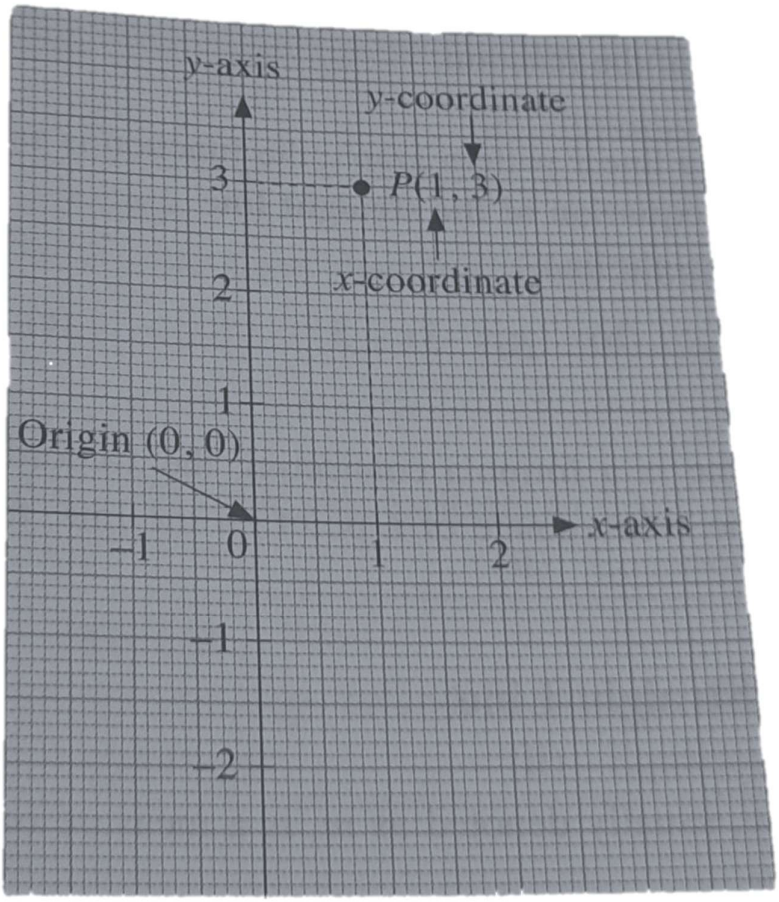
Topic 6: Linear Functions and Graphs

Notes:

The Cartesian Coordinates:



1. The **Rectangular Plane** or **Cartesian Coordinates System** consists of:

(a) A vertical y**-axis**

(b) A horizontal x**-axis**

(c) The **origin**, which is the point of intersection of the x and y axes and has the coordinates (0,0). The point of origin is often labeled as O.

2. The ordered number pair assigned to a point is called the **coordinates** of a point. It is written in a fixed order as **(x,y)** where x is the **x-coordinate** representing the horizontal distance left or right of the y-axis and y is the **y-coordinate** which represents the vertical distance above or below the x-axis. The coordinates of a point allow the position of a point to be located.

For example: Point P(1,3) is located at 1 unit to the right of the y-axis (x coordinate) and 3 units above the x-axis (y coordinate)

3. When plotting points on a Cartesian Coordinate System, observe the following:
   - Select a convenient scale to allow the points to be read or marked off clearly to a high degree of accuracy. The scales are given as;
     - x-axis: _____ cm : _____ units
     - y-axis: _____ cm : _____ units
   - Labeling of axes, origin, points, lines, and objects must be shown.

Linear Function and its Graph:

4. A **function** is a relationship between an independent variable x (aka the input) and a dependent variable y (aka the output.) A function can be represented using an equation, a table, or a graph.

5. Every set of coordinates of point (x,y) that satisfies the equation of a function can be represented as a point on the graph of the function. Conversely, any point on

the graph of the function has its coordinates (x,y) satisfying the equation of the function.

6. If a set of points conform to a linear pattern graph, the function is said to be linear and the function of the graph is known as the **equation of the straight line**.

For example: (Equations start with y)

(i) $y = 2x$
(ii) $y = -3x + 6$
(iii) $y = \frac{2}{5}x + 0.25$

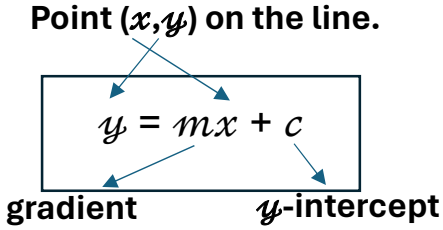
7. The formula of a straight line is a formula connecting the x-coordinates and y-coordinates of all the points on the line. For the x-coordinate of a point on the line, there will exist only one unique y-coordinate.

Gradient and Equation of a Straight Line:

8. The gradient of a straight line, usually denoted as m, is a measure of the degree of steepness of the slope of the line. It is given as;

Gradient, $m = \frac{vertical\ change}{horizonta\ \ change}$ or $\frac{rise}{run}$

9. The general linear function or the equation of a straight line is in the form $y = mx + c$.



Note:

(a) m is the gradient of the line,

(b) c is the y-intercept of a line i.e. where the line crosses the y-axis.

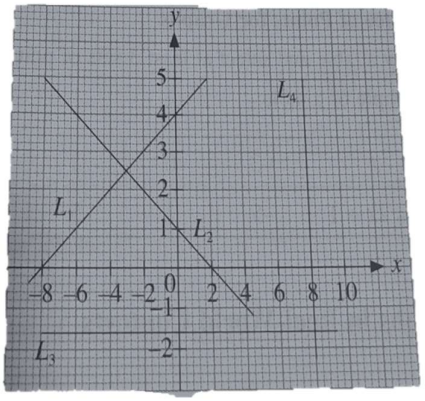
10. There are 4 different forms of straight lines:

(a)"Uphill line" e.g. L_1

(b) "Downhill line e.g. L_2

(c) Horizontal line e.g L_3

(d) Vertical line e.g L_4



Types of Straight Lines
(a) Positive Slope or Gradient Lines that have positive slope slant "uphill" from left to right (Refer to L_1) (i)Gradient of $L_1 = \frac{4}{8} = \frac{1}{2}$ (ii) Equation of L_1 is $y = \frac{1}{2}x + 4$
(b) Negative Slope or Gradient Lines that have negative slope slant "downhill" from left to right (Refer to L_2) (i)Gradient of $L_2 = \frac{-1}{2} = -\frac{1}{2}$ (ii) Equation of L_2 is $y = -\frac{1}{2}x + 1$
(c) Zero Slope or Gradient Lines that are horizontal have zero slope (Refer to L_3) (i)Gradient of $L_3 = 0$ (ii) Equation of L_3 is $y = -1.6$
(d) Slope or Gradient Undefined Lines that are vertical have no slope or undefined slope (Refer to L_4) (i)Gradient of L_4 = undefined (ii) Equation of L_4 is $x = 8$

Drawing the Graph of a Linear Function (i.e. Straight Line):

11. When plotting a straight line, the following rules need to be considered:

(a) Choice of Appropriate Scales for Graphs

If the scale of the graph to be drawn is not given, use the following guidelines:

- Use a convenient scale for both axes i.e. x-axis and y-axis.

  For example:

  (i) 1cm to represent 1 unit on the x-axis and 1 cm to represent 2 units on the y-axis

  (ii) 1 cm to represent 5 units on both axes

  (iii) 1cm to represent 10 units on both axes

- The scale used for x-axis and y-axis can be different depending on the spread of values in x and y.
- Choose a suitable large scale so that the graph displayed will be more accurate for reading values. A good size for a graph should occupy at least half the size of the given graph paper.

(b) Choice of Points for Graphs

Draw a table and select at least 3 sets of points that spread out to plot the graph.

For example:

x	-3	0	2
$y = 2x + 1$	-5	1	5

Linear Graphs in Real-World Contexts:

12. Distance-Time Graph (One type of Travel Graph)

Travel graphs are usually line graphs that describe the motion of moving objects such as cars, trains, cyclists and walkers or joggers. For distance-time graphs, the distance travelled is represented on the vertical y-axis and the time taken to travel that distance is represented on the horizontal x-axis.

For each line segment of the motion of the moving object:

Gradient of the line, m = Speed of the moving object during that time interval.

13. Expenditure and Conversion Graphs

Linear graphs can also be applied to perform conversions such as currencies, to represent expenditure and usage such as utility charges as well as making comparisons between different packages such as wage schemes.