FUNCTION
- one to one (invertible function)
- one to many (non-invertible function)
- many to one (non-function)

DOMAIN
Set of 'allowed' values of \(x\)

RANGE
Set of \(y\) values associated with a domain.

DOMAIN & RANGE

(a) What is the range of \(f(x) = \frac{3}{x+1} + 2\), domain \(x \neq -1\)?

(b) If \(f(x) = (x+3)^2 - 2\), domain \(x \in \mathbb{R}\), what is the domain of \(f^{-1}(x)\)? Fact: domain of \(f^{-1}(x) = \text{range of } f(x)\)

Parent graph: \(f(x) = x^2\)

\[\begin{align*}
\text{parent graph: } f(x) = x^2 \\
x &\rightarrow \\
&\rightarrow \\
&\rightarrow \\
\end{align*}\]

\(x\) coords change, \(y\) coords change

\[
\begin{align*}
\text{range of } f(x) : \\
\text{domain of } f^{-1}(x) : \\
\end{align*}
\]
**Composite Functions**

**Function**
A mapping of one or more objects in one set to a unique object in another set.

**Composite Function**
A combination of functions formed by making the output of one function become the input of another function.

**Algebraically**

\[ f(x) = \frac{x-1}{2-x}, \quad x \in \mathbb{R}, \quad x \neq 2 \]

(a) Find & simplify \( f^2(x) \)

\[ g(x) = \sqrt{x}, \quad x \in \mathbb{R}, \quad x > 0 \]

(b) Find the range of \( gf(x) \)

\[ f^2(x) = \text{[Expression]} \]

Common exam Q:
(a) \( \text{solve } g(x) = f^{-1}(x) \)
(b) \( \text{solve } fg(a) = a \) (1 mark)

**Notation Alert!**
\( f^2(x) \) does not mean \([f(x)]^2\), it means \(ff(x)\)

- Sketch \( f(x) \) to find range
  - \( 2-x=0 \)
  - \( x \) intercept
  - \( y \) intercept
  - *Key point*
  - Range

- Sketch \( g(x) \) to find range
**Inverse Functions**

1. Find the inverse of \( f(x) = \frac{3x+1}{x-2} \) \( x \in \mathbb{R}, x \neq 2 \)

2. Sketch the inverse of \( f(x) = (x-1)^2 + 4 \) and state its domain.

The inverse only exists if the function is not a function.

\[
\begin{align*}
y &= \quad \quad \\
x &= \quad \quad \\
\end{align*}
\]
Translation of \( y = mx + c \)
- Track both axis intercepts as they are translated
- Draw \( y = mx + c \) and label the axis intercepts
- Reflect the negative part of the line up over the x-axis
- Sketch of \( a' \) parallel to the y-axis about the x-axis (may involve reflection if \( a \) is negative)
- Translate by 'b' up/down

Translation of \( y = |mx + c| + b \)
- Track the origin as it is translated
- Build the journey of \( x \), draw \( y = |x| \)
- Translate \( y = |x| \) by c left/right

Sketch the graph of \( y = 4 - 3|x-1| \), \( x \in \mathbb{R} \)

Translate \( y = 2x - 1 \)

Translate \( y = |x| \)
**WARNING**

| ax + b | doesn't mean (ax+b) and -(ax+b)

It's not that simple!

The algebra will think graphs meet when actually the mod graph doesn't intersect the other graph.

**WHAT TO DO**

Sketch both graphs, look for intersections, then form equations.

**MODULUS EQUATIONS & INEQUALITIES**

\[ f(x) = |x-4| + 6 \]

- \( f(x) = g(x) \) has no real solutions
- Find the range of possible values of \( k \)

**Domain of both functions is \( x \in \mathbb{R} \)**

\[ g(x) = 2x + k \]

- Solve \( f(x) < 4x + 6 \)

**ANS:**

\[ y = x - 4 \]

\[ y = 2x + k \]

\[ 6 = 2(4) + k \]

\[ k = -2 \]

**ANS:**

\[ f(x) < 4x + 6 \]

\[ 3(-x + 4) + 6 = 4x + 6 \]
The line $2x-y-k=0$ is tangent to the curve $x=2t-t^3$, $y=2-t^2$ at the point where $t=1$. Prove that the tangent intersects the curve when $t=\frac{-2-k}{2}$ and at no other point.

Finding intersections of a cartesian curve & a parametric curve.

**Range**
All new $y$ values coming from the permitted $t$ values
ie) range of $y(t)$

**Domain**
All the $x$ values coming from the permitted $t$ values
ie) range of $x(t)$

**Tangent**
$m(x-a)=y-b$

**KEY IDEA**
About a tangent

$2(2t-t^3)-(2-t^2)-k=0$

$4t-2t^3-2+t^2-k=0$

$2t^3-t^2-4t+2+k=0$

$(t-1)(\quad)=0$

We know $(t-1)$ is a factor because...

$(t-1)^2(\quad)=0$

$(t^2-2t+1)(2t+2+k)=0$
Find the Cartesian equation of each of these functions

(a) \[ \begin{align*}
    x &= t - \frac{1}{t} \\
    y &= t + \frac{1}{t}
\end{align*} \]

(b) \[ \begin{align*}
    x &= 3 \cos\theta \\
    y &= 4 \sec\theta + 1
\end{align*} \]

(c) \[ \begin{align*}
    x &= \tan\theta \\
    y &= \cot^2\theta
\end{align*} \]

**Clever Tricks**

- **Trig (angles match)**
  - Circle the trig functions in the equations, and their reciprocals.
  - Choose 3 write down the Pythagorean formulae.
  - Sub into the trig formula.

- **Trig (angles don't match)**
  - Make angles match using trig identities.

**NOT Trig**

- Make one say \( t = \ldots \) and sub into the other one.
- \( \sin^2 \theta + \cos^2 \theta = 1 \) and \( \sec^2 \theta - \tan^2 \theta = 1 \)
- Use the Pythagorean hexagon.