Counting Events

1 million flips : How many have at least one H ?

HTTT ...... T
THTT ...... T
HHTT .... T
⋮
HHHH ...... HT

→ Huge # of favorable events!

no H → TTT.... T

x ——— x

01 1 1 1
⋮
1 1 1 1 1

unfavorable event = 01
favorable event = 1024 − 1 = 1023

Let's do this for 10 flips

10 bits. Total possible combinations : $2^{10} = 1024$

But why this?

H — 0
T — 1

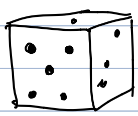
1 1 1 1

2 for this → 2^2
2 for this → 2^3
2 for this → 2^4
⋮
2^{10}

1 —— 10

2^{10}

Move from Binary to decimal:



Dice → 6 sides.

"with replacement"

1 2 4 1 2 6

6 combinations → 6^2
6 → 6^3
6

$6 \times 5 \times 4 \times 3 \times 2 \times 1$

$\boxed{6!}$

Without rep.

n^k : n possible values.
k draws.

Er... why were we doing this again?

Passwords revisited:

4-character password

G 9 C H

$n = 26 + 26 + 10$

$k = 4$

Total number of possible passwords:

$$62^4 = 14,776,336$$

Probability that your password is random guessed:

$$1 / 14\text{ m} = 6.77 \times 10^{-8}$$

If you have 8-characters: $\boxed{4.58 \times 10^{-15}}$

Let's do a slightly different case:

Deck of Cards: 4 suits, 13 ranks (52 total)

How many ways to pick 52 cards?

Do we care about the order?

No: Just one way

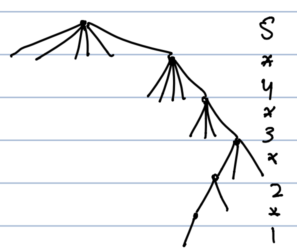
Yes: First pick : 52 choices

Second pick : 51 choices

Third pick : 50 choices

⋮

52^{nd} pick : 01 choice



$= 52!$

How about if we want to pick only 51?

Ordered: $\underbrace{52 * 51 * 50 * \ldots\ldots 2}$ 1 missing last pick. $52!$

Unordered: complicated but:

→ let's solve for picking 01 : 52 ways

In general: You have a set $\{1, 2, 3, \ldots\ldots 52\}$

with cardinality 'n'. $n = 52$

You wish to pick k elements

2, 4, A, A, A, A, A

"with replacement" → n^k ←

"without replacement":

→ ordered: ${}^nP_k = \boxed{n! / (n-k)!}$

→ unordered: ${}^nC_k = n! / (n-k)!\, k!$

$52 * 51 * \ldots\ldots 3$

$$\frac{n!}{(n-k)!} = \frac{n!}{(52-50)!} = \frac{n!}{2!}$$

$\underbrace{1, 2, 3, 4, 5}$

$5 * 4 * 3$

$$\frac{5!}{(5-3)!} = \frac{120}{2} = 60$$

ordered

unordered

1, 2, 3 → 1, 2, 3

3, 2, 1

3, 1, 2

2, 1, 3

2, 3, 1

1, 3, 2

$3 * 2 * 1 = k!$

$$\frac{n!}{(n-k)!} \Big/ k!$$

$$\frac{n!}{(n-k)!\, k!}$$

R, G, B, O

$${}^{4}P_{2} = \frac{4!}{(4-2)!} = \frac{24}{2} = 12$$

R, G

$$\boxed{2 \times 1 = 2!}$$

$$\frac{12}{2!} = \frac{12}{2} = 6$$

R, G	B, R
R, B	B, G
R, O	B, O
G, R	O, R
G, B	O, G
G, O	O, B

R, G

R, B

R, O

G, B

G, O

B, O