Quantifying Chances

- This is a thinking lesson!!

Problem:

- A disease is prevalent in 0.2% of a population.
- We have a test that, given to a sick person, gives a +ve result 85% of the time.
- Of all the people ever tested, 8% were positive.

Q: If Nazo is tested and test comes back positive, what are the chances that she actually has the disease?

☐ 85% ☐ 77% ☐ 21% ☑ 2%

Quantifying Chances

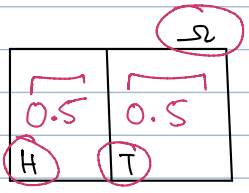
— Flip a coin: ~~hard~~ X

Q: "What are the chances that it will land on its head?"

Not: "If I flip it 10 times, how many will be heads?"

↳ easy

H, T → Events



Everything possible

Universe

probability

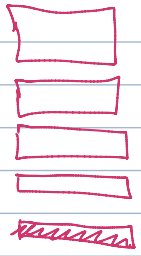
Chance of it being heads = P(H) = 0.5

" " " " tails = P(T) = 0.5

The two axioms of Probability:

1 — must lie in : [0 — 1]

2 — Sum of all events must be 1



~~P(H) = 1.2~~

~~P(H) = 0.7~~ P(T) = 0.2

→ Flip it twice:

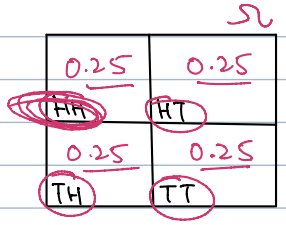
{ HH, HT, TH, TT }

Truth tables

→ The two flips are "independent".

P(HH)

"Both flips are heads"



$P(\Omega) = 1$

favorable

$$P(\text{favorable event}) = \frac{\#\text{ of ways event can occur}}{\#\text{ of all possible outcomes}}$$

$$= \frac{1}{4} = 0.25 = 25\%$$

→ "Any of the flips is a head."

A = { HH, HT, TH }

$$P(A) = 3/4 = 0.75$$

Alternate way / Intuition

"Both flips are heads"

H → 1

T → 0

AND

0 — 0	— 0
0 — 1	— 0
1 — 0	— 0
1 — 1	— 1

P(H H)

→ First flip is H and second flip is H

$$P(H) * P(H) = 0.5 * 0.5 = 0.25$$

"Any of the flips is a head."

First flip is H or second flip is H

$$P(H) + P(H) = 0.5 + 0.5 = 1 - 0.25 = 0.75$$

OR

H 0	H 0	0
H 0	T 1	1
T 1	H 0	1
T 1	T 1	1

Remove the overlap

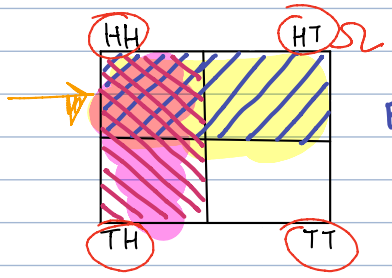
$$P(H) + P(H) - P(HH)$$

$$= 0.5 + 0.5 - 0.25$$

$$= 0.75$$

A and B are not independent!

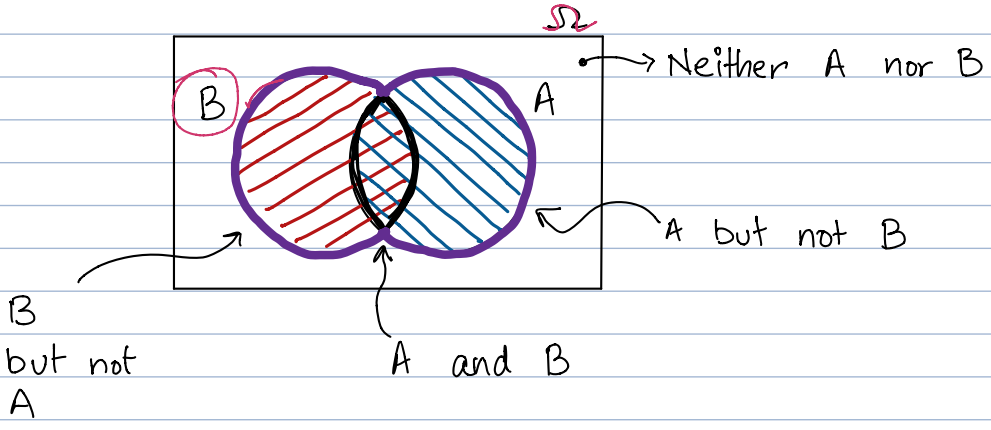
Another view:



first flip H A

Second flip H B

Venn Diagram



Notation:

A and B = $A \cap B$

A or B = $A \cup B$

Let's scale this up to 1 million flips!

Jupyter notebook!