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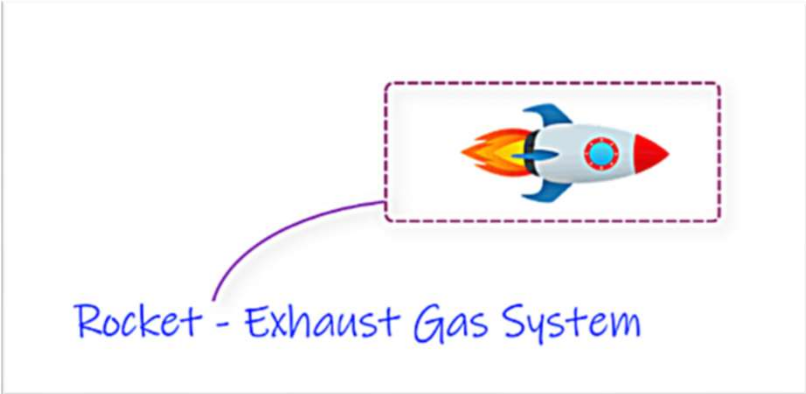
CUBE NOTES

Class 11/12 | AP Physics | IIT JEE | NEET

Tsiolkovsky's Rocket Equation

Key Idea

When a rocket expels exhaust gases at high velocity, it gets propelled in the opposite direction.



Conservation of Linear Momentum in Rockets:

- The law applies to a system of masses, such as a rocket and its exhaust gases.
- As the rocket burns fuel, its mass decreases and converts into exhaust gases

Momentum before = Momentum after

$$mv = (m - \Delta m)(v + \Delta v) + \Delta m (v - u)$$

Deriving the Thrust Equation

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Initial momentum = Final momentum

$$mv = (m - \Delta m)(v + \Delta v) + \Delta m (v - u)$$

$$\cancel{mv} = \cancel{mv} + m\,\Delta v - \Delta m\, v - \cancel{\Delta m\,\Delta v} + \cancel{\Delta m\, v} - \Delta m\, u$$

$m\,\Delta v = u\,\Delta m$ (1) Dividing both sides by Δt

$m\,(\Delta v / \Delta t) = u\,(\Delta m / \Delta t)$

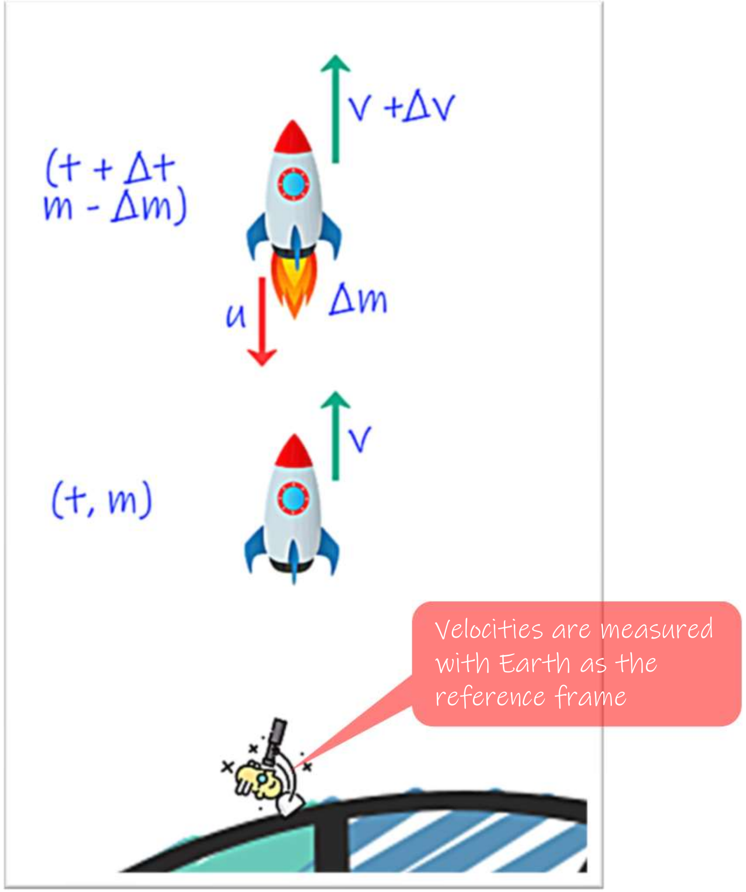
$m\,(dv / dt) = u\,(dm / dt)$ infinitesimally small Δv, Δt & Δm

$m\,a = u\,(dm / dt)$ (2) (dv/dt) is acceleration

$T_f = uR$ 1st Rocket equation

T_f = Thrust Force

R = The rate at which gases are emitted (dm/dt)

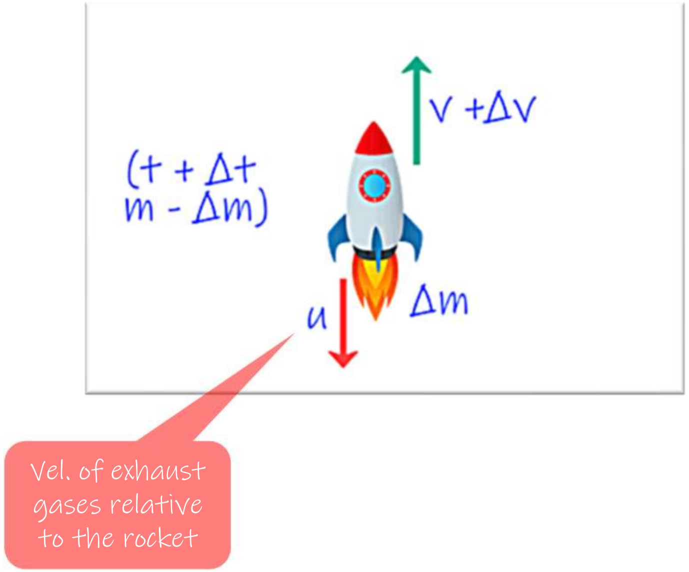


Understanding Thrust and Acceleration

$$ma = T_f = uR$$

T_f = Thrust Force

R = is the rate at which gases are emitted (dm/dt)



- Thrust is proportional to the relative speed u of the ejected fuel and the mass of fuel ejected per unit time.
- A powerful rocket, burns fuel at a rapid rate (large R or dm/dt) and ejects exhaust gases at a high relative speed (u).

Common Misconceptions

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- Misconception: If thrust is constant, acceleration should also be constant

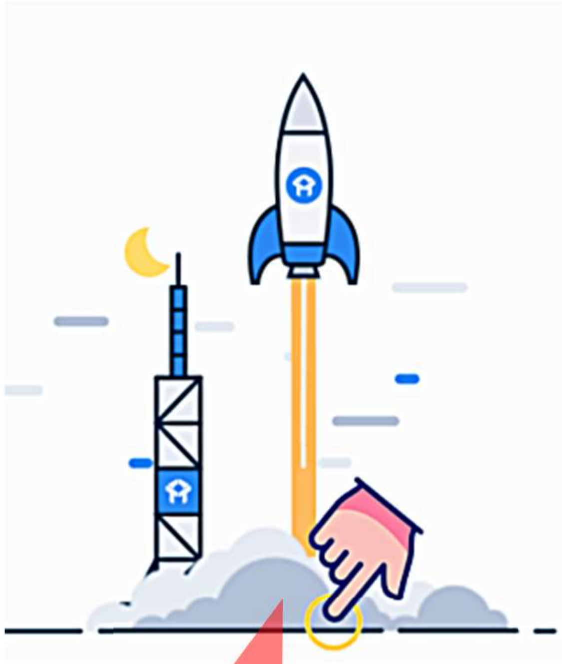
While the thrust is constant, the rocket's acceleration increases because the rocket's mass is decreasing.

$$a = \frac{u}{m}(dm/dt)$$ From (eq. 2) Slide 2

- Misconception: Rockets push against the ground to rise

Rockets do not push against the ground to rise; It is the law conservation of momentum. That is, gases eject in one direction, therefore to preserve momentum, rocket moves in the other.

Rockets work more efficiently in outer space where there is nothing to push against



The direction of momentum of gases that is downwards makes the rocket move upwards.

Tsiolkosky's Rocket Equation

$m\,dv = u\,dm$ From (eq. 1) Slide 2

$dv = u\,(1/m)\,(-dm)$

$$\int_{v_0}^{v} dv = -u \int_{m_0}^{m} (1/m)\,dm$$

$v - v_0 = -u \ln(m/m_0)$ or

$$v - v_0 = u \ln(m_0/m)$$

Engineers try to maximize the ratio m_0/m for max. velocity gain (LHS of the equation)

v: velocity at some time t when the mass is m

v_0: initial velocity when the mass is m_0

u: velocity of the exhaust gases relative to the rocket

Konstantin Tsiolkovsky (1857 - 1935), Russian rocket scientist who pioneered astronautics

Illustration

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If a rocket has an initial mass (m_0) of 1000 kg and final mass (m) of 500 kg, with an exhaust velocity (u) of 2000 m/s, calculate the change in velocity.

Calculation:

Given: $m_0 = 1000$ kg, $m = 500$ kg, $u = 2000$ m/s

Using the second rocket equation: $v - v_0 = u \ln(m_0 / m)$

$v - v_0 = 2000 \ln(1000 / 500)$

$v - v_0 = 2000 \ln(2)$

$v - v_0 \approx 2000 * 0.693$

$v - v_0 \approx 1386$ m/s

The rocket's velocity increases by approximately 1386 m/s due to the fuel burning and exhaust gas expulsion.