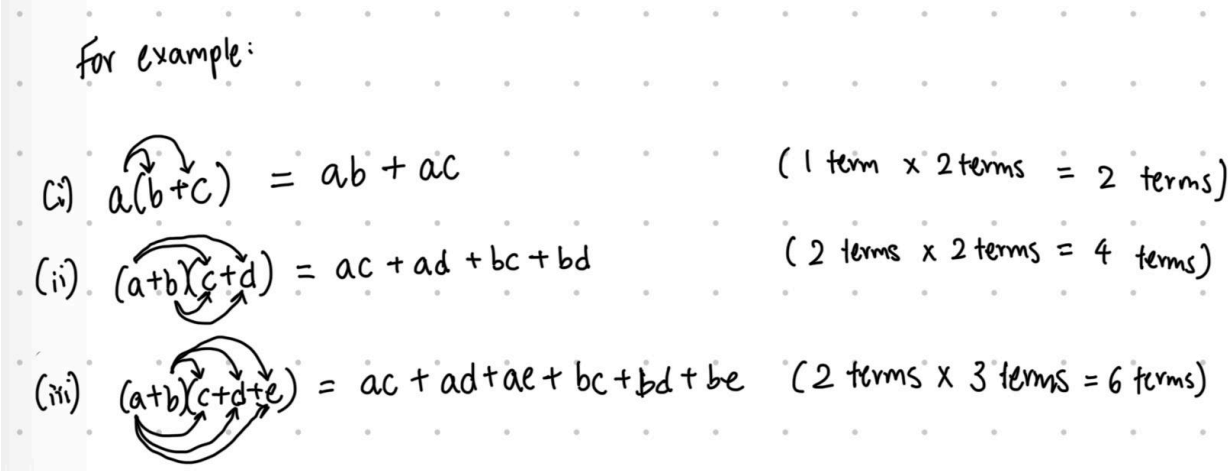
NOTES:

3: Expansion and Factorization of Algebraic Expressions

Expansion of Algebraic Expressions:

1, To perform the expansion of two algebraic expressions in a systematic manner, we multiply each term of the first expression by each term of the second expression. We start with the first term in the first expression and repeat the process for the rest of the terms in the first expression.



Factorization by Grouping:

2. To factorize algebraic expressions, we often need to identify the common factors. At times, the common factors can be identified by grouping the terms of an algebraic expression. Some examples of factorization done by grouping are shown in the table below.

Type	Method	Example
1	Group the terms in the original expression to find the common factor	$m^2 - mn + xm - xn = m(m - n) + x(m - n)$ $= (m - n)(m + x)$
2	Regroup the terms to find the common factor	$x^2 + yz + xz + xy = x^2 + xz + xy + yz$ $= x(x + z) + y(x + z)$ $= (x + z)(x + y)$
3	Change the sign of the factor if necessary, e.g. $b - a = -(a - b)$	$ap - bp + bx - ax = p(a - b) + x(b - a)$ $= p(a - b) - x(a - b)$ $= (a - b)(p - x)$

Quadratic Expression:

3. The general form of **a quadratic expression** in one variable, say x, is $ax^2 + bx + c$, where a, b, and c are constants and $a \neq 0$.

4. In the quadratic expression $ax^2 + bx + c$, the **degree**, which denotes the highest power of x, is 2. The constants a and b are known as **coefficients** of x^2 and x respectively while constant c is known as the **constant term**.

For example:

Consider the quadratic expression $2x^2 - 3x + 5$. The coefficient of x^2 is 2, the coefficient of x is -3 and the constant term is 5.

Expansion of Quadratic Expressions:

5. If a, b, c, and d are constants, then

(i) $ax \times bx = a \times b \times x^2 = abx^2$

For example:

(a) $2x \times 3x = 6x^2$

(b) $-3x \times (-5x) = 15x^2$

(ii) $d(ax^2 + bx + c) = adx^2 + bdx + cd$

(iii) $ax(bx + c) = abx^2 + acx$

(iv) $(ax + b)(cx + d) = ax(cx + d) + b(cx + d)$

$= acx^2 + adx + bcx + bd$

$= acx^2 + (ad + bc)x + bd$

Factorization of Quadratic Expressions:

6. Factorization of an algebraic expression is the process of expressing it into a product of two or more factors. It is the reverse process of expansion. When we factorize a quadratic expression, we will obtain at most 2 linear factors, ie. $ax^2 + bx + c = (ex + f)(gx + h)$.

7. There are generally 2 methods of performing factorization for quadratic expressions:
   (a) 'Multiplication frame' method
   (b) 'Cross-multiplication' method

[not taught in these notes]

Factorization of Trinomials Using Easiest Method:

8. Watch this video to understand: https://youtu.be/c40QJYnKltM

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