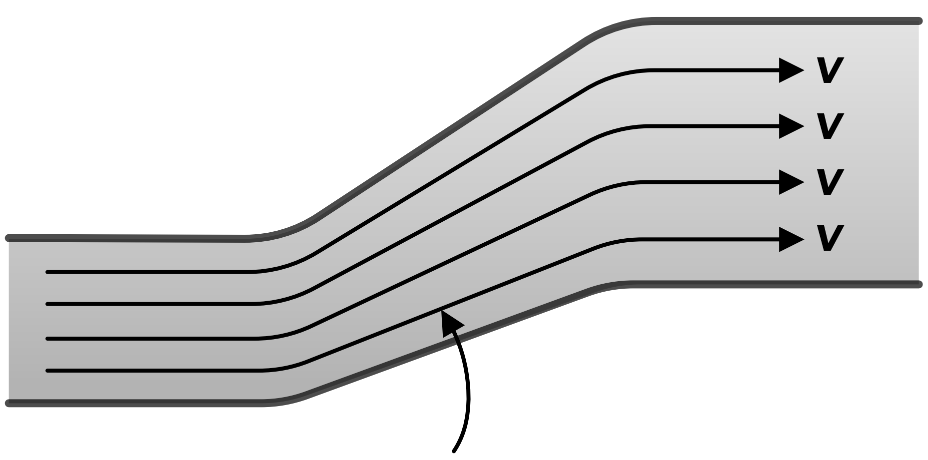
FLOW & BERNOULLI'S EQUATION

Fluid Flow

- The previous sections on fluids have described **hydrostatics**, which is when a fluid is static and not moving.
- This section will focus on **hydrodynamics**, which is when a fluid is moving or "flowing". We're going to assume the fluids we're working with are ideal fluids which we covered in a previous section, but we're going to add one more definition regarding laminar flow.
- An **ideal fluid**:
  1. Is **completely incompressible**: it does not change volume or density when a force is applied to it, regardless of the pressure of the fluid. The particles in an ideal fluid do not get closer or farther apart.
  2. Has **no viscosity**: it does not resist flow and there is no friction between the fluid particles or between the particles and the container.
  3. Moves with **laminar flow**: all of the fluid flows smoothly in one direction, parallel to the tube (the opposite of laminar flow would be turbulent flow).

Laminar flow

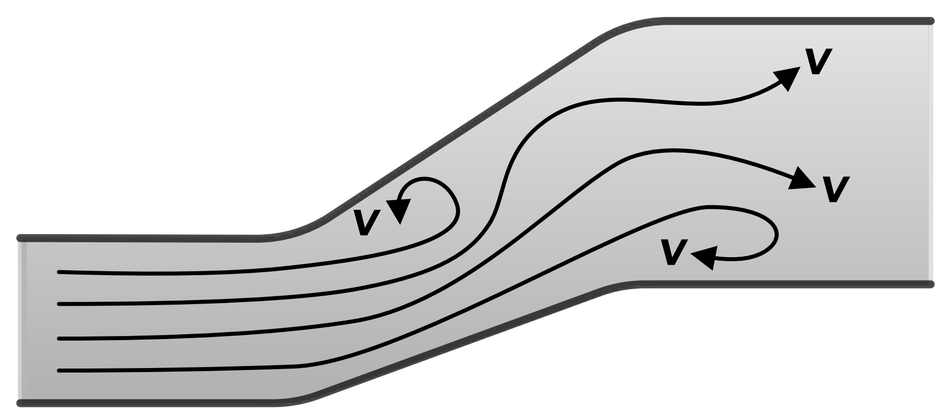
The fluid particles move smoothly in one direction, parallel to the tube



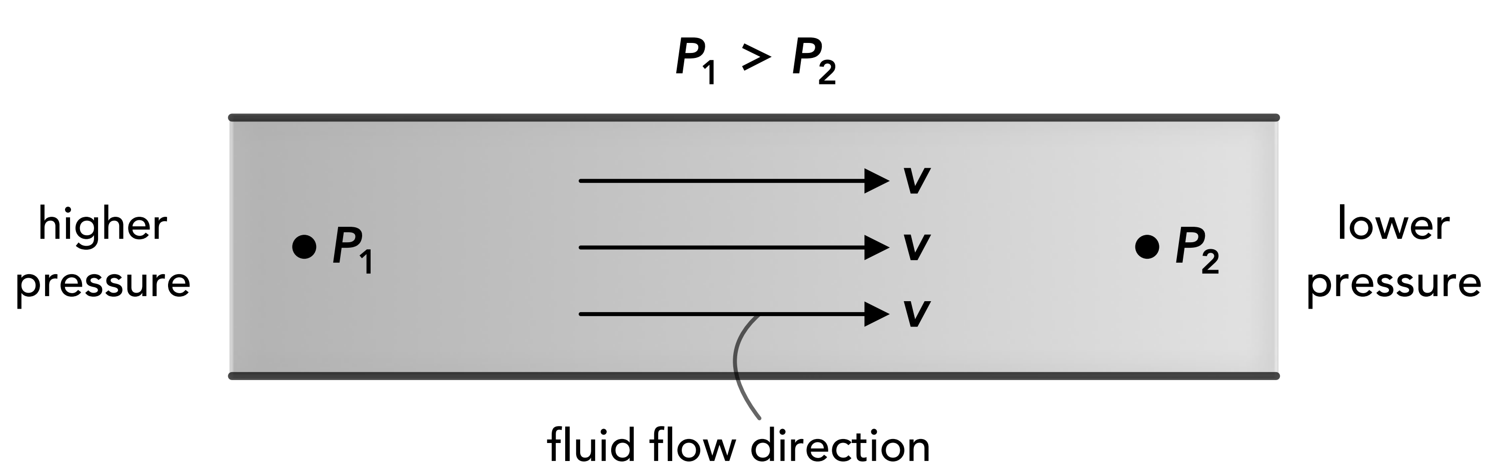
we're going to assume all fluid flow is laminar for this course

Turbulent flow

The fluid particles generally move in one direction but they also follow irregular flow patterns



- Fluid flow is caused by **a difference in pressure** between two points in the fluid.
- A fluid will flow from **higher pressure to lower pressure**.



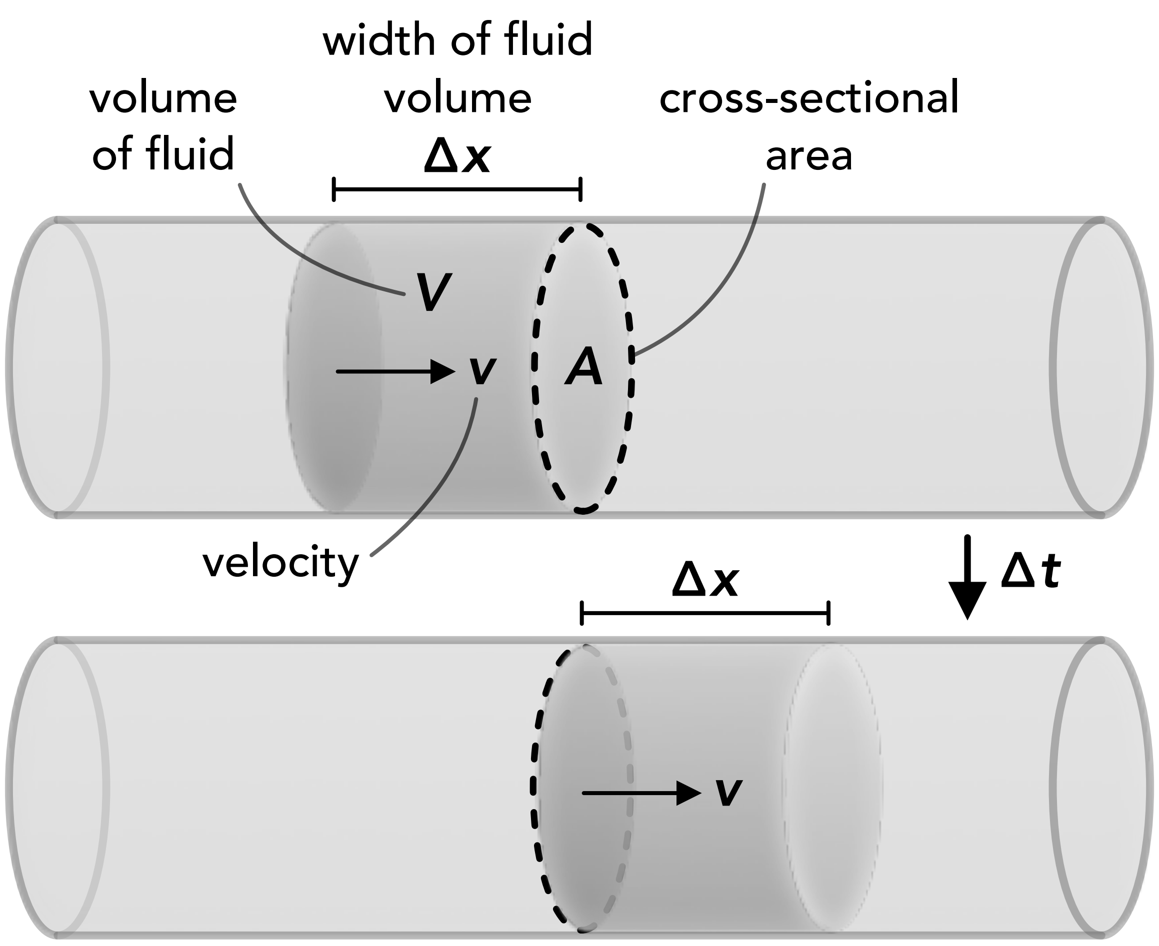
Flow Rate and Conservation of Mass

Values		Unit	Name
g	9.8	$\frac{m}{s^2}$	gravitational acceleration

Variables		SI Unit
V	volume	m^3
v	velocity	$\frac{m}{s}$
A	area	m^2
t	time	s
P	pressure	$Pa = \frac{N}{m^2}$
ρ	density	$\frac{kg}{m^3}$
m	mass	kg
y	height	m

- The size of the tube (or any container) that a fluid is flowing through can change from one end to the other, and different tubes will be different sizes. So instead of only describing the velocity of a fluid, it will be useful to describe the flow rate.
- The **flow rate** of a fluid is the volume of fluid that passes by a point per unit of time, and has an SI unit of m^3/s.
- The flow rate is equal to the velocity of the fluid passing through some cross-sectional area multiplied by that area.

A volume of fluid flows through an area over a period of time



The velocity of the fluid volume is equal to its displacement divided by the period of time

$$v = \frac{\Delta x}{\Delta t}$$

fluid volume:

$$V = A\Delta x$$

$$v = \frac{V}{A\Delta t} \leftarrow \Delta x = \frac{V}{A}$$

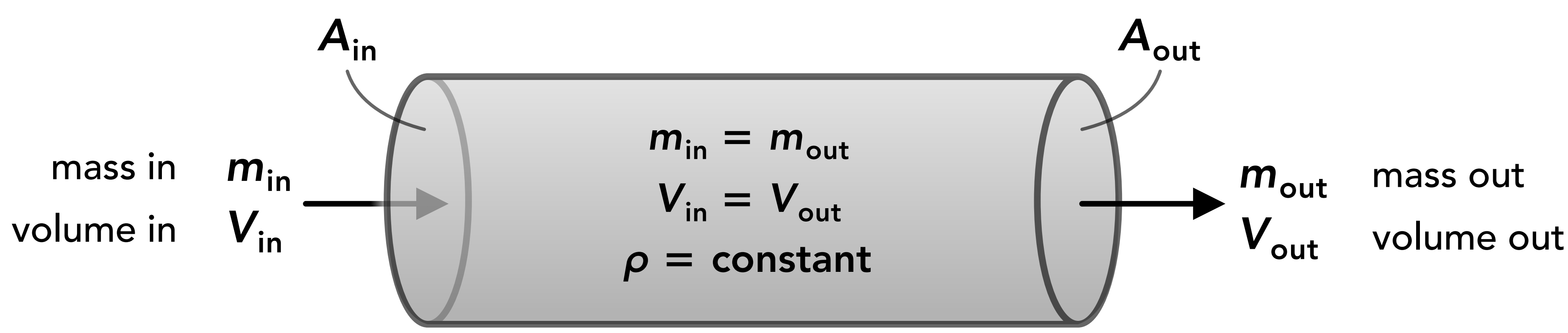
$$Av = \frac{V}{\Delta t}$$

Flow rate

$$\frac{V}{\Delta t} = Av$$

- When studying fluid flow in a tube we assume the tube is completely filled by the fluid. We also assume the fluid is an ideal fluid so it is incompressible (the density of the fluid does not change, and the volume of fluid cannot be compressed or expanded).
- We can also apply **the law of conservation of mass** which states that mass cannot be created or destroyed, it can only be moved. For fluid flow, this means that **the mass of fluid that enters a tube must equal the mass of fluid that exits a tube** during a given period of time.
- Because the fluid fills the tube and is incompressible, this also means that **the volume of fluid that enters a tube must equal the volume of fluid that exits a tube** during a given period of time. Put another way, **the flow rate into a tube must equal the flow rate out of a tube** (or into/out of any section of a tube).
- We can use this relationship to find the velocity of the fluid at different points in a tube that have different cross-sectional areas. The fluid will flow slower through larger cross-sectional areas and faster through smaller cross-sectional areas.

Fluid mass is conserved within a tube, no mass can be added or removed within the tube. The fluid density is constant (we assume the fluid is incompressible) so the fluid volume is also conserved.



$$m_{in} = m_{out}$$

fluid density is constant: $\rho = \dfrac{m}{V} \longrightarrow V_{in} = V_{out}$

$$\frac{V_{in}}{\Delta t} = \frac{V_{out}}{\Delta t}$$

flow rate: $\dfrac{V}{\Delta t} = Av \longrightarrow A_{in}v_{in} = A_{out}v_{out}$

Conservation of flow rate

$$A_1 v_1 = A_2 v_2$$

the conservation of flow rate applies to any two points in the flow, not just the in and out points

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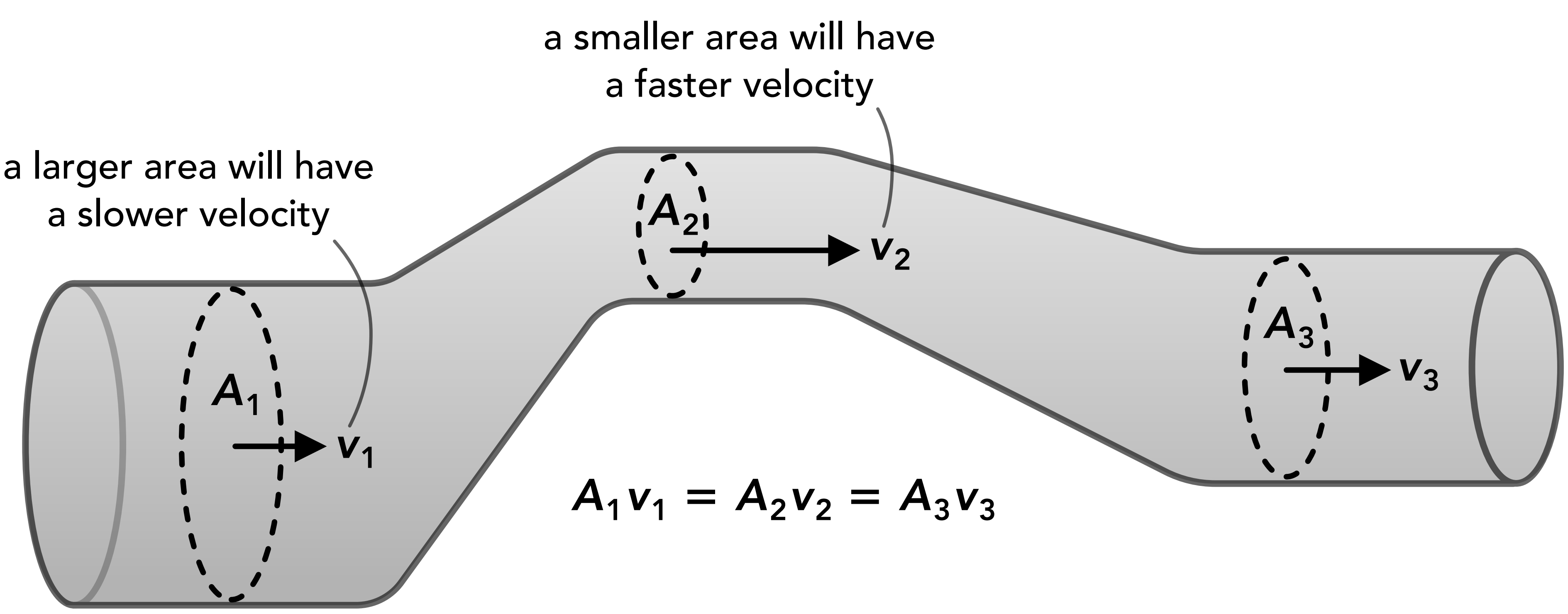
The flow rate is the same at every point in the flow



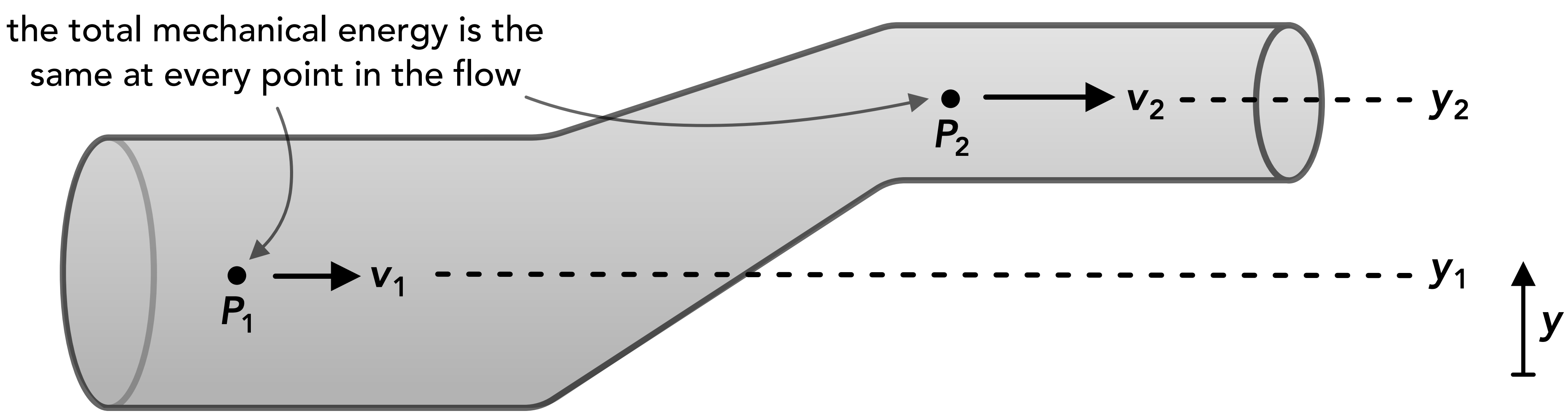
Bernoulli's Equation and Conservation of Energy

- The equation we got from the conservation of mass is useful, but it's not enough to describe everything about fluid flow including pressure and a change in tube height. To do that we need Bernoulli's equation.
- **Bernoulli's equation** relates the pressure, height and velocity of a fluid at different points and can be derived from the conservation of mechanical energy of the fluid.
- The equation tells us that the sum of the three terms shown below is constant throughout the fluid, or we can say the sum of the three terms at one point is equal to the sum of the terms at another point.
- We can use either absolute pressure or gauge pressure in the equation, but we have to use the same type on each side of the equation.

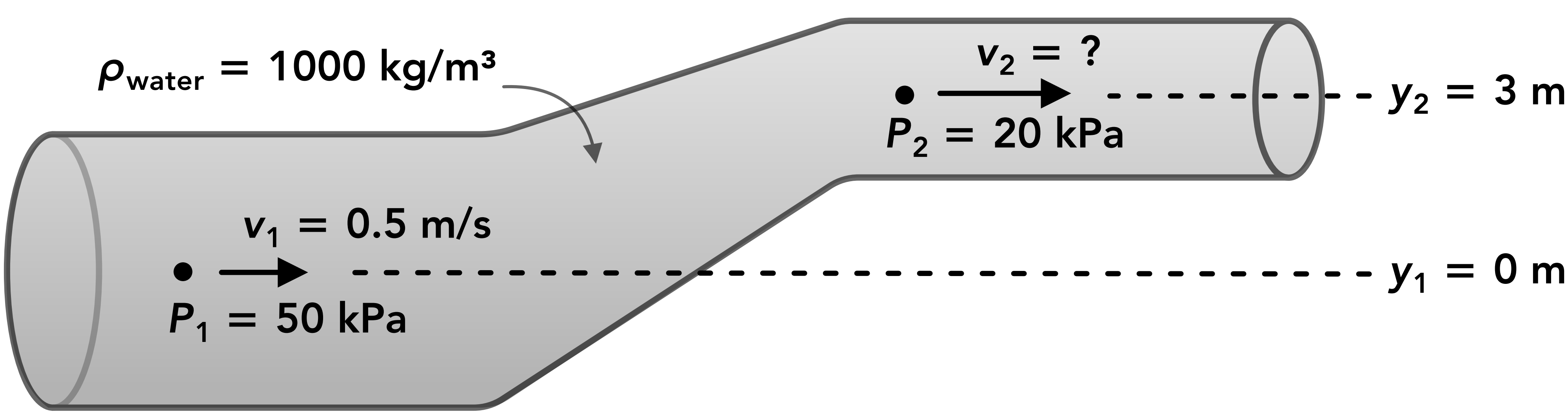
Bernoulli's equation

$$P + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$$

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$



Example: What is the speed of the water at point 2?
(pressures are gauge pressures)



$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$(50{,}000\text{ Pa}) + (1000\text{ kg/m}^3)g(0\text{ m}) + \frac{1}{2}(1000\text{ kg/m}^3)(0.5\text{ m/s})^2 = (20{,}000\text{ Pa}) + (1000\text{ kg/m}^3)g(3\text{ m}) + \frac{1}{2}(1000\text{ kg/m}^3)v_2^2$$

$$1.2\text{ m/s} = v_2$$

- Bernoulli's equation can be derived by applying **the law of conservation of energy** to the fluid as it moves from one point to another.
- The total amount of gravitational potential energy and kinetic energy, plus the work done on the fluid due to the pressure force, is constant throughout the flow. We can divide each energy term by volume to get the terms in Bernoulli's equation so that the equation includes pressure.

work done by pressure force (W) · gravitational potential energy (U_g) · kinetic energy (K)

Conservation of energy of the fluid: $W + U_g + K = \text{constant}$

$$W + mgy + \frac{1}{2}mv^2 = \text{constant}$$

Rewrite work term using pressure: $W = F\Delta x = \frac{F}{A}A\Delta x = PV$

$$PV + mgy + \frac{1}{2}mv^2 = \text{constant}$$

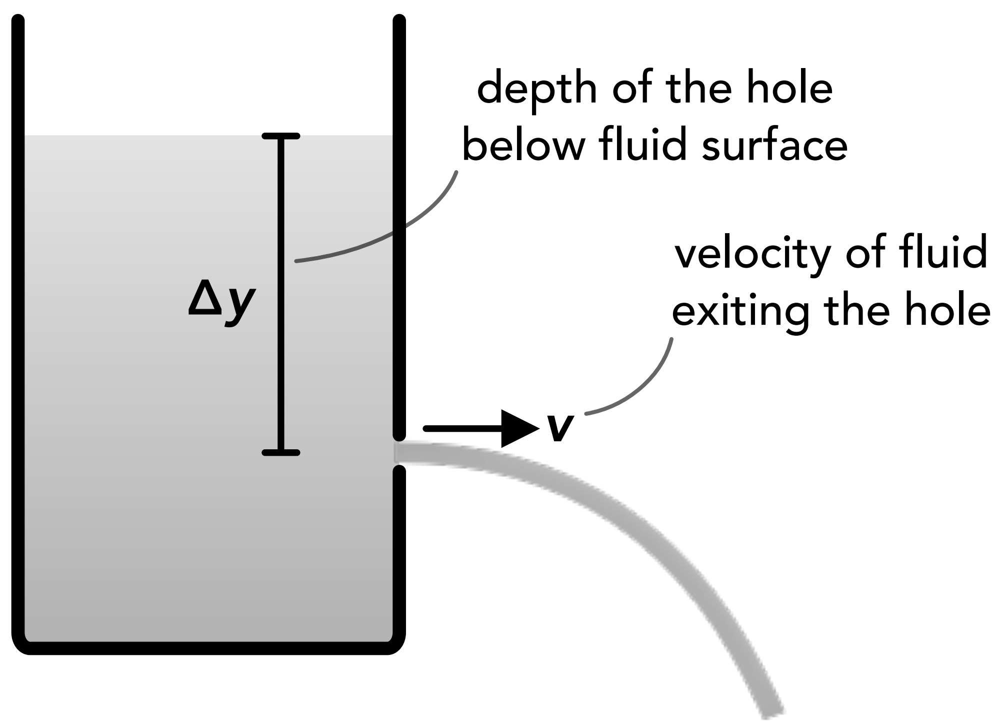
Divide each term by volume: $P = \frac{PV}{V}$ $\rho = \frac{m}{V}$ $\rho = \frac{m}{V}$

Bernoulli's equation: $P + \rho gy + \frac{1}{2}\rho v^2 = \text{constant}$

"work" term · "gravitational potential energy" term · "kinetic energy" term

Torricelli's Theorem

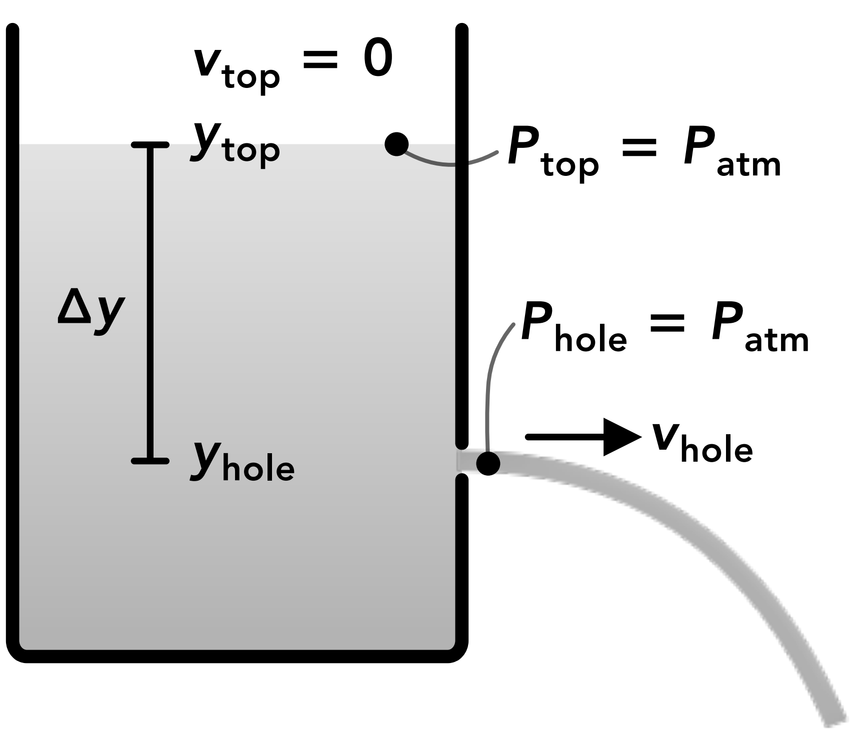
- **Torricelli's theorem** relates the velocity of a fluid exiting a hole in a container to the depth of the hole below the surface of the fluid.
- The pressure in the fluid increases with depth so the velocity of the fluid exiting the hole will be faster at a greater depth.
- This theorem is sometimes combined with kinematics and projectile motion to find things like the distance traveled by the stream of fluid exiting the hole.



Torricelli's theorem

$$v = \sqrt{2g\Delta y}$$

- This is just a specific application of Bernoulli's equation. We assume the velocity of the fluid at the top surface is zero, and that the fluid at the top surface and at the hole are both at the same atmospheric pressure.



$$P_{top} + \rho g y_{top} + \frac{1}{2}\rho v_{top}^2 = P_{hole} + \rho g y_{hole} + \frac{1}{2}\rho v_{hole}^2$$

$$\downarrow P_{top} = P_{hole} = P_{atm}$$

$$\rho g y_{top} + \frac{1}{2}\rho v_{top}^2 = \rho g y_{hole} + \frac{1}{2}\rho v_{hole}^2$$

$$\downarrow v_{top} = 0$$

$$\rho g y_{top} = \rho g y_{hole} + \frac{1}{2}\rho v_{hole}^2$$

$$g y_{top} = g y_{hole} + \frac{1}{2} v_{hole}^2$$

$$g(y_{top} - y_{hole}) = \frac{1}{2} v_{hole}^2$$

$$2g(y_{top} - y_{hole}) = v_{hole}^2$$

$$\sqrt{2g\Delta y} = v_{hole}$$