MATHEMATICS

Module 1

CIVIL ENGINEERING ACADEMY

MATHEMATICS AND STATISTICS (SUB-MODULES)

Module 1A – Analytic Geometry
Module 1B – Single-Variable Calculus
Module 1C – Vector Operations
Module 1D - Statistics

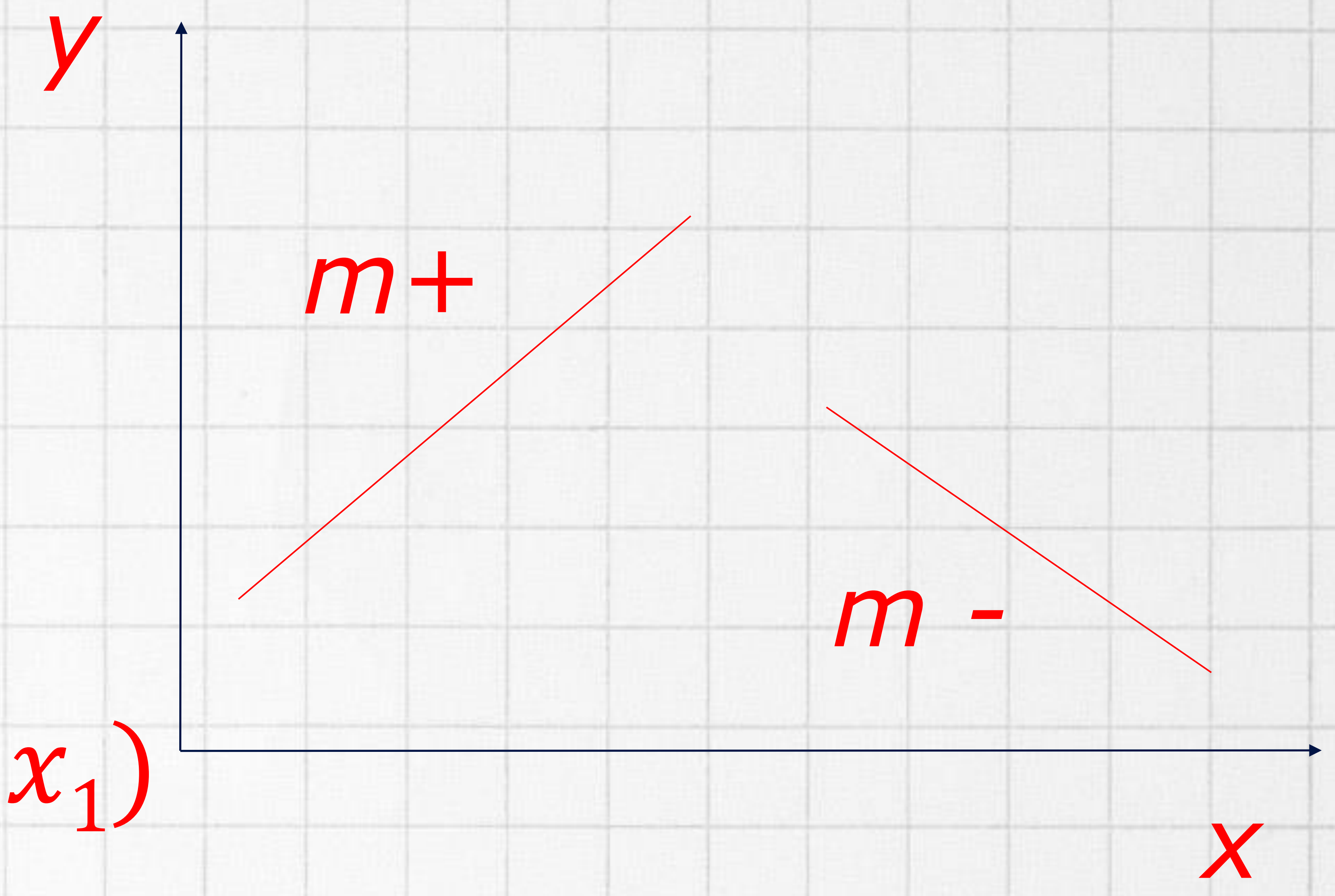
ANALYTIC GEOMETRY

Module 1A

- ✓ Straight Lines
- ✓ Conic Sections
- ✓ Quadric Surface (Sphere)
- ✓ Trigonometry Functions
- ✓ Trigonometry Identities

(FE Reference Handbook)

STRAIGHT LINES



General form of equation : $Ax + By + C = 0$

Standard form of equation : $y = mx + b$

Point-slope form of equation : $y - y_1 = m(x - x_1)$

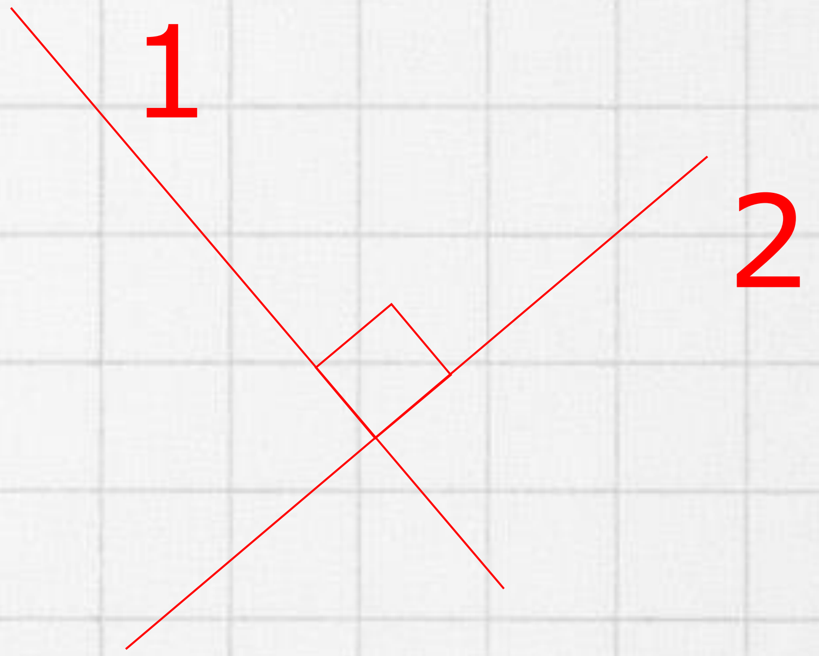
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Parallel lines:

$$m_1 = m_2$$

Distance between two points:

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

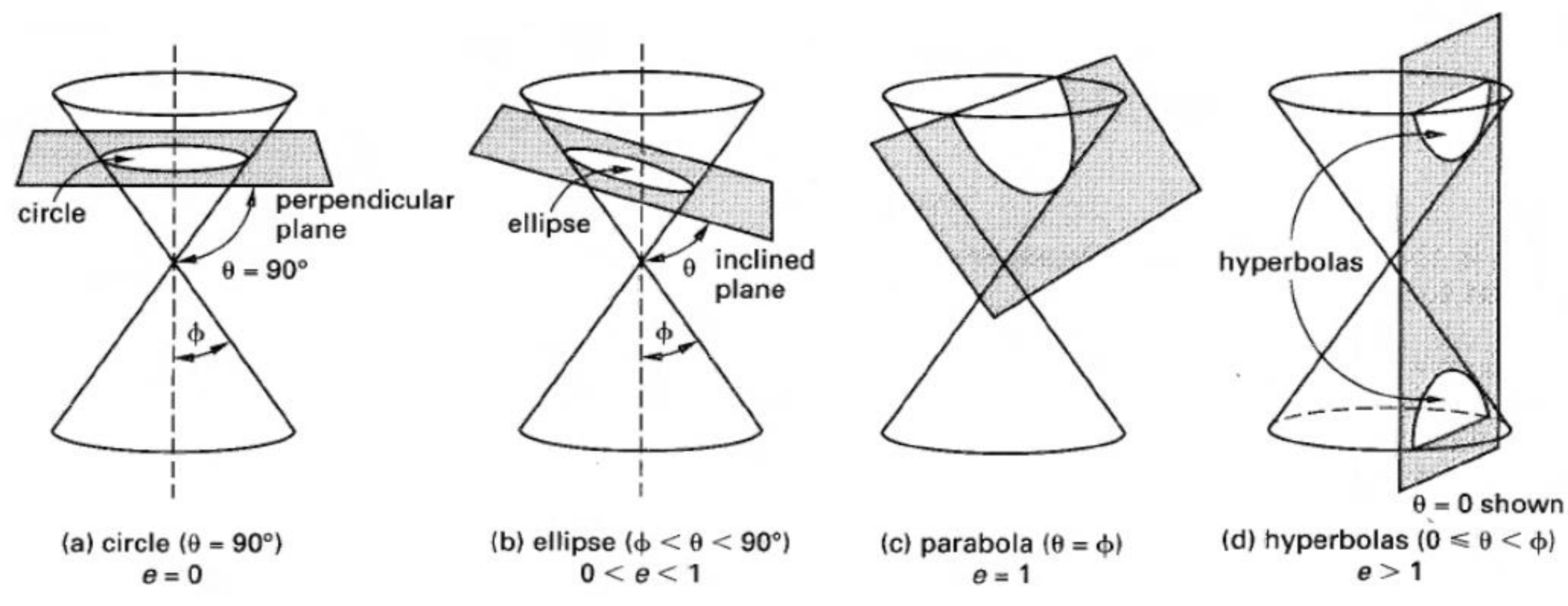


Perpendicular:

$$m_1 = \frac{-1}{m_2}$$

ANALYTIC GEOMETRY & TRIGONOMETRY

CONIC SECTIONS



$$e = \cos\theta \,/\, \cos\phi$$

ANALYTIC GEOMETRY & TRIGONOMETRY

CONIC SECTIONS

General form of equation : $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

If $B^2 - 4AC < 0$ → ellipse

$B^2 - 4AC > 0$ → hyperbola

$B^2 - 4AC = 0$ → parabola

$A = C$ and $B = 0$ → circle

$A = B = C = 0$ → straight line

ANALYTIC GEOMETRY & TRIGONOMETRY

CONIC SECTIONS

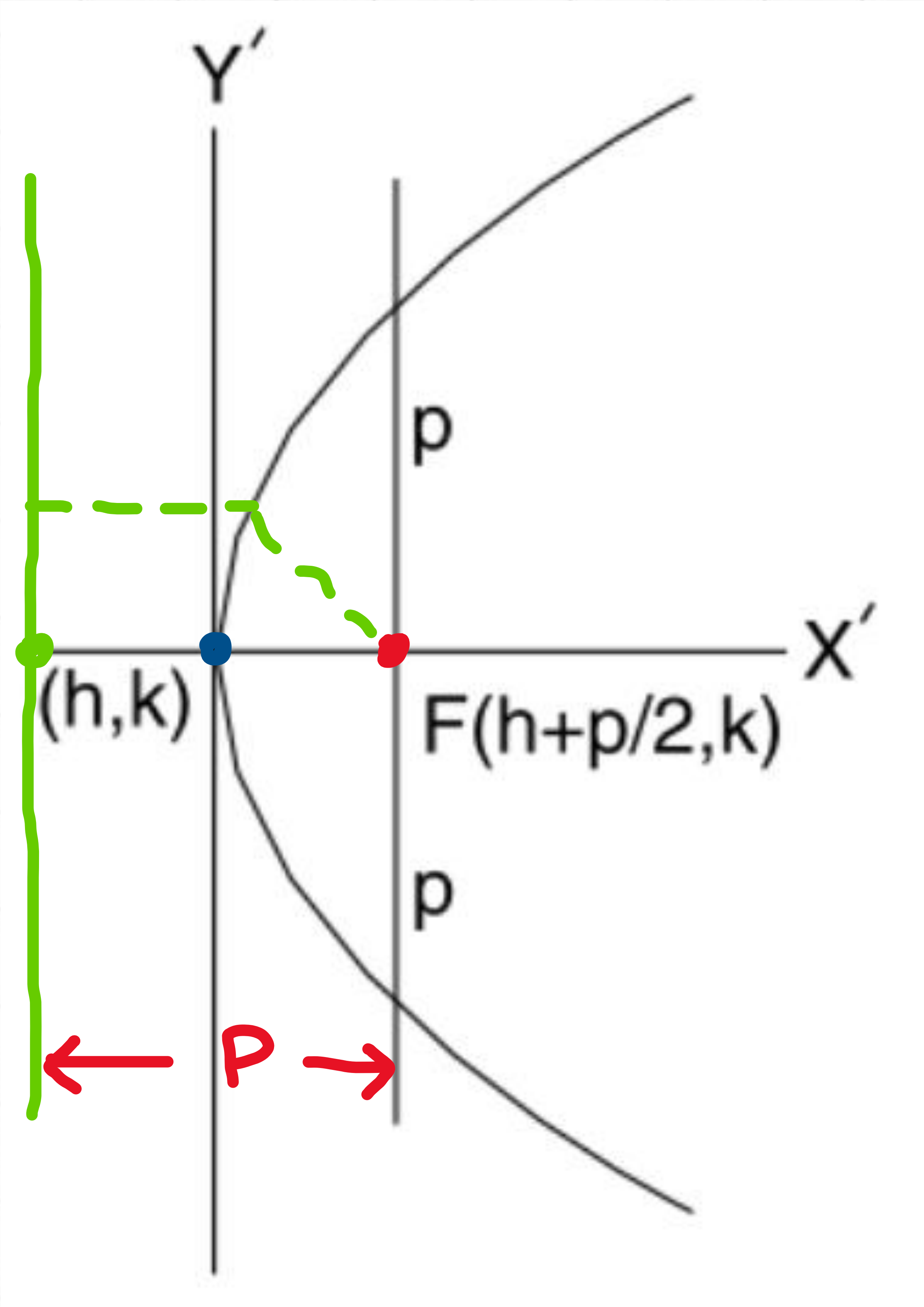
1. Parabola (e=1)

General form of equation :

$$(y-k)^2 = 2p(x-h)$$

Vertex : (h,k)

$$(x-h)^2=2p(y-k)$$



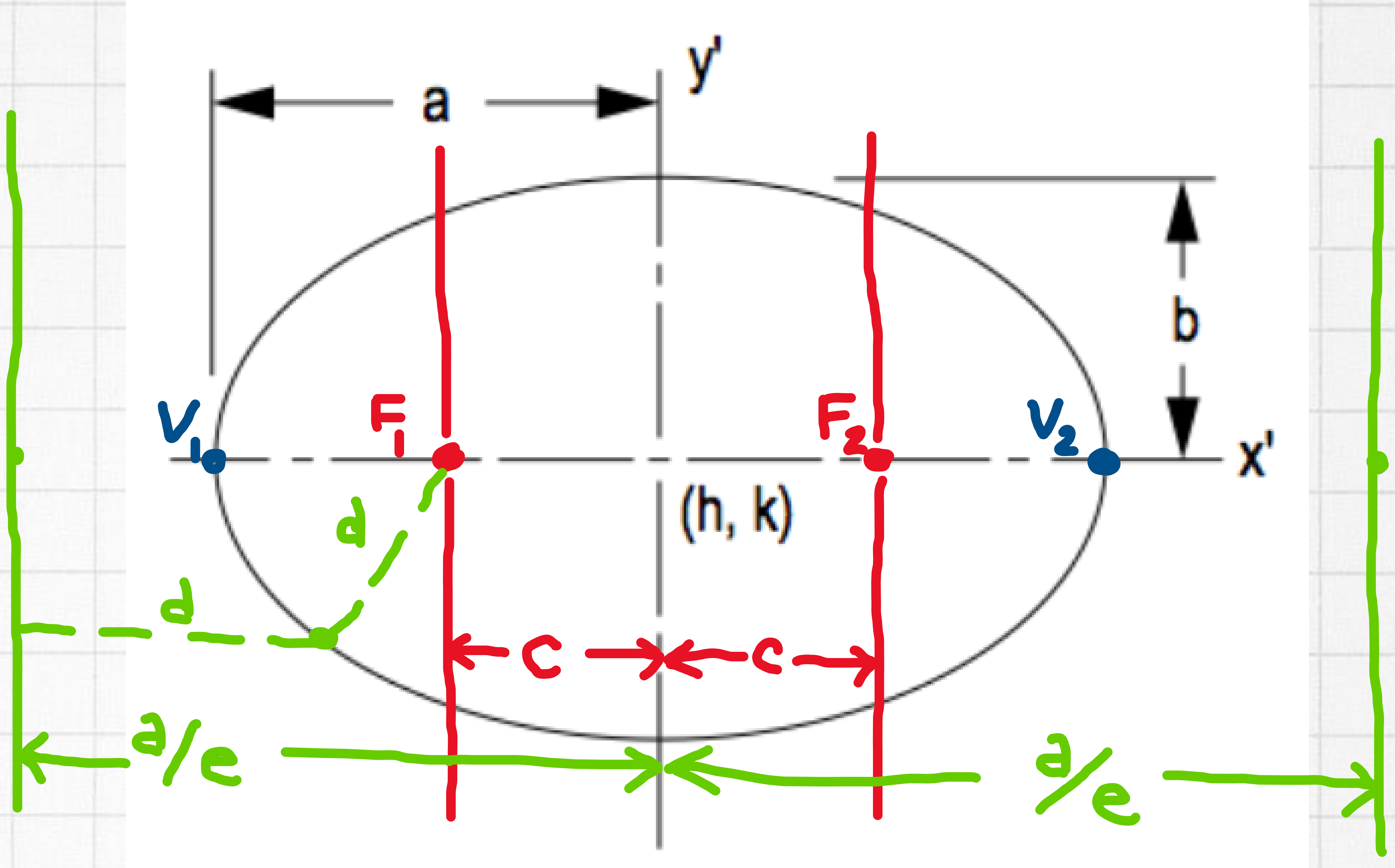
When: $(h,k) = (0,0)$

Focus : $(p/2, 0)$

Directrix: $x = -p/2$

ANALYTIC GEOMETRY & TRIGONOMETRY

2. Ellipse (e<1)



General form of equation :

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Center : (h,k)

When (h,k) = (0,0)

$$b = a\sqrt{1-e^2}$$

Eccentricity:

$$e=\sqrt{1-\left(b^2/a^2\right)}=c/a$$

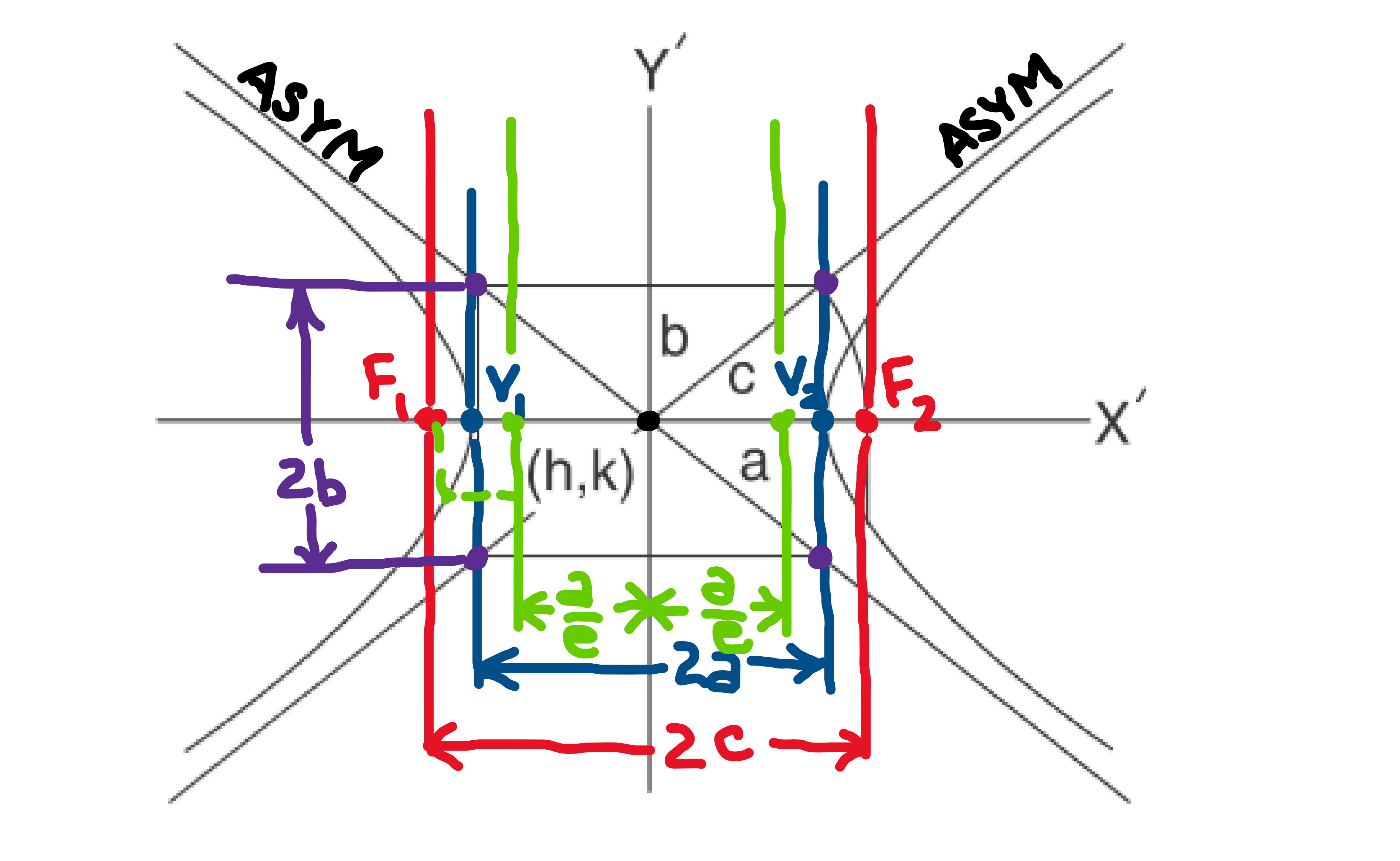
Foci: $(\pm ae, 0)$

Directrix : $x = \pm a/e$

ANALYTIC GEOMETRY & TRIGONOMETRY

CONIC SECTIONS

3. Hyperbola (e>1)



General form of equation :

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Center : (h, k)

When (h,k) = (0,0)

$$b = a\sqrt{e^2 - 1}$$

Eccentricity:

Foci: $(\pm ae, 0)$

$$e = \sqrt{1 + (b^2/a^2)} = c/a$$

Directrix : $x = \pm a/e$

ANALYTIC GEOMETRY & TRIGONOMETRY

4. Circle (e=0)

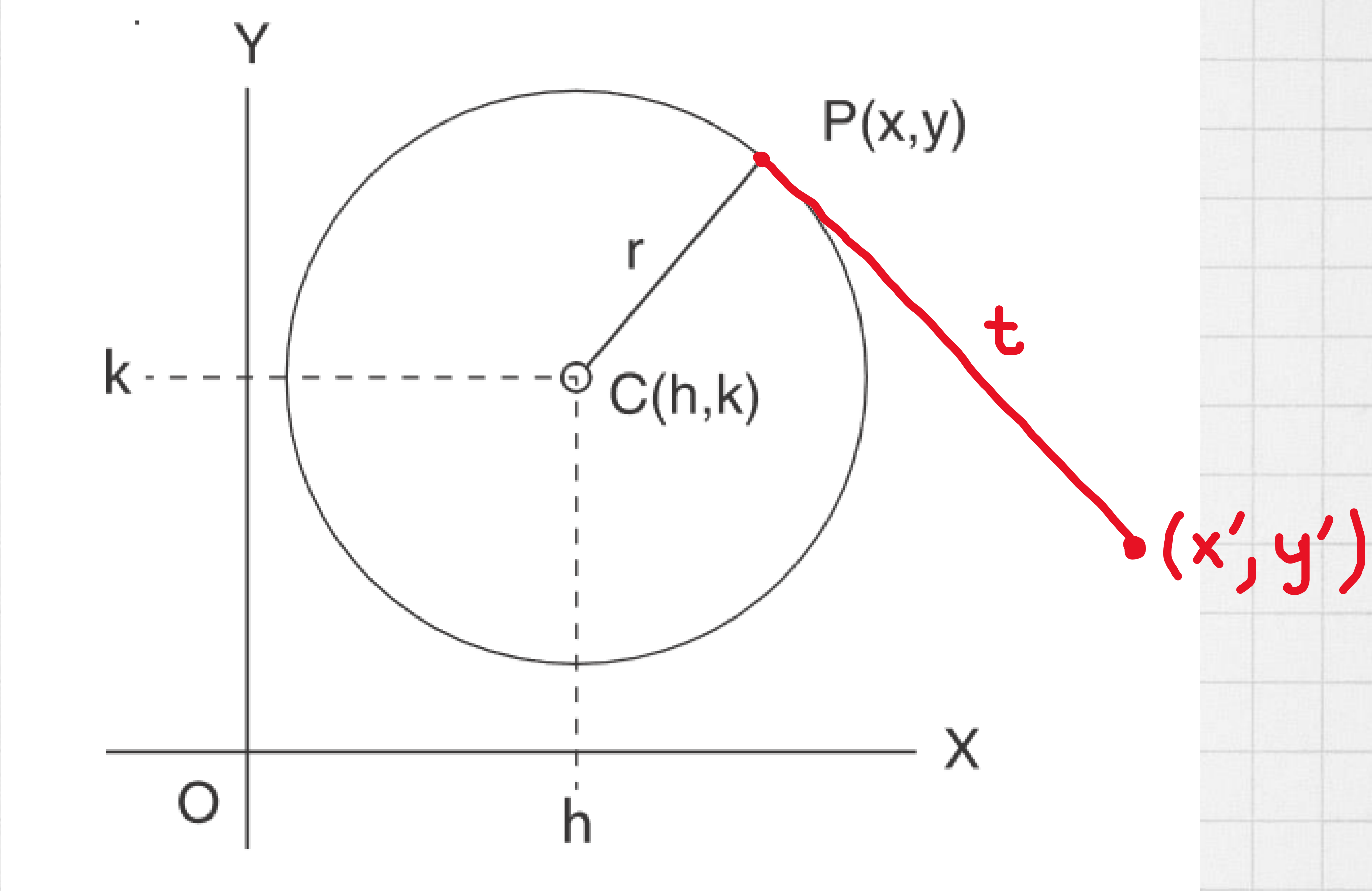
General form of equation :

$$(x-h)^2+(y-k)^2=r^2$$

Center : (h,k)

Length of Tangent (2D space):

$$t^2=(x'-h)^2+(y'-k)^2-r^2$$



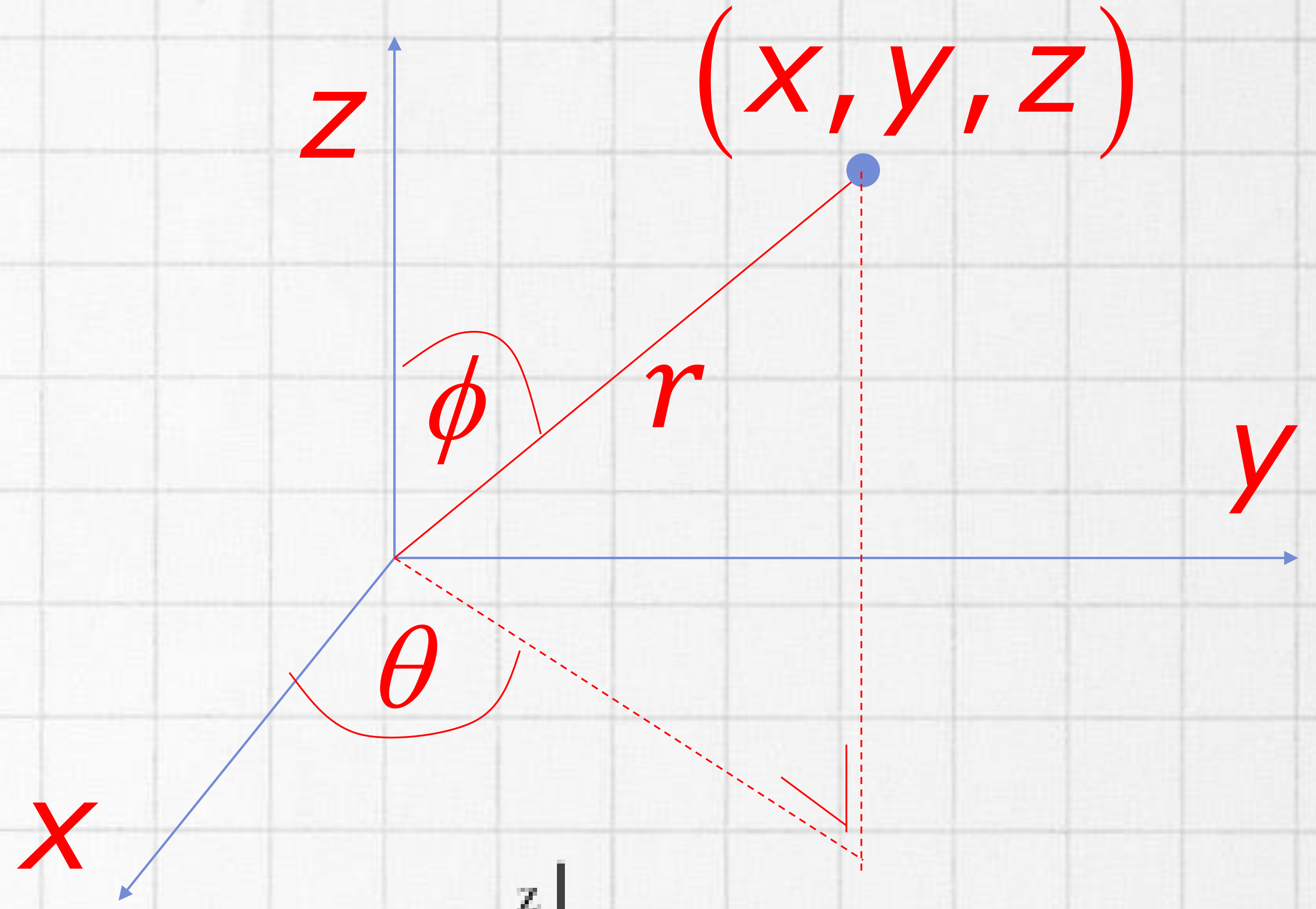
ANALYTIC GEOMETRY & TRIGONOMETRY

QUADRIC SURFACE (SPHERE)

General form of equation : $(x-h)^2+(y-k)^2+(z-m)^2=r^2$

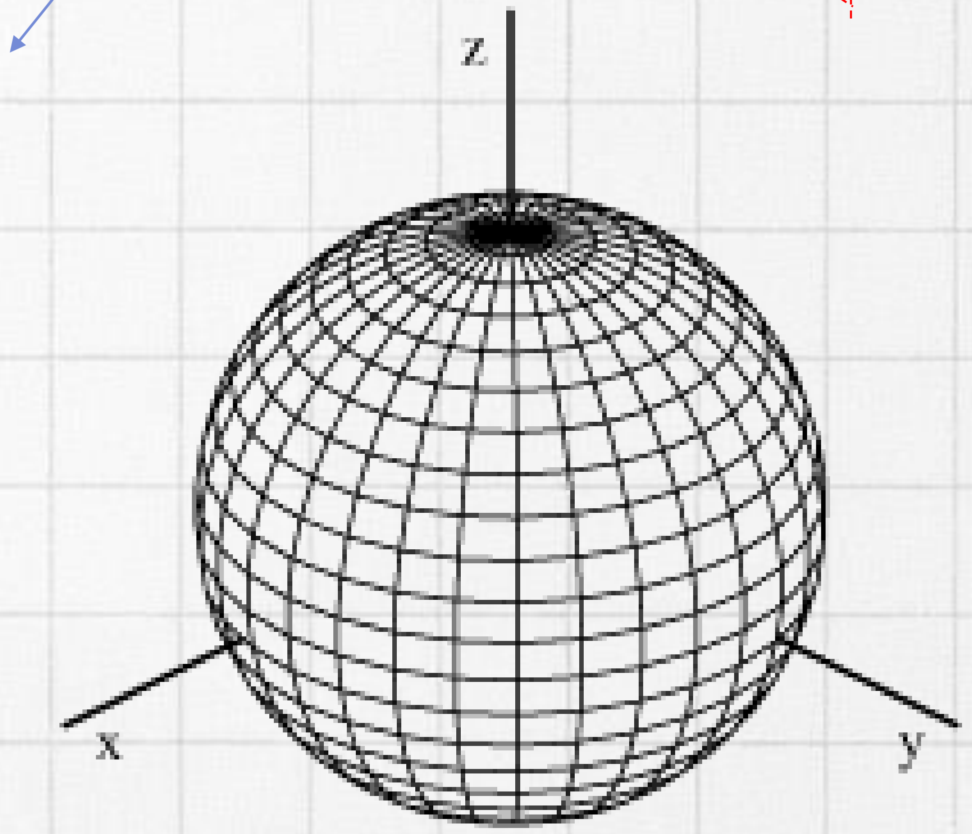
$$x = r\sin\phi\cos\theta$$
$$y = r\sin\phi\sin\theta$$
$$z = r\cos\phi$$

Center : (h,k,m)



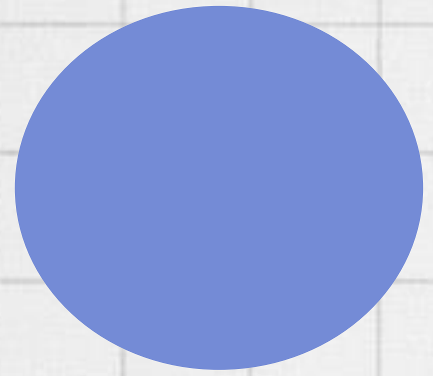
Distance between 2 points:

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$



ANALYTIC GEOMETRY & TRIGONOMETRY

AREA



$$A = \pi r^2 = \frac{\pi}{4} d^2$$

$$A = \frac{1}{2} bh$$

$$A = bh$$

$$A = 4\pi r^2 = \pi d^2$$

VOLUME

$$V = bhl$$

$$V = \pi r^2 h$$

$$V = \frac{4}{3} \pi r^3$$

ANALYTIC GEOMETRY & TRIGONOMETRY

EXAMPLE

Find the coordinate of a circle center and determine its radius if the equation of circle is written as follows:

$$x^2 + y^2 - 4x + 10y + 20 = 0 \quad \rightarrow \mathrm{Ax^2 + Bxy + Cy^2 + Dx + Ey + F}$$

(A) (2,5); 9 (B) (-2,-5); 9 (C) (2,-5); 3 (D) (-2,5); 3

Solution: $\mathrm{x^2 + y^2 - 4x + 10y + 20 = 0} \quad \rightarrow \mathrm{x^2 - 4x + y^2 + 10y = -20}$

$$\rightarrow \mathrm{x^2 - 4x + 4 + y^2 + 10y + 25 = -20 + 4 + 25}$$

$$\mathrm{x^2 - 4x + 4 + y^2 + 10y + 25 = 9}$$

$$\mathrm{(x-2)^2 + (y+5)^2 = 3^2} \quad \rightarrow \mathrm{(x-h)^2 + (y-k)^2} = r^2$$

So, the circle center is at (2,-5) and its radius is 3. **(Answer : C)**

ANALYTIC GEOMETRY & TRIGONOMETRY

EXAMPLE

Find the slope of a straight line perpendicular to another line intercepting the Y-axis at 7 and passing through point (5,-3).

(A) -1/2 (B) 1/2 (C) -2 (D) 2

Solution:

From the problem statement, there are 2 known points: (5,-3) and (0,7)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-(-3)}{0-5} = -2 \quad \rightarrow m_1 = -\frac{1}{m_2} \quad \rightarrow m_1 m_2 = -1$$

Since the second line is perpendicular to first line, the slope will be:

$$m_1 \times m_2 = -1$$

$$-2 \times m_2 = -1$$

$$m_2 = 0.5$$

(Answer : B)

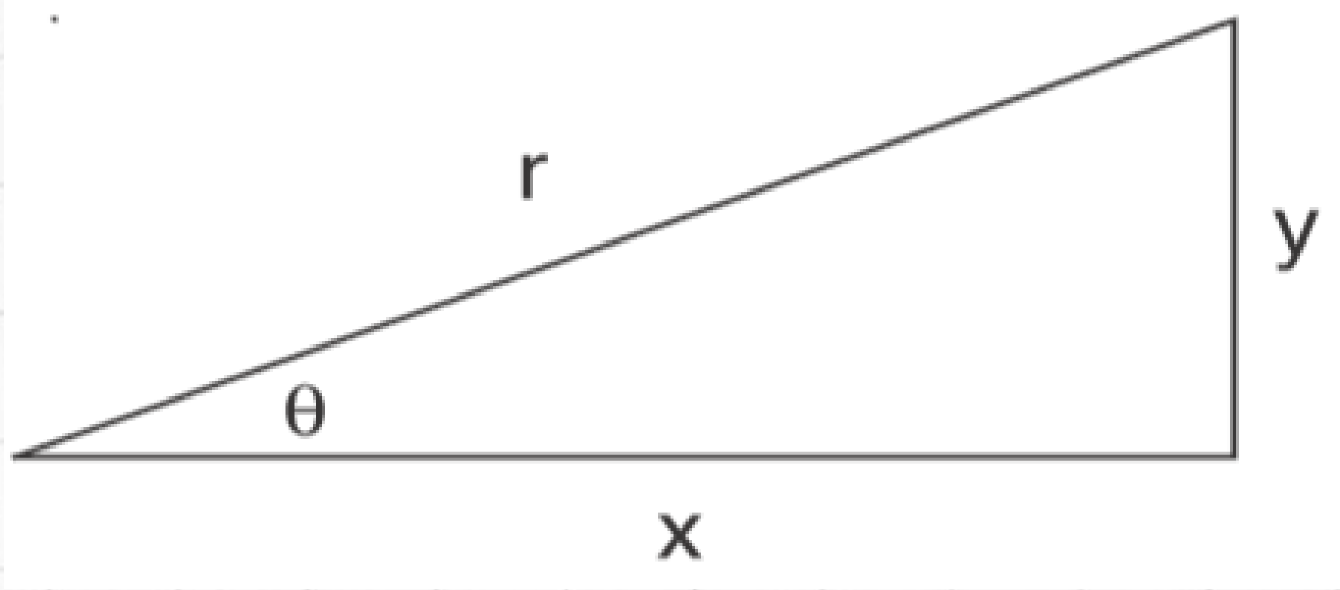
ANALYTIC GEOMETRY & TRIGONOMETRY

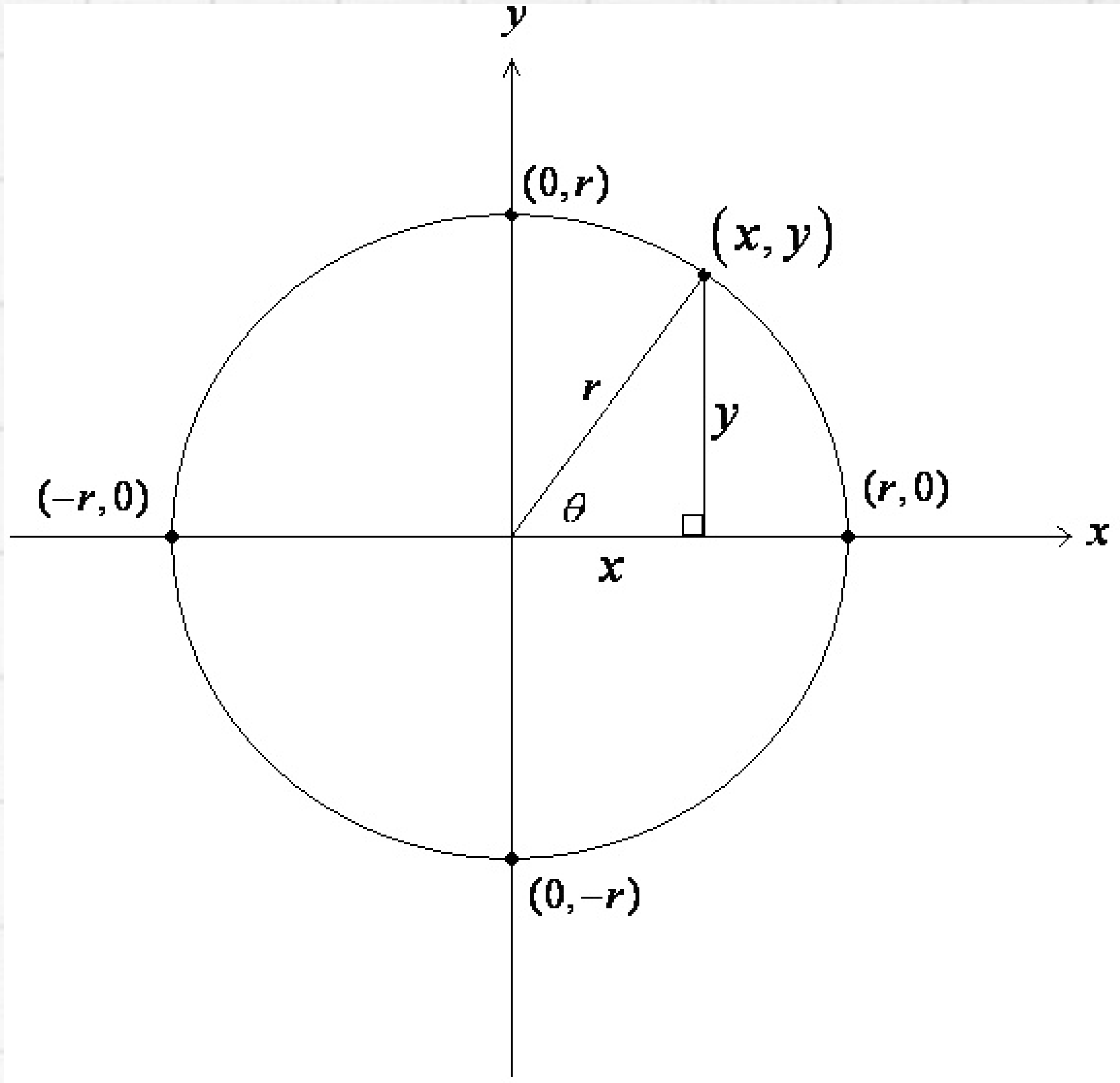
TRIGONOMETRIC FUNCTIONS

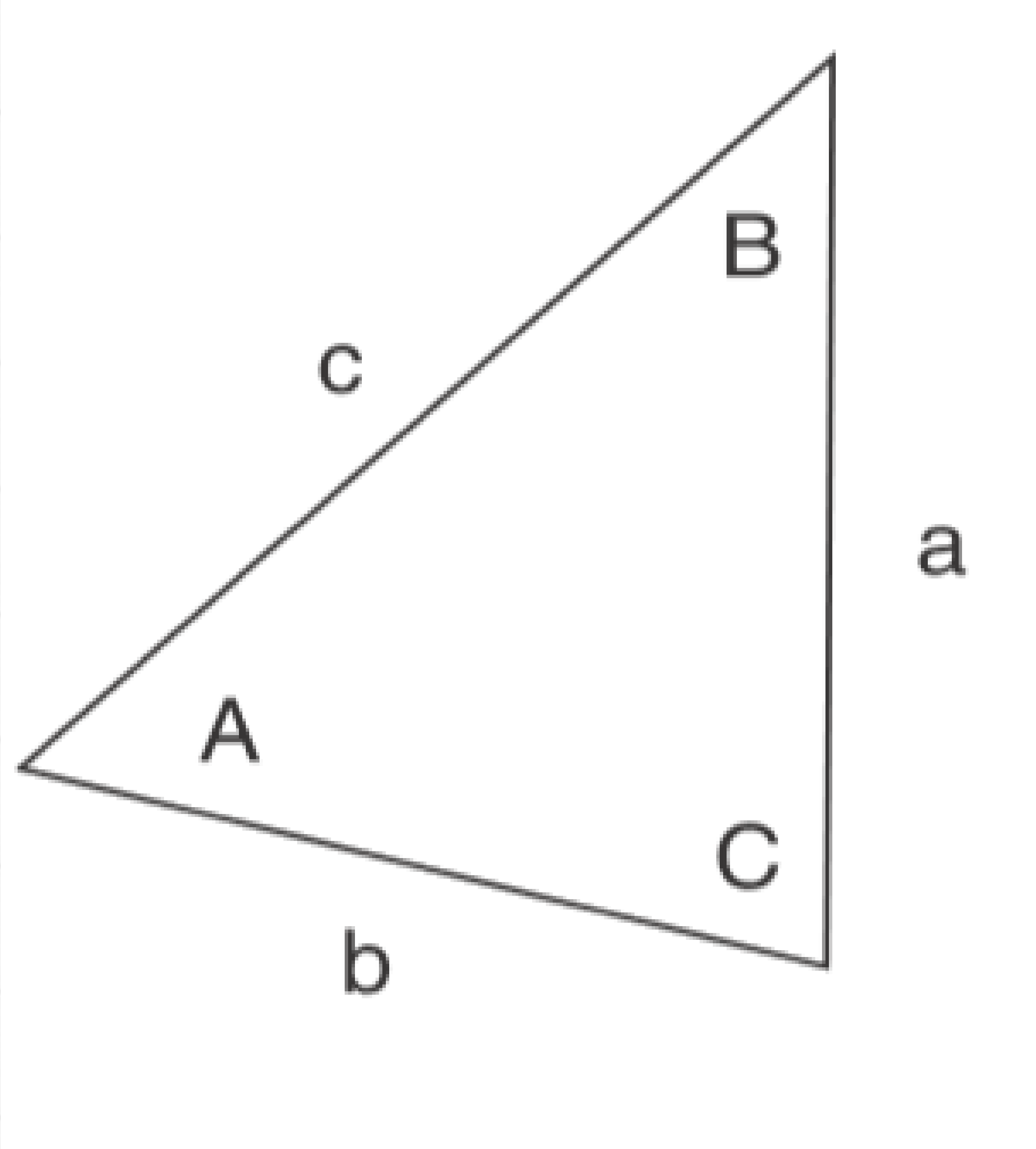
$\sin\theta = y/r \qquad \csc\theta = 1/\sin\theta$

$\cos\theta = x/r \qquad \sec\theta = 1/\cos\theta$

$\tan\theta = y/x \qquad \cot\theta = 1/\tan\theta$







$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$ *Law of Sines*

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$ *Law of Cosines*
$$c^2 = a^2 + b^2 - 2ab\cos C$$

ANALYTIC GEOMETRY & TRIGONOMETRY

TRIGONOMETRIC IDENTITIES

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

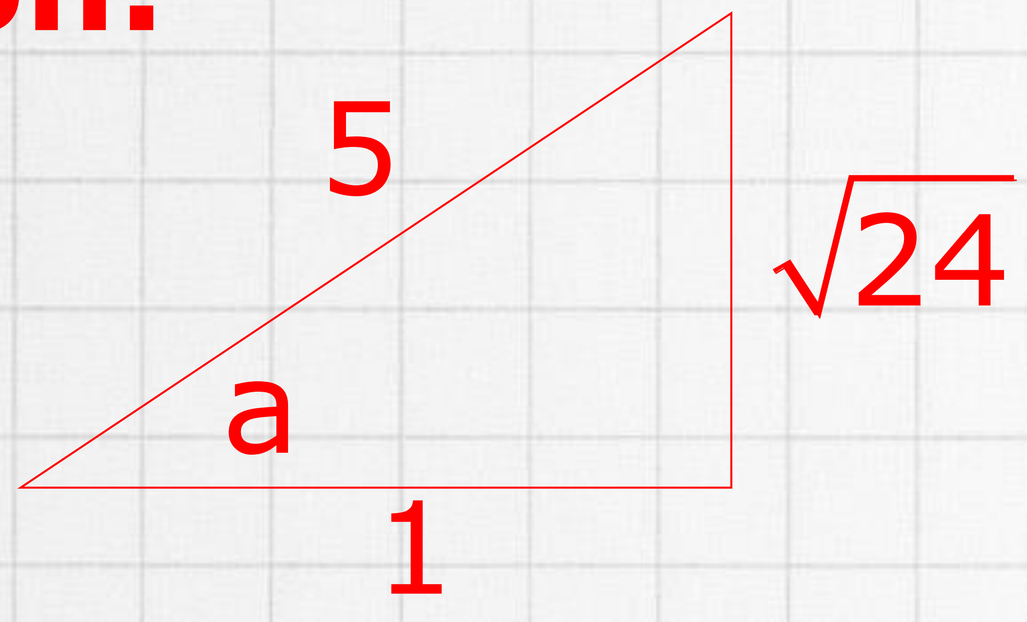
ANALYTIC GEOMETRY & TRIGONOMETRY

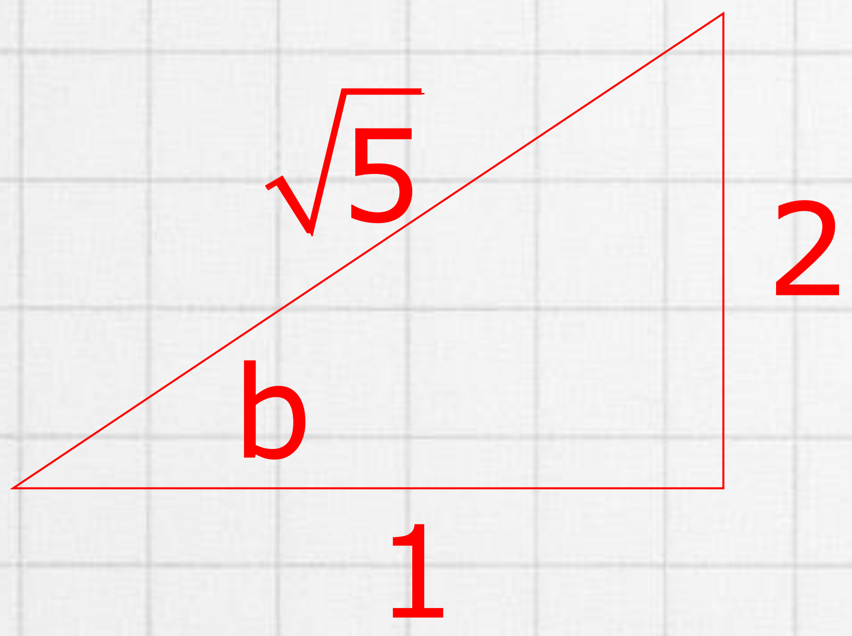
EXAMPLE

Solve the following trigonometry: $\sin\left(\overbrace{\arccos\left(\frac{1}{5}\right)}^{a} - \overbrace{\arctan(2)}^{b}\right)$

(A) 0.46 (B) 0.82 (C) 0.26 (D) 0.62

Solution:





$$\sin a = \frac{\sqrt{24}}{5}$$

$$\cos a = \frac{1}{5}$$

$$\sin b = \frac{2}{\sqrt{5}}$$

$$\cos b = \frac{1}{\sqrt{5}}$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$= \left(\frac{\sqrt{24}}{5}\right)\left(\frac{1}{\sqrt{5}}\right) - \left(\frac{1}{5}\right)\left(\frac{2}{\sqrt{5}}\right)$$

$$= \frac{2\sqrt{6}-2}{5\sqrt{5}} = 0.26$$

(Answer : C)

ANALYTIC GEOMETRY & TRIGONOMETRY

Module 1B – Single-Variable Calculus

COMING UP NEXT…