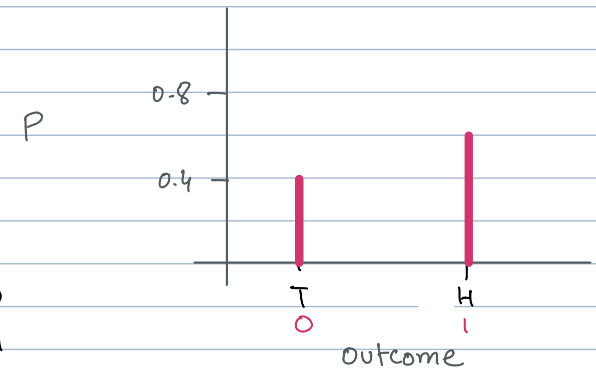
– Summarizing Distributions

– Consider a coin toss with $P(H) = 0.6$

We can summarize this using a function:



$$P(n) = \begin{cases} 0.4 & \text{if } n=0 \\ 0.6 & \text{if } n=1 \end{cases}$$

Generalize:

$$P(n) = \begin{cases} 1-p & \text{if } n=0 \\ p & \text{if } n=1 \end{cases}$$

$(P(A) + P(\bar{A}) = 1$ Law of total probability$)$

Or, we can write this as:

$$P(n\,;\,p) = \underbrace{p^n \cdot (1-p)^{1-n}}$$

model

Takes just one argument!

called the model parameter

Probability density function (PDF)

Set the parameter: $p = 0.6$

The function becomes: $P(n) = 0.6^n \cdot 0.4^{1-n}$

Now: $P(0) = 0.6^0 \cdot 0.4^1$
$= 0.4$

$P(1) = 0.6^1 \cdot 0.4^0$
$= 0.6$

– A PDF calculates probabilities for any value of a RV.

– It is (often) parameterized

– Several common PDFs define well known "distributions"

– The one above is the "Bernoulli Distribution" — single experiment

– "n is bernoulli distributed".

If we do 'n' experiments. Success chance = p

RV : X — number of successes in n experiments

If I flip it 10 times, it can have

(1) — 10 successes

~~P(10)~~ — P(10) needed!

Let's set $n=10$, $p=0.6$

$$P(X=1) = 0.6^1 \times 0.4^9 + 0.4 \times 0.6^1 \times 0.4^8 + 0.4^2 \times 0.6^1 \times 0.4^7 + \dots = 10\,(0.6^1 \times 0.4^9)$$

How about $P(2)$? $\binom{10}{2}$

But we know that this is a common pattern.

X is a RV which belongs to the "Binomial Distribution".

$$P(X=k\,;\,n,p) = \binom{n}{k}\, p^k\, (1-p)^{n-k}$$

$p^{k=2}$ $(1-p)^8$

$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$

${}^nC_k$

Setting parameters : $n=10$, $p=0.6$

$$P(X=k) = \binom{10}{k}\, 0.6^k\, 0.4^{10-k}$$

$X=0$ | 10

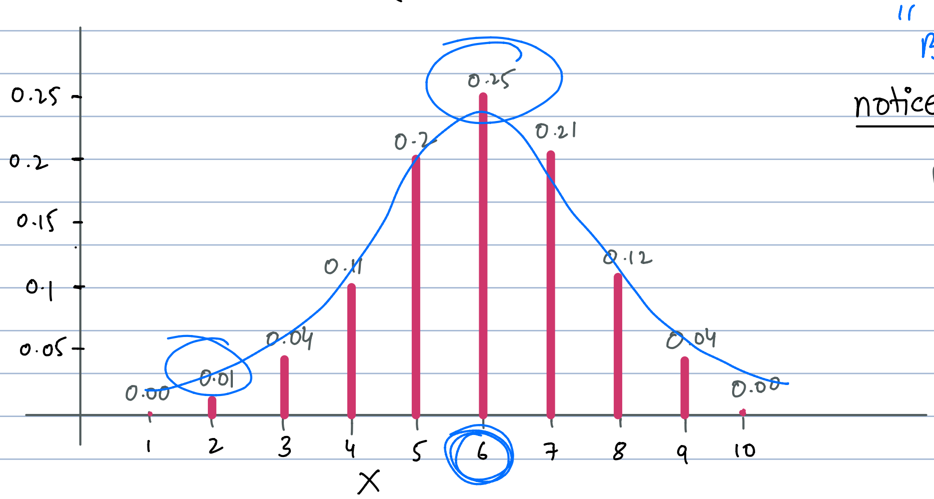
$$P(X=2) = \binom{10}{2}\, 0.6^2 \cdot 0.4^8 = \sim 0.01$$

"Bell curve"

notice the shape!

$P(X=5.5) = ?$

P(X)



Create variations to the model by varying the parameters.

Jupyter Lab!