AEM questions are taken from past exam papers - they have been carefully chosen to represent a typical exam question at each level of difficulty. If you can do these questions, you’re ready to move onto past papers for this topic.

APPRENTICE

Find \( \int \left( 2x^5 - \frac{1}{4x^3} - 5 \right) \, dx \) giving each term in its simplest form.

EXPERT

a. Find \( \int \left( \frac{36}{x^2} + ax \right) \, dx \), where \( a \) is a constant.

b. Hence, given that \( \int_1^3 \left( \frac{36}{x^2} + ax \right) \, dx = 16 \), find the value of the constant \( a \).

MASTER

A curve with equation \( y = f(x) \) passes through the point \((4,9)\).

Given that \( f'(x) = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2, x > 0 \)

a. find \( f(x) \), giving each term in its simplest form.

Point \( P \) lies on the curve.
The normal to the curve at \( P \) is parallel to the line \( 2y + x = 0 \)

b. Find the \( x \) coordinate of \( P \).
AS#39 AREAS UNDER CURVES

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APPRENTICE

The region bounded by the curve \( y = 7x + 6 - \frac{1}{x^2} \), the x-axis and the lines \( x = 1 \) and \( x = 2 \) lies above the x-axis. Show that the area of the region is 16.

EXPERT

The cubic polynomial \( f(x) \) is defined by \( f(x) = x^3 - 19x + 30 \).

a. Given that \( x = 2 \) is a root of the equation \( f(x) = 0 \), express \( f(x) \) as a product of 3 linear factors.

b. Use integration to find the exact value of \( \int_{-5}^{3} f(x) \, dx \).

c. Explain with the aid of a sketch why the answer in part (b) does not give the area enclosed by the curve \( y = f(x) \) and the x-axis for \(-5 \leq x \leq 3\).

MASTER

The Figure shows a sketch of part of the curve with equation

\[ y = 4x^3 + 9x^2 - 30x - 8, \quad -0.5 \leq x \leq 2.2 \]

The curve has a turning point at the point \( A \).

a. Using calculus, show that the x coordinate of \( A \) is 1

The curve crosses the x-axis at the points \( B (2,0) \) and \( C \left( -\frac{1}{4}, 0 \right) \).

The finite region \( R \), shown shaded in Figure 1, is bounded by the curve, the line \( AB \), and the x-axis.

b. Use integration to find the area of the finite region \( R \), giving your answer to 2 decimal places.

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