Topic 4: Basic Algebra and Algebraic Manipulation

Notes:

Notes and Terminology in Algebra:

1. In algebra, letters (like x, y, ab, z^3 etc.) are used to represent numbers or unknown numbers and values that are called **variables**.

2. Mathematical statements like $x + 3y$, $\frac{3a}{5b}$, $x^4 + 2x - 3$, are known as **algebraic expressions**.

3. An **algebraic expression** involves numbers, **variables**, the four operations like addition (+), subtraction (-), multiplication (x) and division (÷). The + and – signs in an algebraic expression separate it into terms and they are used similarly as in arithmetic.

   To give an example:

   (a) $a + b = c$ means that the sum of the two numbers, represented by a and b, is equal to the number which is represented by c.

   (b) $2x - y = z$ means that the difference of the two numbers represented by $2x$ and y equals to the number represented by z.

4. Other notations of algebraic terms are shown below for your information:

(a) a x b = ab

(b) 4 x a = 4a

(c) a x a = a^2

(d) $a \div b = \frac{a}{b}$

(e) $(5b)^2$ = 5b x 5b
= 5^2 x b^2
= 25b^2

(f) a x $b \div c = \frac{ab}{c}$

Representation of Relationships of Mathematics using Algebra:

5. Mathematical relationships in the form of word statements can be translated into algebraic expressions.
For example:

(i) *"Add 5x to 2y"* = 5x + 2y

(ii) *"Subtract 2x from x^3"* = x^3 – 2x

(iii) *"The product of three consecutive numbers with the smallest number represented by m."* = $m(m + 1)(m + 2)$

(iv) *"The cube of d divided by the product of 3b and c."* = $\frac{d^3}{3bc}$

6. An algebraic expression is made up of different parts. Consider this expression; $n^2 \ + 2n - 4m + 5$ and refer to the explanations below.

(i) **Variable** – In algebra, a letter thar represents a number (unknown or not) is called a **variable**. Its value can be varied.

For an example:

n and m are variables.

(ii) **Coefficient** – The number that is multiplied to the **variable** (alphabetic letter.)

Examples:

n^2 → 1 is the **coefficient** of n^2 in n^2
(n^2 x 1 = n^2)

2n → 2 is the **coefficient** of n in 2n
(2 x n = 2n)

-4m → -4 is the **coefficient** of m in -4m
(-4 x m = -4m)

(iii) **Terms** – A **term** can be a number, a variable or even the products or quotients of numbers and **variables**. **Addition and subtraction** separate terms from one another (**not multiplication and division.**)

Examples:

n^2, $2n$, -$4m$ and 5 are **terms**.

There are 4 **terms** in the expression above:
$n^2 \ + 2n - 4m + 5$.

(iv) **Constant** – A **constant** is a term with a fixed numerical value (whole numbers, decimal numbers…) and its value does not change.

For an example:

5 is a **constant**.

(v) **Degree** – A **degree** of an algebraic expression is the highest index (Of) observed in the terms of the expression.

For an example:

Since the highest index is 2 as observed in n^2, the **degree** of this expression is 2.

(vi) **Polynomial** – A **polynomial** refers to an algebraic expression consisting of one or more terms (like the one above.) A **polynomial** is usually arranged in either descending or ascending order with respect to the degree of terms.

For an example:

$n^2 + 2n - 4m + 5$ is a **polynomial** arranged in descending order. (2 is the highest degree followed by 0.)

7. A **linear** expression is an algebraic expression where the degree of the expression is **1** while a **quadratic** expression is an algebraic expression where the degree of the expression is **2**.

   Examples:

   (i) $3x - 4z + 9$ is a linear expression in x and z.
   (ii) $x^2 - xy + 2y^2$ is a quadratic expression.

Evaluation of Algebraic Expressions:

8. Since the variable in an algebraic expression represents an unknown value, the algebraic expression can be **evaluated** by replacing or substituting the variables in the expression with the given numerical values.
   For an example:

   "Evaluate the value of $2x - 4xy + 7$ when $x = 2$ and $y = 3$."

   $2x - 4xy + 7 = 2(2) - 4(2)(3) + 7$
   $= 4 - 24 + 7$
   $= -13$

Addition and Subtraction of Algebraic Expressions:

9. Addition and Subtraction of algebraic terms is done by adding or subtracting variables of the same kind, aka **like terms**. Examples of **like terms** are $3u$ and $7u$, and -5e, eu, and $6eu$. **Unlike terms** are variables which are

the exact opposite of like terms. Examples of unlike terms include 5m and 5n, or xy^3 and x^3y.
Examples of (+) and (-):

(i) 2b + 3b = 5b

(ii) 8m – 5m = 3m

(iii) 2x – 5y + 3x - y = 2x + 3x – 5y – y (Rearrange)

= 5x – 6y

(iv) 3g - 3h – 7g + h = 3g - 7g + h – 3h (Rearrange)

= -4g - 2h

***Do note:**
(a) a + b = b + a (Cumulative Law)
(b) a - b ≠ b - a

Multiplication and Division of Algebraic Expressions:

10. In performing multiplication and division, **coefficients** and **variables** of the respective terms are gathered and multiplied or divided on their own.
Examples of (x) and (÷):

(i) 4e x 5e = 20e^2

(ii) (-3q) x m = -3qm

(iii) -2c^2b x 3cb = 6c^3b^2

(iv) $\frac{12}{3aj} = \frac{4}{j}$

***Do note:**

The product resulted from the multiplication terms of algebraic expressions x and z is written as xz.

(a) $xz = x \,.\, z$
$= x(z)$ or $(x)z$ or $(x)(z)$
$= x \times z$

(b) $xz = zx$ (Commutative Law)

Expansion of Algebraic Expressions:

11. When brackets appear in an algebraic expression, the rules, and priorities by which operations are performed follow the BODMAS rule which is used in normal arithmetic.

(i) Simplify the expression within the brackets first.

Example: $(2a + 5a) \times 3ab = 7a \times 3ab$
$= 21a^2b$

(ii) If an expression in a bracket is being multiplied by a **constant number** or **variable**, each term within the bracket must be multiplied by this **constant number** or **variable** when the bracket is removed.

Examples:
$x(y + z) = xy + xz$ (Distributive Law)

$2(a + 3b - 2c) = 2a + 6b - 4c$

(iii) When an expression consists of more than one pair of brackets, simplify the expression within the INNERMOST pair first before moving to the outermost pair.

Example:

$$2\{c - 3[c - (c + 2)]\} = 2[c - 3(c - c - 2)]$$
$$= 2[c - 3(c - c - 2)]$$
$$= 2(c - 3c + 3c + 6)$$
$$= 2(c - 3c + 3c + 6)$$
$$= 2c - 6c + 6c + 12$$
$$= 2c + 12$$

Important Note*:
In the working, you must change your brackets as you remove them (colour coded) – the curly brackets {} in this scenario changed to square brackets [] as the normal brackets () were removed. The square brackets [] then turned into the normal brackets () and so on.

Simplification of Linear Algebraic Expressions with Fractional Coefficients:

12. A linear algebraic expression with **fractional coefficients** can be viewed as an algebraic term with **fractional coefficient** or an algebraic fraction that involves an algebraic expression in its numerator and with an integral denominator. For example, $\frac{2x}{3}$, , $\frac{2x-3}{5}$.

13. Brackets can also be used in algebraic fractions. For example,

$\frac{1}{4}(2x-3) = \frac{(2x-3)}{4}$

$= \frac{2x-3}{4}$

14. Addition or subtraction of algebraic fractions is done by taking the **LCM** of all the denominators of the algebraic fractions as the common denominator before proceeding with the addition or subtraction.

Factorization

15. **Factorization** is the process of writing an algebraic expression as a product of two or more other algebraic expressions. It is the reverse process of expansion.

    Example: Since expanding $a(b + c) = ab + ac$, therefore factorizing $ab + ac = a(b + c)$

16. To **factorize** an algebraic expression completely, take out the greatest common factor(s) of the terms otherwise the factorization is incomplete.

    Example: Factorize $4ax + 12ay$ completely.

    Solution: $4ax + 12ay = 4a(x + 3y)$ where $4a$ is the greatest common factor of the two terms.