Matrix Coding Activity

NAME:_____ DATE:_____

If a message can be coded by multiplying a matrix that represents it times an encoding matrix, then how can the coded message be decoded?

CREATION

1. Create a message that is 10 characters in length. Count any spaces as a character. Use punctuation at the end if your message needs a tenth character.

___ ___ ___ ___ ___ ___ ___ ___ ___ ___

2. Use the key below to change your message to its numeric equivalent.

0 = _ ; 1 = A ; 2 = B ; 3 = C ; 4 = D ; 5 = E ; 6 = F ; 7 = G ; 8 = H ; 9 = I ; 10 = J ;
11 = K ; 12 = L ; 13 = M ; 14 = N ; 15 = O ; 16 = P ; 17 = Q ; 18 = R ; 19 = S ;
20 = T ; 21 = U ; 22 = V ; 23 = W ; 24 = X ; 25 = Y ; 26 = Z ; 27 = . ; 28 = ?; 29 = !

___ ___ ___ ___ ___ ___ ___ ___ ___ ___

3. To encode the message, right multiply the 1 x 2 matrix formed by each successive pair of numbers by the encoding matrix: $\begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix}$.

4. Use the results from step 3 to write your encoded message as a sequence of numbers.

___ ___ ___ ___ ___ ___ ___ ___ ___ ___

Matrix Coding Activity

0 = _ ; 1 = A ; 2 = B ; 3 = C ; 4 = D ; 5 = E ; 6 = F ; 7 = G ; 8 = H ; 9 = I ; 10 = J ;
11 = K ; 12 = L ; 13 = M ; 14 = N ; 15 = O ; 16 = P ; 17 = Q ; 18 = R ; 19 = S ;
20 = T ; 21 = U ; 22 = V ; 23 = W ; 24 = X ; 25 = Y ; 26 = Z ; 27 = . ; 28 = ?; 29 = !

INVESTIGATION

1. Try to decode the following message if the key above and the encoding matrix, $\begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix}$, was used to encode it.

   4 52

2. Duplicate your process from #1 above to decode the message below. If the result is not readable troubleshoot your processes.

   20 60 -10 96

Matrix Coding Activity

APPLICATION

1. Use the information learned from the investigation process to decode your message. Be sure to show all your work.

Your encoded message: ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

Your decoded message: ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

2. Decode my message: 12 28 12 56 -18 36 9 66 -14 48

My decoded message: ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

What assumptions did you have to make to be able to decode my message?

_____

_____

MathLight

Matrix Coding Activity

CONCLUSIONS

Without changing the key, fully explain at least two ways of increasing the difficulty of "cracking" a coded message. _____

_____

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MathLight

Matrix Coding Activity

NAME: Answer Key DATE: _____

If a message can be coded by multiplying a matrix that represents it times an encoding matrix, then how can the coded message be decoded?

CREATION

1. Create a message that is 10 characters in length. Count any spaces as a character. Use punctuation at the end if your message needs a tenth character.

M	A	T	H	___	R	U	L	E	S

(answers will vary)

2. Use the key below to change your message to its numeric equivalent.

0 = _ ; 1 = A ; 2 = B ; 3 = C ; 4 = D ; 5 = E ; 6 = F ; 7 = G ; 8 = H ; 9 = I ; 10 = J ;
11 = K ; 12 = L ; 13 = M ; 14 = N ; 15 = O ; 16 = P ; 17 = Q ; 18 = R ; 19 = S ;
20 = T ; 21 = U ; 22 = V ; 23 = W ; 24 = X ; 25 = Y ; 26 = Z ; 27 = . ; 28 = ?; 29 = !

13	1	20	8	0	18	21	12	5	19

3. To encod the message, right multiply the 1 x 2 matrix formed by each successive pair of numbers by the encoding matrix: $\begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix}$.

$[13 \;\; 1] \cdot \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix}$

$\begin{vmatrix} (13 - 1) & (26 + 2) \end{vmatrix}$

$[12 \;\; 28]$

$[20 \;\; 8] \cdot \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix}$

$\begin{vmatrix} (20 - 8) & (40 + 16) \end{vmatrix}$

$[12 \;\; 56]$

$[0 \;\; 18] \cdot \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix}$

$\begin{vmatrix} (0 - 18) & (0 + 36) \end{vmatrix}$

$[-18 \;\; 36]$

$[21 \;\; 12] \cdot \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix}$

$\begin{vmatrix} (21 - 12) & (42 + 24) \end{vmatrix}$

$[9 \;\; 66]$

$[5 \;\; 19] \cdot \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix}$

$\begin{vmatrix} (5 - 19) & (10 + 38) \end{vmatrix}$

$[-14 \;\; 48]$

4. Use the results from step 3 to write your encoded message as a sequence of numbers.

12	28	12	56	-18	36	9	66	-14	48

Matrix Coding Activity

0 = _ ; 1 = A ; 2 = B ; 3 = C ; 4 = D ; 5 = E ; 6 = F ; 7 = G ; 8 = H ; 9 = I ; 10 = J ;
11 = K ; 12 = L ; 13 = M ; 14 = N ; 15 = O ; 16 = P ; 17 = Q ; 18 = R ; 19 = S ;
20 = T ; 21 = U ; 22 = V ; 23 = W ; 24 = X ; 25 = Y ; 26 = Z ; 27 = . ; 28 = ?; 29 = !

INVESTIGATION

1. Try to decode the following message if the key above and the encoding matrix, $\begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix}$, was used to encode it.

4 52

First find the inverse of the encoding matrix.

$$E^{-1} = \frac{1}{|E|}\begin{vmatrix} 2 & -2 \\ 1 & 1 \end{vmatrix}$$

$|E| = 2 - (-2) = 4$ $\quad E^{-1} = \frac{1}{4}\begin{vmatrix} 2 & -2 \\ 1 & 1 \end{vmatrix} = \begin{vmatrix} ½ & -½ \\ ¼ & ¼ \end{vmatrix}$

$[4 \;\; 52] \bullet \begin{vmatrix} ½ & -½ \\ ¼ & ¼ \end{vmatrix}$

$|(2 + 13)(-2 + 13)|$

$[15 \;\; 11]$ = **OK**

2. Duplicate your process from #1 above to decode the message below. If the result is not readable troubleshoot your processes.

20 60 -10 96

$[20 \;\; 60] \bullet \begin{vmatrix} ½ & -½ \\ ¼ & ¼ \end{vmatrix}$ $\qquad$ $[-10 \;\; 96] \bullet \begin{vmatrix} ½ & -½ \\ ¼ & ¼ \end{vmatrix}$

$|(10 + 15) \;\; (-10 + 15)|$ $\qquad$ $|(-5 + 24) \;\; (5 + 24)|$

$[25 \;\; 5]$ $\qquad$ $[19 \;\; 29]$

Y E S !

Matrix Coding Activity

APPLICATION

1. Use the information learned from the investigation process to decode your message. Be sure to show all your work.

Your encoded message: 12 28 12 56 -18 36 9 66 -14 48

(answers will vary)

$$[12 \;\; 28] \cdot \begin{vmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} \end{vmatrix}$$

$$\begin{vmatrix} (6 + 7) & (-6 + 7) \end{vmatrix}$$

$$[13 \;\; 1]$$

$$[12 \;\; 56] \cdot \begin{vmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} \end{vmatrix}$$

$$\begin{vmatrix} (6 + 14) & (-6 + 14) \end{vmatrix}$$

$$[20 \;\; 8]$$

$$[-18 \;\; 36] \cdot \begin{vmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} \end{vmatrix}$$

$$\begin{vmatrix} (-9 + 9) & (9 + 9) \end{vmatrix}$$

$$[0 \;\; 18]$$

$$[9 \;\; 66] \cdot \begin{vmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} \end{vmatrix}$$

$$\begin{vmatrix} (9/2 + 33/2) & (-9/2 + 33/2) \end{vmatrix}$$

$$[21 \;\; 12]$$

$$[-14 \;\; 48] \cdot \begin{vmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} \end{vmatrix}$$

$$\begin{vmatrix} (-7 + 12) & (7 + 12) \end{vmatrix}$$

$$[5 \;\; 19]$$

Your decoded message: M A T H ___ R U L E S

2. Decode my message: 12 28 12 56 -18 36 9 66 -14 48

$$[12 \;\; 28] \cdot \begin{vmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} \end{vmatrix}$$

$$\begin{vmatrix} (6 + 7) & (-6 + 7) \end{vmatrix}$$

$$[13 \;\; 1]$$

$$[12 \;\; 56] \cdot \begin{vmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} \end{vmatrix}$$

$$\begin{vmatrix} (6 + 14) & (-6 + 14) \end{vmatrix}$$

$$[20 \;\; 8]$$

$$[-18 \;\; 36] \cdot \begin{vmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} \end{vmatrix}$$

$$\begin{vmatrix} (-9 + 9) & (9 + 9) \end{vmatrix}$$

$$[0 \;\; 18]$$

$$[9 \;\; 66] \cdot \begin{vmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} \end{vmatrix}$$

$$\begin{vmatrix} (9/2 + 33/2) & (-9/2 + 33/2) \end{vmatrix}$$

$$[21 \;\; 12]$$

$$[-14 \;\; 48] \cdot \begin{vmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} \end{vmatrix}$$

$$\begin{vmatrix} (-7 + 12) & (7 + 12) \end{vmatrix}$$

$$[5 \;\; 19]$$

My decoded message: M A T H ___ R U L E S

What assumptions did you have to make to be able to decode my message?

The assumption that the key that changed letters to numbers was the same and that the encoding matrix was the same.

Matrix Coding Activity

CONCLUSIONS

Without changing the key, fully explain at least two ways of increasing the difficulty of "cracking" a coded message. The encoding matrix could contain two, three, four or more digit numbers thus increasing the permutations of possible encoded messages. It would also make it more difficult to determine the decoding matrix (the inverse of the encoding matrix).

If the numbers were grouped into three's the resulting matrix would be a 1 x 3 and the encoding matrix would have to be a 3 x 3. This would increase the permutations of possible encoded messages. It would also make it more difficult to determine the decoding matrix (the inverse of the encoding matrix).

If the numbers were grouped into four number groups then the resulting matrix would be a 1 x 4 and the encoding matrix would have to be a 4 x 4. This would greatly increase the permutations of possible encoded messages. It would also greatly increase the difficulty of determining the decoding matrix (the inverse of the encoding matrix).