# CIRCULAR MOTION

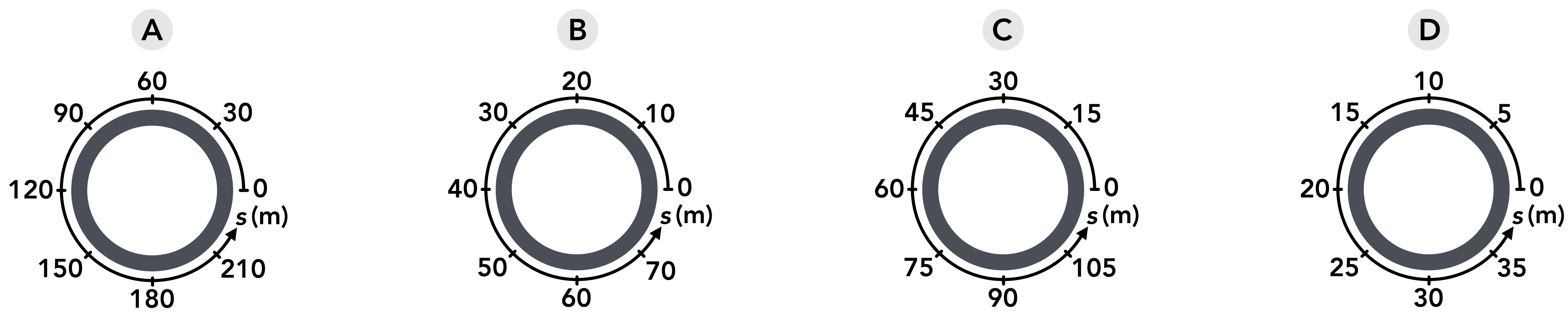
## Tangential position and displacement

1. Which of the following are examples of an object in circular motion? (Select all that apply)
   - A  A person riding on a Ferris wheel (the person)
   - B  A record playing on a record player (the record)
   - C  A spinning ceiling fan (the fan)
   - D  A fly sitting on a blade of a ceiling fan (the fly)

2. What is the circumference of a circle with a radius of 5 m?
   - A  10 m
   - B  15.7 m
   - C  31.4 m
   - D  62.8 m

3. If the circumference of a circle is 12 m, what is the diameter?
   - A  3.14 m
   - B  3.8 m
   - C  6 m
   - D  12 m

4. If the radius of a circular track is 19.1 m, which axis shown below would be labeled correctly?



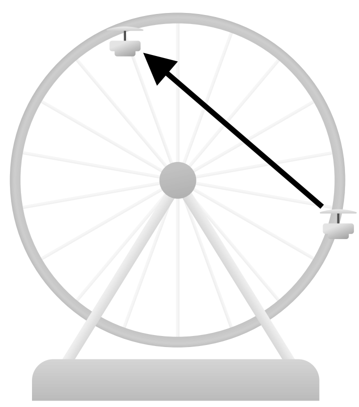

5. What is the SI unit for tangential position?
   - A  rad
   - B  km
   - C  deg
   - D  m

6. What is the SI unit for tangential displacement?
   - A  m
   - B  rad
   - C  km
   - D  rev

7. If the diameter of a circular race track is 0.5 km, what is the circumference of the track in m?
   - A  314 m
   - B  500 m
   - C  785 m
   - D  1571 m

8. If a speed skater races around a circular ice rink counterclockwise for 30 m, the skater's displacement is
   - A Positive
   - B Negative

9. Which of the arrows below shows the length of a positive tangential displacement of a Ferris wheel car?

A

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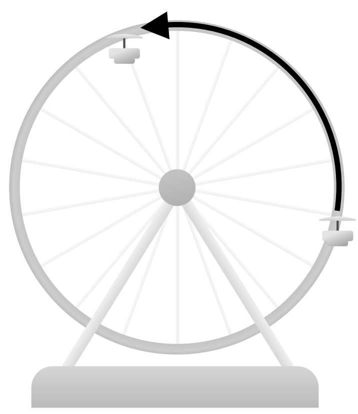
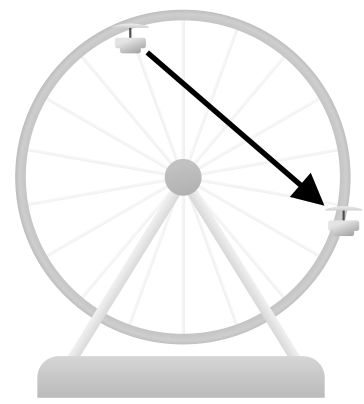
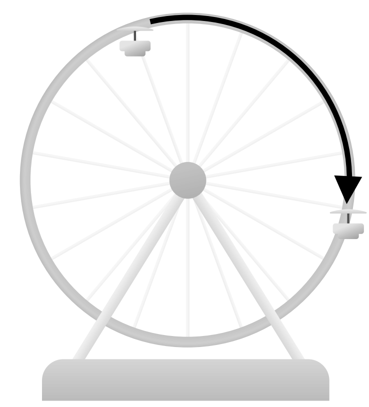
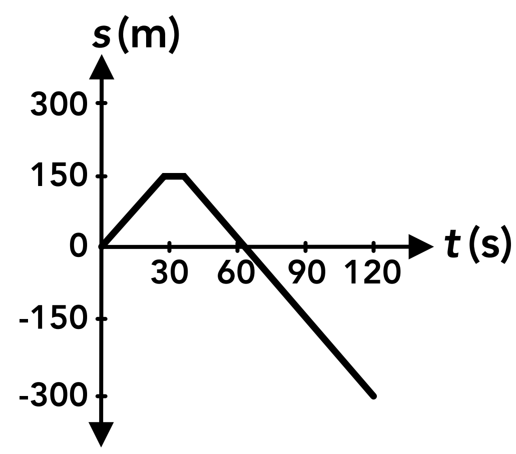
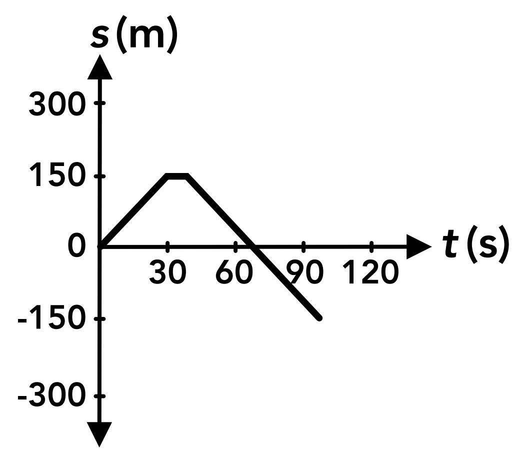
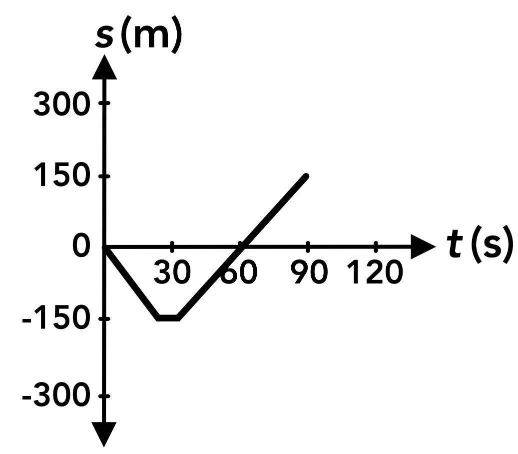
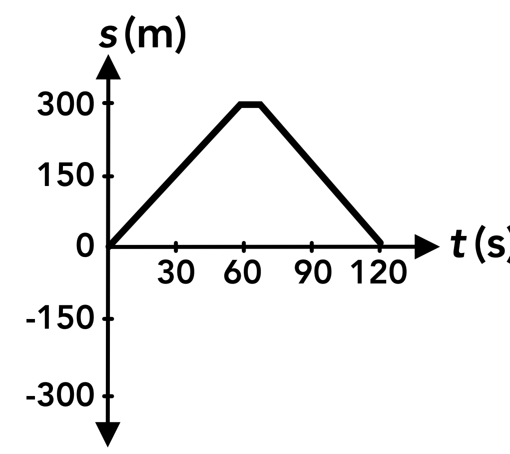

C

D


10. A person is ice skating around the outside of a circular rink with a radius of 24 m. They skate 1 lap counterclockwise, stop, then skate 2 laps clockwise. Which graph represents that motion of the skater?

A


B


C


D



11. A cyclist is riding around a circular track with a circumference of 200 m. If they travel 1 full lap (1 revolution) and end in the same location where they started, what is the cyclist's final position?

12. A satellite is in a circular orbit around the Earth. If the diameter of the Earth is 12,742 km (assuming the Earth is a sphere) and the satellite is 1,500 km above the surface of the Earth, what is the circumference of the satellite's orbit in km?

13. If a race car on circular race track starts at a position of 270 m then drives to a position of 50 m, what is the car's displacement?

14. A runner is on a circular track. If they start at a position of 32 m and run a distance of 57 m clockwise, what is their final position?

15. A car resets its odometer to zero, then drives along half of a circular track and stops. If the car's odometer reads 1.70 km, what is the radius of the track in km?

16. A fly is sitting on a spinning record and follows a circular path. During a brief section of the song, the fly moves from a position of -8 cm to a position of 13 cm. What was the fly's displacement in cm during that time?

17. A running coach has the team perform a drill where they run around a circular track and reverse directions when he blows a whistle. If one runner starts at a position of 10 m, then runs 20 m clockwise, then 35 m counterclockwise, then 5 m clockwise, what is the runner's final position?

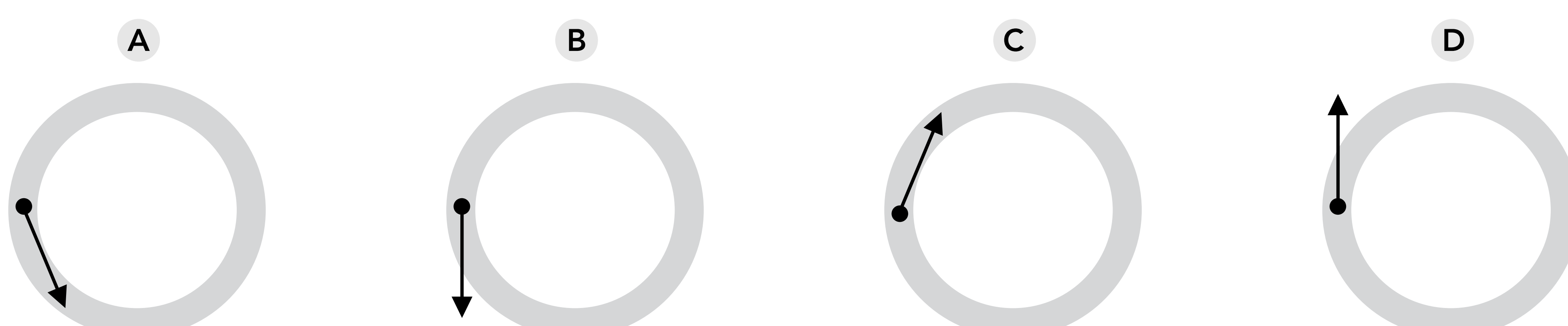
## Tangential speed and velocity

18. What is the SI unit for tangential velocity?
    - A rad/s
    - B m/s
    - C km/h
    - D rpm

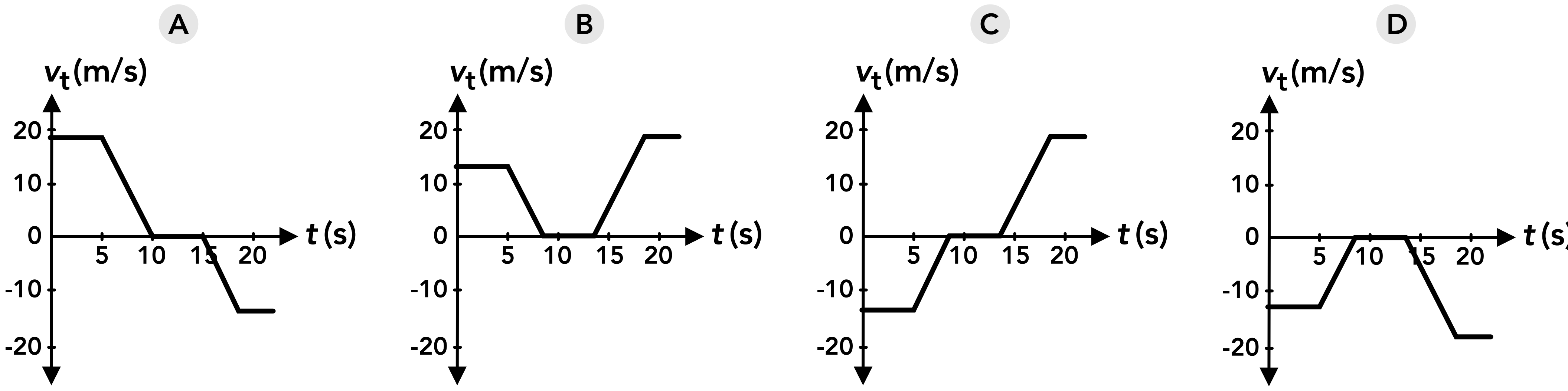
19. If a cyclist is riding around a circular track with a tangential velocity of 5 km/h, what is the velocity in m/s?
    - A 5 m/s
    - B 1.4 m/s
    - C 5000 m/s
    - D 83.3 m/s

20. A figure skater is spinning clockwise with their arms stretched out. The tangential velocity of their hands is
    - A Positive
    - B Negative

21. A car is driving clockwise around a circular track. What is the direction of the instantaneous tangential velocity of the car when it's in the position indicated by the dot shown in the figures below?



22. A car is driving clockwise around a circular track at a speed of 14 m/s when it slows down, stops, turns around, then accelerates to a speed of 19 m/s counterclockwise. Which of these graphs could represent the motion of the car?




23. If a car on a circular track travels 160 m counterclockwise in 7 s, what is its average tangential velocity?

24. If a runner on a circular race track travels from position of 17 m to a position of -5 m over the course of 16 s, what is the average tangential velocity of the runner?

25. A Ferris wheel rotates counterclockwise. When the ride starts, one person on the Ferris wheel is at a tangential position of 12 m relative to the bottom of the Ferris wheel. If the cars move at a constant tangential velocity of 1.5 m/s, what is the position of that person after 15 s?

26. If a car on a circular track is at a position of 35 m and is driving at -15 m/s, how long does it take for the car to reach a position of -50 m?

27. If a car on a circular track travels 230 m in the counterclockwise direction in 9.5 s, what is the average tangential velocity of the car in km/h?

28. A go-kart is driving at a velocity of 6 m/s counterclockwise around a circular race track. What is the position of the go-kart 4 s after it passes the 10 m mark?

29. A truck in circular motion around a track drives from a position of -52 m to a position of 14 m in 5.2 s. What is the average tangential velocity of the truck in km/h?

## Tangential acceleration

30. What is the SI unit for tangential acceleration?
    - A rad/s
    - B m/s
    - C $m/s^2$
    - D $rad/s^2$

31. A car is driving counterclockwise around a circular track. If the car increases its speed, the car's tangential acceleration is
    - A Positive
    - B Negative

32. A car is driving clockwise around a circular track. If the car slows down, the car's tangential acceleration is
    - A Positive
    - B Negative

33. If a sprinter on a circular race track starts from rest and then speeds up to a tangential velocity of 5 m/s over a period of 3 s, what was the sprinter's tangential acceleration?

34. A truck is driving around a section of road that follows a circular path. It's moving at 16 m/s when it hits the brakes and slows down to 12 m/s over period of 3 s. What was the truck's tangential acceleration?

35. If someone says their car can go from 0 to 100 km/h in 5 seconds, what would be the tangential acceleration of their car around a circular track in $m/s^2$ ?

36. If a car has a maximum acceleration of 4 $m/s^2$, how many seconds would it take the car to go from a tangential velocity of 20 km/h to 50 km/h?

37. Two cars line up for the start of a race at the 0 m line. When the gun goes off, one car accelerates at 5 $m/s^2$. What is the car's position after 3 s of accelerating?

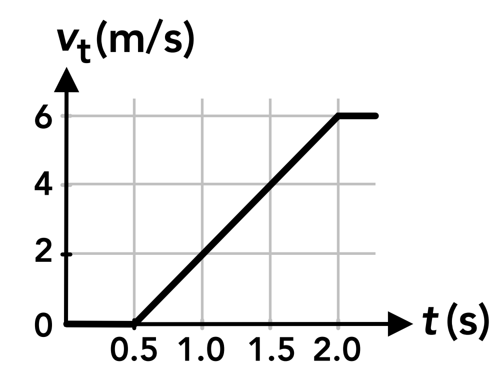
38. A person is riding on a Ferris wheel which is turning slowly at a constant speed. The ride operator then turns up the speed and person experiences a tangential acceleration of 1.5 $m/s^2$ for 6 s. If the person covers a circular distance of 30 m during that time period, what was the initial tangential velocity of the person?

39. A cyclist on a circular track is moving 6 m/s. When they pass the 50 m mark they decide to pedal harder and accelerate. After 4 s of accelerating they pass the 90 m mark. What was the acceleration of the cyclist during that time?

40. A car on a circular track is driving 16 m/s. When it passes the 30 m mark it accelerates at 5 m/s². What is the position of the car when it reaches a tangential velocity of 23 m/s?

41. A bolt near the end of a spinning wind turbine blade traces out a counterclockwise circular path with a tangential velocity of 8 m/s. The wind suddenly stops, and the bolt travels a circular displacement of 112 m as the turbine comes to a stop. What was the tangential acceleration of the bolt while the turbine slowed down?

42. The graph on the right represents the motion of a sprinter that starts from rest then accelerates at the start of a race. What is the acceleration of the sprinter between 0.5 s and 2.0 s?




43. A circular race track has a radius of 0.04 km. A car starts from rest then accelerates at a constant 4 m/s². How long does it take the car to cover 1 lap?

44. The tip of a 1.2 m long helicopter blade is in circular motion with a tangential velocity of 25 m/s. The helicopter increases the speed of the blades in order to lift off, and the tips of the blades experience a tangential acceleration of 6 m/s² for a period of 5.2 s. During this period, how many revolutions does one blade experience?

45. A cyclist is riding around a circular track with a tangential velocity of 27 km/h. When the cyclist is at a position of 25 m, it accelerates at 2 m/s². When the cyclist reaches the 50 m mark, what is the cyclist's tangential velocity in km/h?

46. A Ferris wheel car starts from rest. When the ride begins, the car experiences a tangential accleration of -0.2 m/s² for 1 entire rotation of the Ferris wheel. By the time it completes 1 rotation the car has a tangential velocity of -5 m/s. What is the diameter of the Ferris wheel (the circular path of the car)?

47. During a race around a circular track, a car is driving at 80 km/h. When it passes the 35 m mark on the track, it accelerates until it passes the 140 m mark. If it takes 4 sec to travel between those two marks, what is the final velocity of the car in km/h?

## Answers - Tangential position and displacement

1. Answer: A, D
   A) A person on a Ferris wheel follows a circular path so they're in circular motion
   B) A record rotates around its own center so it's in rotational motion
   C) A fan rotates around its own center so it's in rotational motion
   D) A fly on a ceiling fan blade follows a circular path so it's in circular motion

2. Answer: C
   circumference of a circle: $\boldsymbol{C = 2\pi r = 2\pi\,(5\text{ m})} = 31.4\text{ m}$

3. Answer: B
   circumference of a circle: $\boldsymbol{C = \pi d}$ $\quad\quad$ $\boldsymbol{d = C / \pi = (12\text{ m}) / \pi} = 3.8\text{ m}$

4. Answer: C
   The circumference of the track would be: $\boldsymbol{C = 2\pi r = 2\pi\,(19.1\text{ m})} = 120\text{ m}$

5. Answer: D
   A) rad is the SI unit for angular position and angular displacement
   B) km is a valid unit for tangential position but is not the SI unit
   C) deg is a valid unit for angular position and angular displacement
   D) m is the SI unit for tangential position

6. Answer: A
   A) m is the SI unit for tangential displacement
   B) rad is the SI unit for angular position and angular displacement
   C) km is a valid unit for tangential displacement but is not the SI unit
   D) rev is a valid unit for tangential displacement but is not the SI unit

7. Answer: 1571 m

   $$\frac{\mathbf{0.5\text{ km}}}{}\times\frac{\mathbf{1000\text{ m}}}{\mathbf{1\text{ km}}} = 500\text{ m}$$

   $\boldsymbol{C = \pi d} = \pi\,(500\text{ m}) = 1571\text{ m}$

8. Answer: A
   By convention, the counterclockwise direction is the positive direction.

9. Answer: B
   By convention, the counterclockwise direction is the positive direction. The tangential displacement is the arc length of the path of the car, not the straight line displacement.

10. Answer: B
    The circumference of the rink is: $\boldsymbol{C = 2\pi r = 2\pi(24\text{ m})} = 150.8\text{ m}$
    By convention, counterclockwise is the positive direction. They move 150.8 m in the positive direction, stay at that position for a period of time, then they move 2(150.8 m) = 301.6 m in the negative direction and end at a final position of -150.8 m.

11. Answer: 200 m
    Their initial position would be 0 m and their tangential displacement is 200 m (the circumference of the track) so their final position is 200 m. The displacement is assumed to be positive because no direction was given.

12. Answer: 49,455 km
The diameter of the satellite's circular orbit is: 12,742 km + 2(1500 km) = 15,742 km
The circumference of the orbit is: $C = \pi d = \pi \text{ (15,742 km)}$ = 49,455 km

13. Answer: -220 m
$\Delta s = s_f - s_i = \text{(50 m)} - \text{(270 m)}$ = -220 m

14. Answer: -25 m
$\Delta s = s_f - s_i$ $\quad$ $\text{(-57 m)} = s_f - \text{(32 m)}$ $\quad$ $s_f$ = -25 m

15. Answer: 0.54 km
The circumference of the track is: 2(1.70 km) = 3.40 km
$C = 2\pi r$ $\quad$ $\text{(3.40 km)} = 2\pi r$ $\quad$ $r$ = 0.54 km

16. Answer: 21 cm
$\Delta s = s_f - s_i = \text{(13 cm)} - \text{(-8 cm)}$ = 21 cm

17. Answer: 20 m
$s_f$ = (10 m) + (-20 m) + (35 m) + (-5 m) = 20 m

## Answers - Tangential speed and velocity

18. Answer: B
A) rad/s is the SI unit for angular velocity
B) m/s is the SI unit for tangential velocity
C) km/h is a valid unit for tangential velocity but is not the SI unit
D) rpm is a valid unit for angular velocity

19. Answer: 1.4 m/s

$$\frac{\text{5 km}}{\text{h}} \times \frac{\text{1000 m}}{\text{1 km}} \times \frac{\text{1 h}}{\text{60 min}} \times \frac{\text{1 min}}{\text{60 s}} = 1.4 \text{ m/s}$$

20. Answer: B
By convention, the clockwise direction is the negative direction.

21. Answer: D
A) This arrow does not show a tangential velocity because it is not pointing tangent to the circular path
B) This arrow shows a counterclockwise tangential velocity
C) This arrow does not show a tangential velocity because it is not pointing tangent to the circular path
D) This arrow shows a clockwise tangential velocity

22. Answer: C
By convention, counterclockwise is the positive direction. The car starts with a velocity of -14 m/s. Then it slow down and stop so the speed decreases to 0 m/s. Then it speeds up to a velocity of positive 19 m/s.

23. Answer: 22.9 m/s

$$v_t = \frac{\Delta s}{\Delta t} = \frac{\text{(160 m)}}{\text{(7 s)}} = 22.9 \text{ m/s}$$

24. Answer: -1.38 m/s

$$v_t = \frac{\Delta s}{\Delta t} = \frac{(-5\text{ m}) - (17\text{ m})}{(16\text{ s})} = -1.38\text{ m/s}$$

25. Answer: 34.5 m

$$s_f = s_i + v_t \Delta t = (12\text{ m}) + (1.5\text{ m/s})(15\text{ s}) = 34.5\text{ m}$$

26. Answer: 5.7 s

$$v_t = \frac{\Delta s}{\Delta t} \qquad (-15\text{ m/s}) = \frac{(-50\text{ m}) - (35\text{ m})}{\Delta t} \qquad \Delta t = 5.7\text{ s}$$

27. Answer: 87.2 km/h

$$v_t = \frac{\Delta s}{\Delta t} = \frac{(230\text{ m})}{(9.5\text{ s})} = 24.21\text{ m/s}$$

$$\frac{24.21\text{ m}}{\text{s}} \times \frac{1\text{ km}}{1000\text{ m}} \times \frac{60\text{ s}}{1\text{ min}} \times \frac{60\text{ min}}{1\text{ h}} = 87.2\text{ km/h}$$

28. Answer: 34 m

$$s_f = s_i + v_t \Delta t = (10\text{ m}) + (6\text{ m/s})(4\text{ s}) = 34\text{ m}$$

29. Answer: 45.7 km/h

$$v_t = \frac{\Delta s}{\Delta t} = \frac{(14\text{ m}) - (-52\text{ m})}{(5.2\text{ s})} = 12.69\text{ m/s}$$

$$\frac{12.69\text{ m}}{\text{s}} \times \frac{1\text{ km}}{1000\text{ m}} \times \frac{60\text{ s}}{1\text{ min}} \times \frac{60\text{ min}}{1\text{ h}} = 45.7\text{ km/h}$$

## Answers - Tangential acceleration

30. Answer: C
A) rad/s is the SI unit for angular velocity
B) m/s is the SI unit for tangential velocity
C) $m/s^2$ is the SI unit for tangential acceleration
D) $rad/s^2$ is the SI unit for angular acceleration

31. Answer: A
By convention, the counterclockwise direction is the positive direction. The car’s velocity is counterclockwise and if the velocity increases in magnitude then the acceleration is in the same direction as the velocity.

32. Answer: A
By convention, the counterclockwise direction is the positive direction. The car’s velocity is clockwise and if the velocity decreases in magnitude then the acceleration is in the opposite direction as the velocity.

33. Answer: 1.7 $m/s^2$

$$a_t = \frac{\Delta v_t}{\Delta t} = \frac{(5\text{ m/s}) - (0\text{ m/s})}{(3\text{ s})} = 1.7\text{ m/s}^2$$

34. Answer: -1.3 $m/s^2$

$$a_t = \frac{\Delta v_t}{\Delta t} = \frac{(12\text{ m/s}) - (16\text{ m/s})}{(3\text{ s})} = -1.3\text{ m/s}^2$$

35. Answer: 5.6 m/s²

$$\frac{100\text{ km}}{\text{h}} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ h}}{60\text{ min}} \times \frac{1\text{ min}}{60\text{ s}} = 27.78\text{ m/s}$$

$$a_t = \frac{\Delta v_t}{\Delta t} = \frac{(27.78\text{ m/s})}{(5\text{ s})} = 5.6\text{ m/s}^2$$

36. Answer: 2.1 s

$$\frac{20\text{ km}}{\text{h}} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ h}}{60\text{ min}} \times \frac{1\text{ min}}{60\text{ s}} = 5.56\text{ m/s}$$

$$\frac{50\text{ km}}{\text{h}} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ h}}{60\text{ min}} \times \frac{1\text{ min}}{60\text{ s}} = 13.89\text{ m/s}$$

$$a_t = \frac{\Delta v_t}{\Delta t} \qquad (4\text{ m/s}^2) = \frac{(13.89\text{ m/s}) = (5.56\text{ m/s})}{\Delta t} \qquad \Delta t = 2.1\text{ s}$$

37. Answer: 22.5 m

$$s_f = s_i + v_{ti}t + \frac{1}{2}a_t t^2 = (0\text{ m}) + (0\text{ m/s})(3\text{ s}) + \frac{1}{2}(5\text{ m/s}^2)(3\text{ s})^2 = 22.5\text{ m}$$

38. Answer: 0.5 m/s

$$\Delta s = v_{ti}\, t + \frac{1}{2}a_t t^2 \qquad (30\text{ m}) = v_{ti}\,(6\text{ s}) + \frac{1}{2}(1.5\text{ m/s}^2)(6\text{ s})^2 \qquad v_{ti} = 0.5\text{ m/s}$$

39. Answer: 2 m/s²

$$s_f = s_i + v_{ti}t + \frac{1}{2}a_t t^2 \qquad (90\text{ m}) = (50\text{ m}) + (6\text{ m/s})(4\text{ s}) + \frac{1}{2}a_t\,(4\text{ s})^2 \qquad a_t = 2\text{ m/s}^2$$

40. Answer: 57.3 m

$$v_{tf}^2 = v_{ti}^2 + 2a_t(s_f - s_i) \qquad (23\text{ m/s})^2 = (16\text{ m/s})^2 + 2(5\text{ m/s}^2)(s_f - 30\text{ m}) \qquad s_f = 57.3\text{ m}$$

41. Answer: -0.29 m/s²

$$v_{tf}^2 = v_{ti}^2 + 2a_t(s_f - s_i) \qquad (0\text{ m/s})^2 = (8\text{ m/s})^2 + 2\,a_f\,(112\text{ m}) \qquad a_t = -0.29\text{ m/s}^2$$

42. Answer: 4 m/s²

The acceleration is the slope of the velocity-time graph:

$$a_t = \text{slope} = \frac{(6\text{ m/s}) - (0\text{ m/s})}{(2.0\text{ s}) - (0.5\text{ s})} = 4\text{ m/s}^2$$

43. Answer: 11.2 s

The circumference of the track is: $C = 2\pi r = 2\pi\,(40\text{ m}) = 251.33\text{ m}$

$$s_f = s_i + v_{ti}t + \frac{1}{2}a_t t^2 \qquad (251.33\text{ m}) = (0\text{ m}) + (0\text{ m/s})t + \frac{1}{2}(4\text{ m/s}^2)t^2 \qquad t = 11.2\text{ s}$$

44. Answer: 28 rev

The circumference of the circular path is: $C = 2\pi r = 2\pi\,(1.2\ \mathrm{m}) = 7.54\ \mathrm{m}$

$s_f = s_i + v_{ti}t + \frac{1}{2}a_t t^2$ $\Delta s = (25\ \mathrm{m/s})(5.2\ \mathrm{s}) + \frac{1}{2}(6\ \mathrm{m/s^2})(5.2\ \mathrm{s})^2$ $\Delta s = 211.12\ \mathrm{m}$

$\text{revolutions} = \frac{\Delta s}{C} = \frac{(211.12\ \mathrm{m})}{(7.54\ \mathrm{m})} = 28\ \mathrm{rev}$

45. Answer: 45 km/h

$\frac{27\ \mathrm{km}}{\mathrm{h}} \times \frac{1000\ \mathrm{m}}{1\ \mathrm{km}} \times \frac{1\ \mathrm{h}}{60\ \mathrm{min}} \times \frac{1\ \mathrm{min}}{60\ \mathrm{s}} = 7.5\ \mathrm{m/s}$

$v_{tf}^2 = v_{ti}^2 + 2a_t(s_f - s_i)$ $v_{tf}^2 = (7.5\ \mathrm{m/s})^2 + 2(2\ \mathrm{m/s^2})(50\ \mathrm{m} - 25\ \mathrm{m})$ $v_{tf} = 12.5\ \mathrm{m/s}$

$\frac{12.5\ \mathrm{m}}{\mathrm{s}} \times \frac{1\ \mathrm{km}}{1000\ \mathrm{m}} \times \frac{60\ \mathrm{s}}{1\ \mathrm{min}} \times \frac{60\ \mathrm{min}}{1\ \mathrm{h}} = 45\ \mathrm{km/h}$

46. Answer: 19.9 m

$v_{tf}^2 = v_{ti}^2 + 2a_t(s_f - s_i)$ $(-5\ \mathrm{m/s})^2 = (0\ \mathrm{m/s})^2 + 2(-0.2\ \mathrm{m/s^2})\,\Delta s$ $\Delta s = 62.5\ \mathrm{m}$

$C = \pi d$ $(62.5\ \mathrm{m}) = \pi d$ $d = 19.9\ \mathrm{m}$

47. Answer: 109.1 km/h

$\frac{80\ \mathrm{km}}{\mathrm{h}} \times \frac{1000\ \mathrm{m}}{1\ \mathrm{km}} \times \frac{1\ \mathrm{h}}{60\ \mathrm{min}} \times \frac{1\ \mathrm{min}}{60\ \mathrm{s}} = 22.22\ \mathrm{m/s}$

$s_f = s_i + v_{ti}t + \frac{1}{2}a_t t^2$ $(140\ \mathrm{m}) = (35\ \mathrm{m}) + (22.22\ \mathrm{m/s})(4\ \mathrm{s}) + \frac{1}{2}a_t\,(4\ \mathrm{s})^2$ $a_t = 2.02\ \mathrm{m/s^2}$

$a_t = \frac{\Delta v_t}{\Delta t}$ $(2.02\ \mathrm{m/s^2}) = \frac{v_{tf} - (22.22\ \mathrm{m/s})}{(4\ \mathrm{s})}$ $v_{tf} = 30.3\ \mathrm{m/s}$

$\frac{30.3\ \mathrm{m}}{2} \times \frac{1\ \mathrm{km}}{1000\ \mathrm{m}} \times \frac{60\ \mathrm{s}}{1\ \mathrm{min}} \times \frac{60\ \mathrm{min}}{1\ \mathrm{h}} = 109.1\ \mathrm{km/h}$