	
	•[5] complete the square	•[5] $-1(x-1)^2+16$	
	Method 2	**Method 2**	
	•[4] expand completed square form and equate coefficients	•[4] $px^2+2pqx+pq^2+r$ and $p=-1$,	
	•[5] process for q and r and write in required form	•[5] $q=-1$ and $r=16$ Note if $p=1$ •[5] is not available	**3**

Notes:

2. Accept $16-(x-1)^2$ or $-\left[(x-1)^2-16\right]$ at $\bullet^5$.

Commonly Observed Responses:

Candidate A	**Candidate B**	**Candidate C**
$-(x^2-2x-15)$ $\bullet^4$ ✓ $-(x^2-2x+1-1-15)$ $-(x-1)^2-16$ $\bullet^5$ ✗	$15+2x-x^2$ $\bullet^3$ ✓ $x^2-2x-15$ $\bullet^4$ ✗ $px^2+2pqx+pq^2+r$ and $p=1$ $q=-1 \quad r=-16$ $\bullet^5$ $\boxed{✓ 2}$ eased	$-x^2+2x+15$ $\bullet^3$ ✓ $-(x+1)^2...$ $\bullet^4$ ✗ $-(x+1)^2+14$ $\bullet^5$ ✗
Candidate D	**Candidate E**	**Candidate F**
$15+2x-x^2$ $\bullet^3$ ✓ $x^2-2x-15$ $\bullet^4$ ✗ $(x-1)^2-16$ $\bullet^5$ $\boxed{✓ 2}$ eased Eased, unitary coefficient of x^2 (lower level skill)	$15+2x-x^2$ $\bullet^3$ ✓ $x^2-2x-15$ $\bullet^4$ ✓ $(x-1)^2-16$ so $15+2x-x^2=-(x-1)^2+16$ $\bullet^5$ ✓	$-x^2+2x+15$ $\bullet^3$ ✓ $-(x+1)^2...$ $\bullet^4$ ✗ $-(x+1)^2+16$ $\bullet^5$ $\boxed{✓ 1}$

2(c)

$\bullet^6$ identify critical condition $\bullet^7$ identify critical values	$\bullet^6$ $-1(x-1)^2+16=0$ or $f((g(x))=0$ $\bullet^7$ 5 and -3	**2**

Notes:

3. Any communication indicating that the denominator cannot be zero gains $\bullet^6$.

4. Accept $x=5$ and $x=-3$ or $x\neq 5$ and $x\neq -3$ at $\bullet^7$.

5. If $x=5$ and $x=-3$ appear without working award 1/2.

Commonly Observed Responses:

Candidate A	**Candidate B**
$\dfrac{1}{-(x-1)^2+16}$ $\bullet^6$ ✓ $x\neq 5$ $\bullet^7$ ∧	$\dfrac{1}{f(g(x))}$ $f(g(x))>0$ $\bullet^6$ ✗ $x=-3,\ x=5$ $\bullet^7$ ✓ $-3<x \quad x<5$

3(a)

$\bullet^1$ determine the value of the required term	$\bullet^1$ $22\frac{3}{4}$ or $\frac{91}{4}$ or $22\cdot75$	**1**

Notes:

1. Do not penalise the inclusion of incorrect units.
2. Accept rounded and unsimplified answers following evidence of correct substitution.

Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
3 (b)			
	Method 1 **(Considering both limits)**	**Method 1**	
	$\bullet^2$ know how to calculate limit	$\bullet^2$ $\frac{32}{1-\frac{1}{3}}$ or $L=\frac{1}{3}L+32$	
	$\bullet^3$ know how to calculate limit	$\bullet^3$ $\frac{13}{1-\frac{3}{4}}$ or $L=\frac{3}{4}L+13$	
	$\bullet^4$ calculate limit	$\bullet^4$ 48	
	$\bullet^5$ calculate limit	$\bullet^5$ 52	
	$\bullet^6$ interpret limits and state conclusion	$\bullet^6$ $52>50$ $\therefore$ toad will escape	
	Method 2 **(Frog first then numerical for toad)**	**Method 2**	
	$\bullet^2$ know how to calculate limit	$\bullet^2$ $\frac{32}{1-\frac{1}{3}}$ or $L=\frac{1}{3}L+32$	
	$\bullet^3$ calculate limit	$\bullet^3$ 48	
	$\bullet^4$ determine the value of the highest term less than 50	$\bullet^4$ $49\cdot803...$	
	$\bullet^5$ determine the value of the lowest term greater than 50	$\bullet^5$ $50\cdot352...$	
	$\bullet^6$ interpret information and state conclusion	$\bullet^6$ $50\cdot352>50$ $\therefore$ toad will escape	
	Method 3 **(Numerical method for toad only)**	**Method 3**	
	$\bullet^2$ continues numerical strategy	$\bullet^2$ numerical strategy	
	$\bullet^3$ exact value	$\bullet^3$ $30\cdot0625$	
	$\bullet^4$ determine the value of the highest term less than 50	$\bullet^4$ $49\cdot803...$	
	$\bullet^5$ determine the value of the lowest term greater than 50	$\bullet^5$ $50\cdot352...$	
	$\bullet^6$ interpret information and state conclusion	$\bullet^6$ $50\cdot352>50$ $\therefore$ toad will escape	
	Method 4 **(Limit method for toad only)**	**Method 4**	
	$\bullet^2$ & $\bullet^3$ know how to calculate limit	$\bullet^2$ & $\bullet^3$ $\frac{13}{1-\frac{3}{4}}$ or $L=\frac{3}{4}L+13$	
	$\bullet^4$ & $\bullet^5$ calculate limit $\bullet$	$\bullet^4$ & $\bullet^5$ 52	
	$\bullet^6$ interpret limit and state conclusion	$\bullet^6$ $52>50$ $\therefore$ toad will escape	**5**

Notes:
3. •[6] is unavailable for candidates who do not consider the toad in their conclusion.
4. For candidates who only consider the frog numerically award 1/5 for the strategy.

Commonly Observed Responses:

Error with frogs limit - Frog Only	**Using Method 3 - Toad Only**	**Using Method 3- Toad Only**	**Using Method 3 - Toad Only**
$L_F = \dfrac{34}{1-\frac{1}{3}}$ •[2] × •[3] × $L_F = 51$ •[4] ✓1 $51 > 50$ •[5] ✓1 $\therefore$ frog will escape. •[6] ✓1	•[2] ✓ •[3] ✓ •[4] missing ^ •[5] $50 \cdot 352...$ ✓ •[6] $50.352 > 50$ so the toad escapes. ✓	•[2] ✓ •[3] ✓ •[4] missing ^ •[5] $50 \cdot 1..$rounding error × •[6] $50.1 > 50$ ✓1 so the toad escapes.	•[2] ✓ •[3] ✓ •[4] $49 \cdot 7..$ rounding error × •[5] $50 \cdot 1...$ ✓1 •[6] $50.1 > 50$ ✓1 so the toad escapes.

Toad Conclusions

Limit = 52

This is greater than the height of the well and so the toad will escape – award •[6].

However

Limit =52 and so the toad escapes - •[6] ^.

Iterations		
$f_1 = 32$	$t_1 = 13$	
$f_2 = 42 \cdot 667$	$t_2 = 22 \cdot 75$	
$f_3 = 46 \cdot 222$	$t_3 = 30 \cdot 0625$	
$f_4 = 47 \cdot 407$	$t_4 = 35 \cdot 547$	
$f_5 = 47 \cdot 802$	$t_5 = 39 \cdot 660$	
$f_6 = 47 \cdot 934$	$t_6 = 42 \cdot 745$	
$f_7 = 47 \cdot 978$	$t_7 = 45 \cdot 059$	
$f_8 = 47 \cdot 993$	$t_8 = 46 \cdot 794$	
$f_9 = 47 \cdot 998$	$t_9 = 48 \cdot 096$	
	$t_{10} = 49 \cdot 072$	
	$t_{11} = 49 \cdot 804$	
	$t_{12} = 50 \cdot 353$	

Question	Generic Scheme	Illustrative Scheme	Max Mark
4 (a)			
	•[1] know to equate $f(x)$ and $g(x)$ •[2] solve for x	•[1] $\frac{1}{4}x^2-\frac{1}{2}x+3=\frac{1}{4}x^2-\frac{3}{2}x+5$ •[2] $x=2$	**2**

Notes:

1. •[1] and •[2] are not available to candidates who: (i) equate zeros, (ii) give answer only without working, (iii) arrive at $x=2$ with erroneous working.

Commonly Observed Responses:

<table>
<tr>
<td>

Candidate A

$y=\frac{1}{4}x^2-\frac{1}{2}x+3$

$y=\frac{1}{4}x^2-\frac{3}{2}x+5$ •[1] ✓

subtract to get

$0=x-2$ •[2] ✓

$x=2$

</td>
<td>

Candidate B

$\frac{1}{4}x^2-\frac{1}{2}x=-3$

$\frac{1}{4}x^2-\frac{3}{2}x=-5$ •[1] ✗

$x=2$ •[2] ✗

In this case the candidate has equated zeros

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Candidate C

$f(x)=\frac{1}{4}x^2-\frac{1}{2}x+3$ $g(x)=\frac{1}{4}x^2-\frac{3}{2}x+5$

$f'(x)=\frac{1}{2}x-\frac{1}{2}$ $g'(x)=\frac{1}{2}x-\frac{3}{2}$

$x=1$ $x=3$ •[1] ✓ •[2] ✓

$\therefore x=2$

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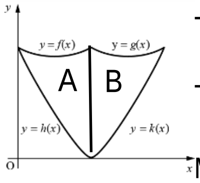
Question	Generic Scheme	Illustrative Scheme	Max Mark
4 (b)			
	●[3] know to integrate	●[3] $\int$	
	●[4] interpret limits	●[4] $\int_0^2$	
	●[5] use ‘upper – lower’	●[5] $\int_0^2 \left(\frac{1}{4}x^2 - \frac{1}{2}x + 3\right) - \left(\frac{3}{8}x^2 - \frac{9}{4}x + 3\right)\,dx$	
	●[6] integrate	●[6] $-\frac{1}{24}x^3 + \frac{7}{8}x^2$ accept unsimplified integral	
	●[7] substitute limits	●[7] $\left(-\frac{1}{24}\times 2^3 + \frac{7}{8}\times 2^2\right) - 0$	
	●[8] evaluate area between $f(x)$ and $h(x)$	●[8] $\frac{19}{6}$	
	●[9] state total area	●[9] $\frac{19}{3}$	**7**

Notes:

2. If limits $x = 0$ and $x = 2$ appear ex nihilo award ●[4].
4. If a candidate differentiates at ●[6] then ●[6], ●[7] and ●[8] are not available. However, ●[9] is still available.
5. Candidates who substitute at ●[7], without attempting to integrate at ●[6], cannot gain ●[6], ●[7] or ●[8]. However, ●[9] is still available.
6. Evidence for ●[8] may be implied by ●[9].
7. ●[9] is a strategy mark and should be awarded for correctly multiplying their solution at ●[8], or for any other valid strategy applied to previous working.
8. For ●[5] both a term containing a variable and the constant term must be dealt with correctly.
9. In cases where ●[5] is not awarded, ●[6] may be gained for integrating an expression of equivalent difficulty ie a polynomial of at least degree two. ●[6] is unavailable for the integration of a linear expression.
10. ●[8] must be as a consequence of substituting into a term where the power of x is not equalto 1 or 0.

Commonly Observed Responses:	
Candidate A - Valid Strategy Candidates who use the strategy:  Total Area = Area A + Area B Then mark as follows: Mark Area A for $\bullet^3$ to $\bullet^8$ then mark Area B for $\bullet^3$ to $\bullet^8$ and award the higher of the two. $\bullet^9$ is available for correctly adding two equal areas.	**Candidate B - Invalid Strategy** For example, candidates who integrate each of the four functions separately within an invalid strategy $\bullet^3$ ✓ Gain $\bullet^4$ if limits correct on $\int f(x)$ and $\int h(x)$ or $\int g(x)$ and $\int k(x)$ $\bullet^5$ is unavailable Gain $\bullet^6$ for calculating either $\int f(x)$ or $\int g(x)$ and $\int h(x)$ or $\int k(x)$ Gain $\bullet^7$ for correctly substituting at least twice Gain $\bullet^8$ for evaluating at least two integrals correctly $\bullet^9$ is unavailable
Candidate C $\int_0^2 (\frac{1}{4}x^2 - \frac{1}{2}x + 3 - \frac{3}{8}x^2 - \frac{9}{4}x + 3)\ dx$ $\int_0^2 (-\frac{1}{8}x^2 - \frac{11}{4}x)\ dx$ $\bullet^5$ ✓ $\frac{-1}{24}x^3 - \frac{11}{8}x^2$ $\bullet^6$ ×	**Candidate D** $\int_0^2 (\frac{1}{4}x^2 - \frac{1}{2}x + 3 - \frac{3}{8}x^2 - \frac{9}{4}x + 3)\ dx$ $\int_0^2 (-\frac{1}{8}x^2 - \frac{11}{4}x + 6)\ dx$ $\bullet^5$ × $-\frac{1}{24}x^3 - \frac{11}{8}x^2 + 6x$ $\bullet^6$ $\boxed{✓1}$
Candidate E $\int \ldots = -\frac{1}{3}$ cannot be negative so $= \frac{1}{3}$ $\bullet^8$ × however, $= -\frac{1}{3}$ so Area $= \frac{1}{3}$ $\bullet^8$ ✓	**Candidate F** $\int_0^2 (\frac{1}{4}x^2 - \frac{1}{2}x + 3 - \frac{3}{8}x^2 - \frac{9}{4}x + 3)\ dx$ $\int_0^2 (-\frac{1}{8}x^2 + \frac{7}{4}x)\ dx$ $\bullet^5$ ✓ $-\frac{1}{24}x^3 + \frac{7}{8}x^2$ $\bullet^6$ ✓

Question	Generic Scheme	Illustrative Scheme	Max Mark
5(a)			
	$\bullet^1$ state centre of C_1 $\bullet^2$ state radius of C_1 $\bullet^3$ calculate distance between centres of C_1 and C_2 $\bullet^4$ calculate radius of C_2	$\bullet^1$ $(-3,-5)$ $\bullet^2$ 5 $\bullet^3$ 20 $\bullet^4$ 15	**4**

Notes:

1. For $\bullet^4$ to be awarded radius of C_2 must be greater than the radius of C_1.
2. Beware of candidates who arrive at the correct solution by finding the point of contact by an invalid strategy.
3. $\bullet^4$ is for $\text{Distance}_{c1c2} - r_{c1}$ but only if the answer obtained is greater than r_{c1}.

Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
5 (b)			
	•[5] find ratio in which centre of C_3 divides line joining centres of C_1 and C_2	•[5] $3:1$	
	•[6] determine centre of C_3	•[6] $(6,7)$	
	•[7] calculate radius of C_3	•[7] $r=20$ (answer must be consistent with distance between centres)	
	•[8] state equation of C_3	•[8] $(x-6)^2+(y-7)^2=400$	4

Notes:

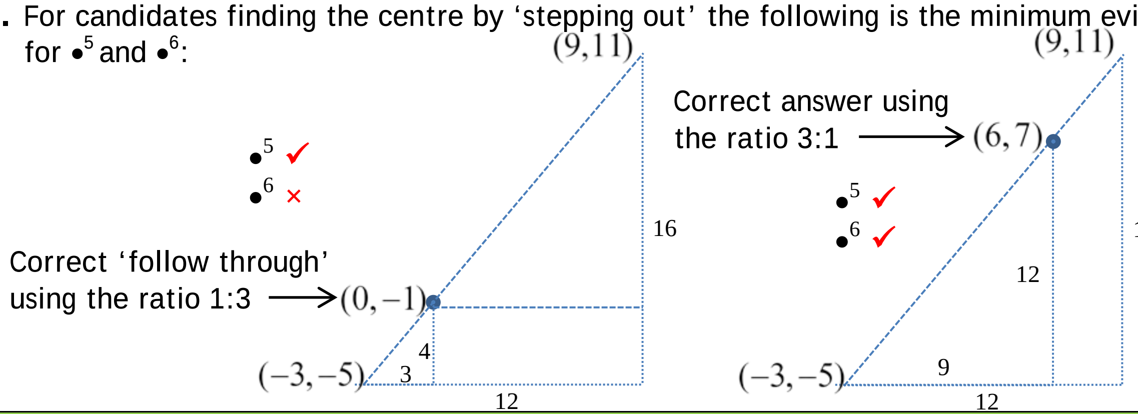
4. For •[5] accept ratios $\pm3:\pm1$, $\pm1:\pm3$, $\mp3:\pm1$, $\mp1:\pm3$ (or the appearance of $\frac{3}{4}$).

5. •[7] is for $r_{c2}+r_{c1}$.

6. Where candidates arrive at an incorrect centre or radius from working then •[8] is available. However •[8] is not available if either centre or radius appear ex nihilo (see note 5).

7. Do not accept 20^2 for •[8].

8. For candidates finding the centre by 'stepping out' the following is the minimum evidence for •[5] and •[6]:



Commonly Observed Responses:

Candidate A

using the mid-point of centres: •[5] ×

centre $C_3=(3,3)$ •[6] ✓ 2

radius of $C_3=20$ •[7] ✓

$(x-3)^2+(y-3)^2=400$ •[8] ✓ 1

Candidate B

$C_1=(-3,-5)$ $C_2(9,11)$ $r=20$

$1:3$

$C_3=\frac{1}{4}\begin{pmatrix}0\\-4\end{pmatrix}$ •[5] ✓ note 4

$C_3=(0,-1)$ •[6] ✓ 2

$x^2+(y+1)^2=400$ •[7] ✓ •[8] ✓ 1

Candidate C - touches C_1 internally only

•[5] ×

•[6] centre $C_3=(3,3)$ ×

•[7] radius of $C_3=$ radius of $C_2=15$ ✓ 1

•[8] $(x-3)^2+(y-3)^2=225$ ✓ 1

Candidate D - touches C_2 internally only

•[5] ×

•[6] centre $C_3=(3,3)$ ×

•[7] radius of $C_3=$ radius of $C_1=5$ ✓ 1

•[8] $(x-3)^2+(y-3)^2=25$ ✓ 1

Candidate E - centre C_3 collinear with C_1,C_2

•[5] ×

•[6] e.g. centre $C_3=(21,27)$ ×

•[7] radius of $C_3=45$ (touch C_1 internally only) ✓ 1

•[8] $(x-21)^2+(y-27)^2=2025$ ✓ 1

Question	Generic Scheme	Illustrative Scheme	Max Mark
6 (a)			
	•[1] Expands •[2] Evaluate $\mathbf{p.q}$ •[3] Completes evaluation	•[1] $\mathbf{p.q}+\mathbf{p.r}$ •[2] $4\frac{1}{2}$ •[3] $...+0=4\frac{1}{2}$	**3**
Notes:			
1. For $\mathbf{p}.(\mathbf{q}+\mathbf{r})=\mathbf{pq}+\mathbf{pr}$ with no other working •[1] is not available.			
Commonly Observed Responses:			
6 (b)			
	•[4] correct expression	•[4] $-\mathbf{q}+\mathbf{p}+\mathbf{r}$ or equivalent	**1**
6 (c)			
	•[5] correct substitution •[6] start evaluation •[7] find expression for $\lvert\mathbf{r}\rvert$	•[5] $-\mathbf{q.q}+\mathbf{q.p}+\mathbf{q.r}$ •[6] $-9+....+3\lvert\mathbf{r}\rvert\cos 30^\circ=9\sqrt{3}-\frac{9}{2}$ •[7] $\lvert\mathbf{r}\rvert=\frac{3\sqrt{3}}{\cos 30}$	**3**
Notes:			
2. Award •[5] for $-\mathbf{q}^2+\mathbf{q.p}+\mathbf{q.r}$			
Commonly Observed Responses:			
Candidate A $-\mathbf{q.q}+\mathbf{q.p}+\mathbf{q.r}=9\sqrt{3}-\frac{9}{2}$ •[5] ✓ $-9+\frac{9}{2}+3\lvert\mathbf{r}\rvert\cos 150^\circ=9\sqrt{3}-\frac{9}{2}$ •[6] ✗ $\lvert\mathbf{r}\rvert=\frac{3\sqrt{3}}{\cos 150}$ •[7] $\boxed{✓1}$		**Candidate B** $-\mathbf{q.q}+\mathbf{q.p}+\mathbf{q.r}=9\sqrt{3}-\frac{9}{2}$ •[5] ✓ $-9+\frac{9}{2}+3\lvert\mathbf{r}\rvert\cos 30^\circ=9\sqrt{3}-\frac{9}{2}$ •[6] ✓ $\lvert\mathbf{r}\rvert=6$ •[7] ✓	

Question	Generic Scheme	Illustrative Scheme	Max Mark
7 (a)			
	•[1] integrate a term •[2] complete integration with constant	•[1] $\frac{3}{2}\sin 2x$ OR x •[2] $x+c$ OR $\frac{3}{2}\sin 2x+c$	2

Notes:

Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
7 (b)			
	•[3] substitute for cos 2*x* •[4] substitute for 1 and complete	•[3] $3\left(\cos^2 x-\sin^2 x\right)...$ or $...(\sin^2 x+\cos^2 x)$ •[4] $...\left(\sin^2 x+\cos^2 x\right)=4\cos^2 x-2\sin^2 x$	2

Notes:

1. Any valid substitution for $\cos 2x$ is acceptable for •[3].
2. Candidates who show that $4\cos^2 x-2\sin^2 x=3\cos 2x+1$ may gain both marks.
3. Candidates who quote the formula for $\cos 2x$ in terms of A but do not use in the context of the question cannot gain •[3].

Commonly Observed Responses:

Candidate A

$$3\cos 2x+1=(2\cos^2 x-1)+(2\cos^2 x-1)+(1-2\sin^2 x)+1$$ •[3] ✓

$$=4\cos^2 x-2\sin^2 x$$ •[4] ✓

Candidate B

$$4\cos^2 x-2\sin^2 x=2(\cos 2x+1)-(1-\cos 2x)$$ •[3] ✓

$$=3\cos 2x+1$$ •[4] ✓

Question	Generic Scheme	Illustrative Scheme	Max Mark
7 (c)			
	•[5] interpret link •[6] state result	•[5] $-\frac{1}{2}\int ...$ •[6] $-\frac{3}{4}\sin 2x-\frac{1}{2}x+c$	2

Notes:

Commonly Observed Responses:

Candidate A

$$\int \sin^2 x-2\cos^2 x\,dx$$

$$=\int (3\cos 2x+1)\;dx$$ •[5] ✗

$$\frac{3}{2}\sin 2x+x+c$$ •[6] ✗

Question	Generic Scheme	Illustrative Scheme	Max Mark
8 (a) (i)			
•¹ calculate T when $x = 20$		•¹ 10·4 or 104	**1**
8 (a) (ii)			
•² calculate T when $x = 0$		•² 11 or 110	**1**

Notes:

1. Accept correct answers with no units.
2. Accept $5\sqrt{436}$ or $10\sqrt{109}$ or equivalent for $T(20)$.
3. For correct substitution alone, with no calculation •¹ and •² are not available.
4. For candidates who calculate T when $x = 0$ at •¹ then •² is available as follow through for calculating T when $x = 20$ in part(ii).

Commonly Observed Responses:

a)	**(i)**	$10\cdot4$	•¹ ✓ **See note 1**
	(ii)	110	•² ✓
b)	leading to $9\cdot8$ seconds		•¹⁰ × **See note 7**

Question	Generic Scheme	Illustrative Scheme	Max Mark
8 (b)			
	$\bullet^3$ write function in differential form	$\bullet^3$ $5(36+x^2)^{\frac{1}{2}} + \ldots$	
	$\bullet^4$ start differentiation of first term	$\bullet^4$ $5 \times \frac{1}{2}\left(\ \right)^{-\frac{1}{2}} \ldots$	
	$\bullet^5$ complete differentiation of first term	$\bullet^5$ $\ldots\ldots\ldots\ldots \times 2x \ldots\ldots\ldots$	
	$\bullet^6$ complete differentiation and set candidate's derivative $= 0$	$\bullet^6$ $5x(36+x^2)^{-\frac{1}{2}} - 4 = 0$	
	$\bullet^7$ start to solve	$\bullet^7$ $5x = 4(36+x^2)^{\frac{1}{2}}$ or $\dfrac{5x}{(36+x^2)^{\frac{1}{2}}} = 4$	
	$\bullet^8$ know to square both sides	$\bullet^8$ $25x^2 = 16(36+x^2)$ or $\dfrac{25x^2}{(36+x^2)} = 16$	
	$\bullet^9$ find value of x	$\bullet^9$ $x = 8$	
	$\bullet^{10}$ calculate minimum time	$\bullet^{10}$ T = 9·8 or 98 no units required	8

Notes:

5. Incorrect expansion of $(\ldots)^{\frac{1}{2}}$ at stage $\bullet^3$ only $\bullet^6$ and $\bullet^{10}$ are available as follow through.
6. Incorrect expansion of $(\ldots)^{-\frac{1}{2}}$ at stage $\bullet^7$ only $\bullet^{10}$ is available as follow through.
7. Where candidates have omitted units, then $\bullet^{10}$ is only available if the implied units are consistent throughout their solution.
8. $\bullet^{10}$ is only available as a follow through for a value which is less than the values obtained for the two extremes.

Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
9.			
	$\bullet^1$ use compound angle formula $\bullet^2$ compare coefficients $\bullet^3$ process for k $\bullet^4$ process for a $\bullet^5$ equates expression for h to 100 $\bullet^6$ write in standard format and attempt to solve $\bullet^7$ solve equation for $1\cdot5t$ $\bullet^8$ process solutions for t	$\bullet^1$ $k\sin 1\cdot5t\cos a - k\cos 1\cdot5t\sin a$ $\bullet^2$ $k\cos a = 36$, $k\sin a = 15$ **stated explicitly** $\bullet^3$ $k = 39$ $\bullet^4$ $a = 0{\cdot}39479\ldots\text{rad}$ or $22\cdot6^\circ$ $\bullet^5$ $39\sin(1\cdot5t - 0\cdot39479...) + 65 = 100$ $\bullet^6$ $\sin(1\cdot5t - 0\cdot39479...) = \frac{35}{39}$ $\Rightarrow 1\cdot5t - 0\cdot39479... = \sin^{-1}\left(\frac{35}{39}\right)$ (columns $\bullet^7$ and $\bullet^8$) $\bullet^7$ $1\cdot5t = 1\cdot508$ and $2\cdot422$ $\bullet^8$ $t = 1\cdot006$ and $1\cdot615$	8

Notes:

1. Treat $k\sin 1\cdot5t\cos a - \cos 1\cdot5t\sin a$ as bad form only if the equations at the $\bullet^2$ stage both contain k.
2. $39\sin 1\cdot5t\cos a - 39\cos 1\cdot5t\sin a$ or $39(\sin 1\cdot5t\cos a - \cos 1\cdot5t\sin a)$ is acceptable for $\bullet^1$ and $\bullet^3$.
3. Accept $k\cos a = 36$ and $-k\sin a = -15$ for $\bullet^2$.
4. $\bullet^2$ is not available for $k\cos 1\cdot5t = 36$ and $k\sin 1\cdot5t = 15$, however, $\bullet^4$ is still available.
5. $\bullet^3$ is only available for a single value of k, $k > 0$.
6. $\bullet^4$ is only available for a single value of a.
7. The angle at $\bullet^4$ must be consistent with the equations at $\bullet^2$ even when this leads to an angle outwith the required range.
8. Candidates who identify and use any form of the wave equation may gain $\bullet^1$, $\bullet^2$ and $\bullet^3$, however, $\bullet^4$ is only available if the value of a is interpreted for the form $k\sin(1\cdot5t - a)$.
9. Candidates who work consistently in degrees cannot gain $\bullet^8$.
10. Do not penalise additional solutions at $\bullet^8$.
11. On this occasion accept any answers which round to $1\cdot0$ and $1\cdot6$ (2 significant figures required).

Commonly Observed Responses:

Response 1: Missing information in working.

Candidate A	Candidate B	Candidate C
$39\cos a = 36$ $-39\sin a = -15$ $\tan a = \frac{15}{36}$ $a = 0{\cdot}39479\ldots\text{rad or } 22{\cdot}6^\circ$ $\bullet^1$ $\wedge$ $\bullet^2$ ✓ $\bullet^3$ ✓ $\bullet^4$ ✓	$\cos a = 36$ $\sin a = 15$ $\tan a = \frac{15}{36}$ $a = 0{\cdot}39479\ldots\text{rad or } 22{\cdot}6^\circ$ Does not satisfy equations at $\bullet^2$ $\bullet^1$ $\wedge$ $\bullet^2$ ✗ $\bullet^3$ $\wedge$ $\bullet^4$ ✗	$k\sin 1{\cdot}5t\cos a - k\cos 1{\cdot}5t\sin a$ $k\cos a = 36,\quad k\sin a = 15$ $k = 39 \text{ or } -39$ $\tan a = \frac{15}{36}$ $a = 0{\cdot}39479\ldots\text{rad or } 22{\cdot}6^\circ$ *or* $a = 3{\cdot}53638\ldots\text{rad or } 202{\cdot}6^\circ$ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✗ $\bullet^4$ ✗

Response 2: Correct expansion of $k\sin(x + a)^\circ$ and possible errors for $\bullet^2$ and $\bullet^4$

Candidate D	Candidate E	Candidate F
$k\cos a = 36$ $k\sin a = 15$ $\tan a = \frac{36}{15}$ $a = 1{\cdot}176\ldots\text{rad or } 67{\cdot}4^\circ$ $\bullet^2$ ✓ $\bullet^4$ ✗	$k\cos a = 15$ $k\sin a = 36$ $\tan a = \frac{36}{15}$ $a = 1{\cdot}176\ldots\text{rad or } 67{\cdot}4^\circ$ $\bullet^2$ ✗ $\bullet^4$ $\boxed{✓1}$	$k\cos a = 36$ $k\sin a = -15$ $\tan a = \frac{-15}{36}$ $a = 5{\cdot}888\ldots\text{rad or } 337{\cdot}4^\circ$ $\bullet^2$ ✗ $\bullet^4$ $\boxed{✓1}$

Response 3: Labelling incorrect, $\sin(A - B) = \sin A\cos B - \cos A\sin B$ from formula list.

Candidate G	Candidate H	Candidate I
$k\sin A\cos B - k\cos A\sin B$ $k\cos a = 36$ $k\sin a = 15$ $\tan a = \frac{15}{36}$ $a = 0{\cdot}39479\ldots\text{rad or } 22{\cdot}6^\circ$ $\bullet^1$ ✗ $\bullet^2$ ✓ $\bullet^4$ ✓	$k\sin A\cos B - k\cos A\sin B$ $k\cos 1{\cdot}5t = 36$ $k\sin 1{\cdot}5t = 15$ $\tan 1{\cdot}5t = \frac{15}{36}$ $1{\cdot}5t = 0{\cdot}39479\ldots\text{rad or } 22{\cdot}6^\circ$ $\bullet^1$ ✗ $\bullet^2$ ✗ $\bullet^4$ $\boxed{✓1}$	$k\sin A\cos B - k\cos A\sin B$ $k\cos B = 36$ $k\sin B = 15$ $\tan B = \frac{15}{36}$ $B = 0{\cdot}39479\ldots\text{rad or } 22{\cdot}6^\circ$ $\bullet^1$ ✗ $\bullet^2$ $\boxed{✓1}$ $\bullet^4$ $\boxed{✓1}$

Candidate J	Candidate K
$39\sin(1{\cdot}5t - 0{\cdot}395) = 100$ $\sin(1{\cdot}5t - 0{\cdot}395) = \frac{100}{39}$ $1{\cdot}5t - 0{\cdot}395 = \sin^{-1}\frac{100}{39}$ $\bullet^5$ ✗ $\bullet^6$ $\boxed{✓1}$ $\bullet^7$ ✗ $\bullet^8$ ✗	$39\sin(1{\cdot}5t - 0{\cdot}395) = 100$ $1{\cdot}5t - 0{\cdot}395 = \sin^{-1}\frac{39}{100}$ $\bullet^6$ ✗ $\bullet^7$ ✗ $\bullet^8$ ✗

[END OF MARKING INSTRUCTIONS]