complex numbers - 1

answers on next page

3 questions – progressing from 'accessible' to 'discriminating'

1. For each complex number z given below, find the modulus, $|z|$, and the principal argument, $\arg z$. [***no** calculator*]

(a) $z = \dfrac{1-\mathrm{i}}{1+\mathrm{i}}$ (b) $z = \dfrac{-1-7\mathrm{i}}{4+3\mathrm{i}}$

2. Find all complex solutions to the equation $z^4 + 81 = 0$ and express them in Cartesian form; that is, in the form $z = a + \mathrm{i}b$. [***no** calculator*]

3. (a) Using de Moivre's theorem, find identities for $\sin 4\theta$ and $\cos 4\theta$ expressed only in terms of $\sin\theta$ and $\cos\theta$.

(b) Hence, show that $\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + 4\tan^4\theta}$, [***no** calculator*]

complex numbers - 1

Answers

1. (a) $|z| = 1, \ \arg z = -\frac{\pi}{2}$ (b) $|z| = \sqrt{2}, \ \arg z = -\frac{3\pi}{4}$

2. 4 solutions: $z = \frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}\mathrm{i}, \ z = -\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}\mathrm{i}, \ z = \frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}\mathrm{i}, \ z = -\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}\mathrm{i}$

3. (a) $\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$ and $\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$