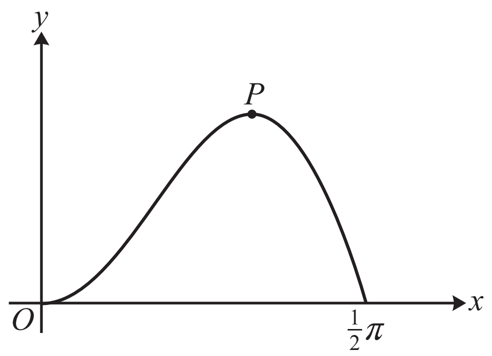
4



The diagram shows the part of the curve $y = 3x\sin 2x$ for which $0 \leqslant x \leqslant \frac{1}{2}\pi$.

The maximum point on the curve is denoted by P.

(a) Show that the x-coordinate of P satisfies the equation $\tan 2x + 2x = 0$. **[3]**

(b) Use the Newton-Raphson method, with a suitable initial value, to find the x-coordinate of P, giving your answer correct to 4 decimal places. Show the result of each iteration. **[4]**

(c) The trapezium rule, with four strips of equal width, is used to find an approximation to

$$\int_0^{\frac{1}{2}\pi} 3x\sin 2x\,\mathrm{d}x.$$

Show that the result can be expressed as $k\pi^2(\sqrt{2}+1)$, where k is a rational number to be determined. **[4]**

(d) **(i)** Evaluate $\int_0^{\frac{1}{2}\pi} 3x\sin 2x\,\mathrm{d}x$. **[1]**

(ii) Hence determine whether using the trapezium rule with four strips of equal width gives an under- or over-estimate for the area of the region enclosed by the curve $y = 3x\sin 2x$ and the x-axis for $0 \leqslant x \leqslant \frac{1}{2}\pi$. **[1]**

(iii) Explain briefly why it is not easy to tell from the diagram alone whether the trapezium rule with four strips of equal width gives an under- or over-estimate for the area of the region in this case. **[1]**

5 **In this question you must show detailed reasoning.**

(a) Prove that $(\cot\theta + \operatorname{cosec}\theta)^2 = \dfrac{1+\cos\theta}{1-\cos\theta}$. **[4]**

(b) Hence solve, for $0 < \theta < 2\pi$, $3(\cot\theta + \operatorname{cosec}\theta)^2 = 2\sec\theta$. **[5]**