equations of lines (vectors) - 1

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4 questions – progressing from 'accessible' to 'discriminating'

1. Show that the point with position vector $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$ lies on the line L that has the vector equation $\vec{\mathbf{r}} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} + t\begin{pmatrix} -1 \\ 3 \end{pmatrix}$. [***no*** *calculator*]

2. Write the equation of the line $2x + 3y = 7$ in vector equation form of a line; that is, in the form $\vec{\mathbf{r}} = \vec{\mathbf{a}} + t\vec{\mathbf{b}}$, such that the components of $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are integers.
[***no*** *calculator*]

3. The two lines with vector equations $\vec{\mathbf{r}} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -1 \\ -3 \end{pmatrix}$ and $\vec{\mathbf{r}} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \mu\begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix}$ intersect at point P. Find the coordinates of P. [***no*** *calculator*]

4. Find the two points on the line with Cartesian equation $x - 5 = \frac{y+4}{-4} = \frac{z-6}{3}$ which are a distance of 5 units from the origin. [***no*** *calculator*]

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Answers

1. when $t=-3$: $\vec{\mathbf{r}}=\begin{pmatrix}-1\\4\end{pmatrix}+(-3)\begin{pmatrix}-1\\3\end{pmatrix}=\begin{pmatrix}-1\\4\end{pmatrix}+\begin{pmatrix}3\\-9\end{pmatrix}=\begin{pmatrix}2\\-5\end{pmatrix}$ *Q.E.D.*

2. infinite possible answers – including $\vec{\mathbf{r}}=\begin{pmatrix}2\\1\end{pmatrix}+t\begin{pmatrix}3\\-2\end{pmatrix}$ and $\vec{\mathbf{r}}=\begin{pmatrix}5\\-1\end{pmatrix}+t\begin{pmatrix}3\\-2\end{pmatrix}$

3. $(2,-2,-5)$

4. $(4,0,3)$ and $(3,4,0)$