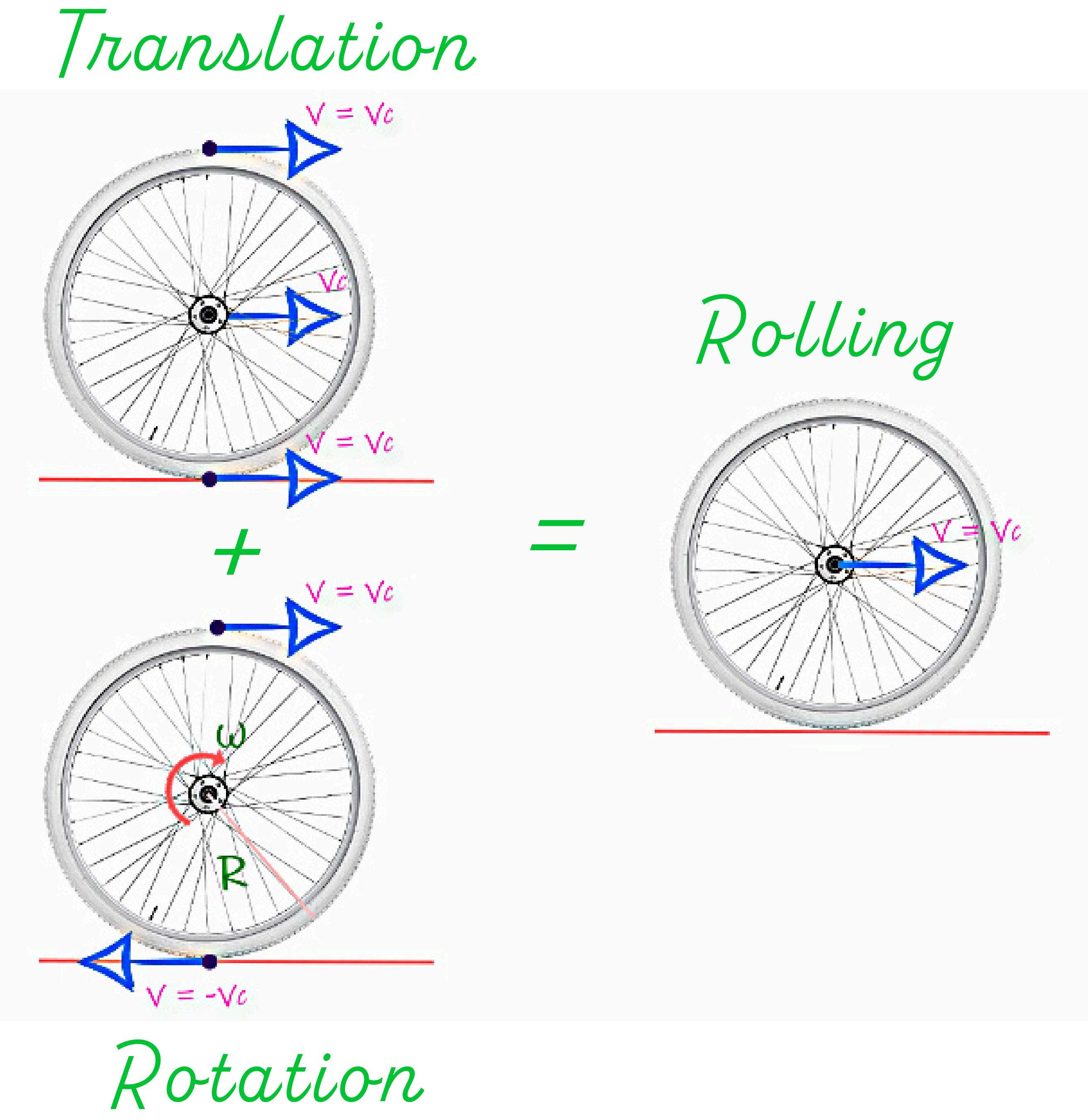
Rolling without slipping

1. The concept
2. Cycle wheel as an example
3. Condition for rolling without slipping
4. Example of rolling <u>with</u> slipping
5. Rotation + Translation = Rolling
6. Velocity analysis
7. Key formulas and equations
8. Common mistakes and misconceptions

WHAT IS ROLLING WITHOUT SLIPPING?

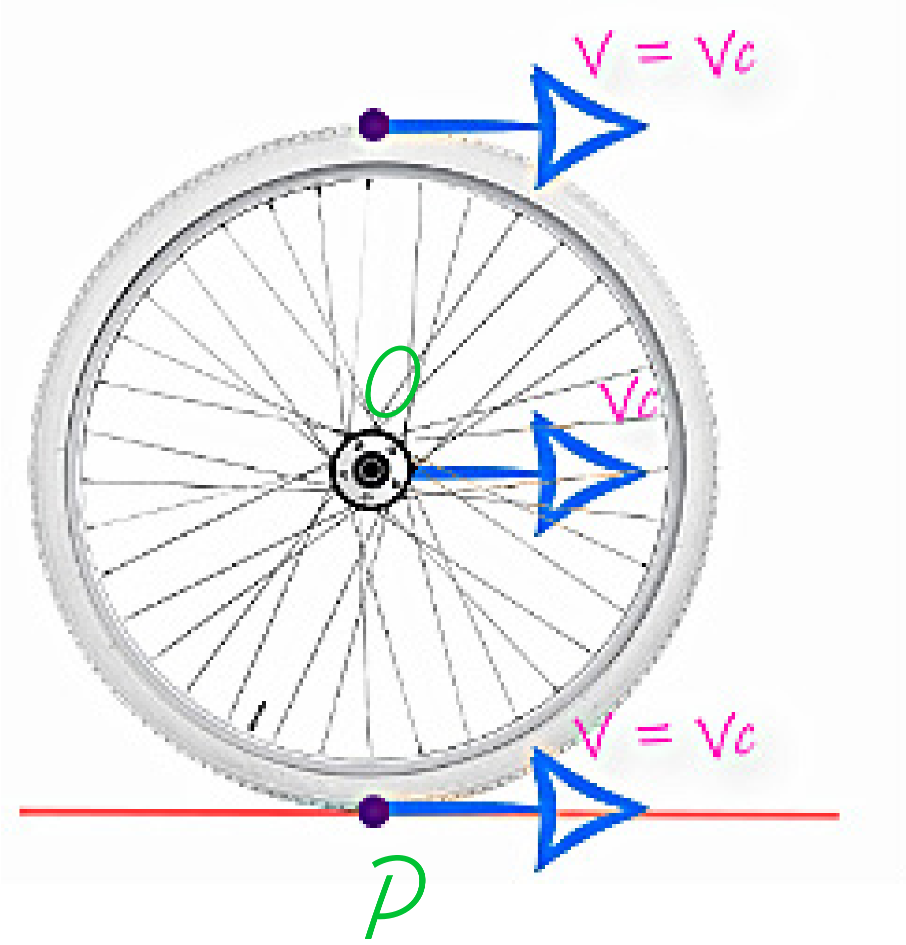
1. Rolling motion: Movement without slipping, sliding, or bouncing.
2. Key Condition: No relative motion at the contact point, i.e velocity relative to the surface = 0
3. Components:
   - Translation: Straight-line motion of the center of mass.
   - Rotation: Spinning motion around an axis.



UNDERSTANDING ROLLING WITHOUT SLIPPING

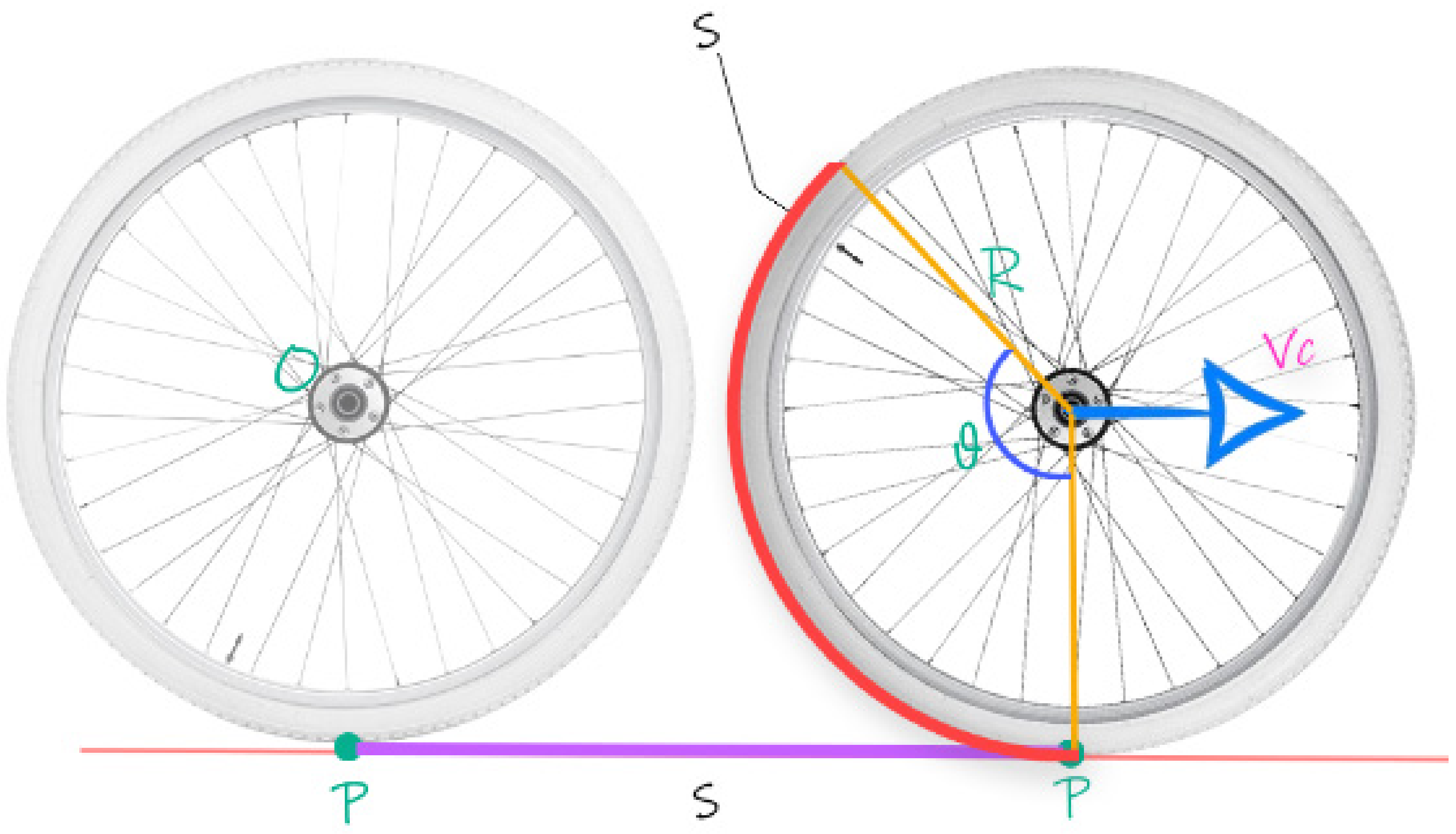
Using a Bicycle Wheel as an example-

1. Point O: The center of the wheel moves forward at a constant speed, denoted by v_c (velocity of the center of mass).
2. Point P: The contact point on the road also moves forward at the same speed, v_c (and so do all points), ensuring the wheel moves as a single unit.



UNDERSTANDING ROLLING WITHOUT SLIPPING...CONT.

1. Over a time interval, t:
   - Both O and P move a distance, s
   - The wheel also rotates through an ang



2. The arc length point P turns is $s = \theta R$

3. Differentiating with respect to time:

   $ds/dt = (d\theta/dt) \times R$

   Here:

   ds/dt = Linear speed of the center (v_c)

   $d\theta/dt$ = Angular speed of the wheel (ω)

4. Therefore,

   $v_c = \omega R$

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CONDITION FOR ROLLING WITHOUT SLIPPING

1. The equation

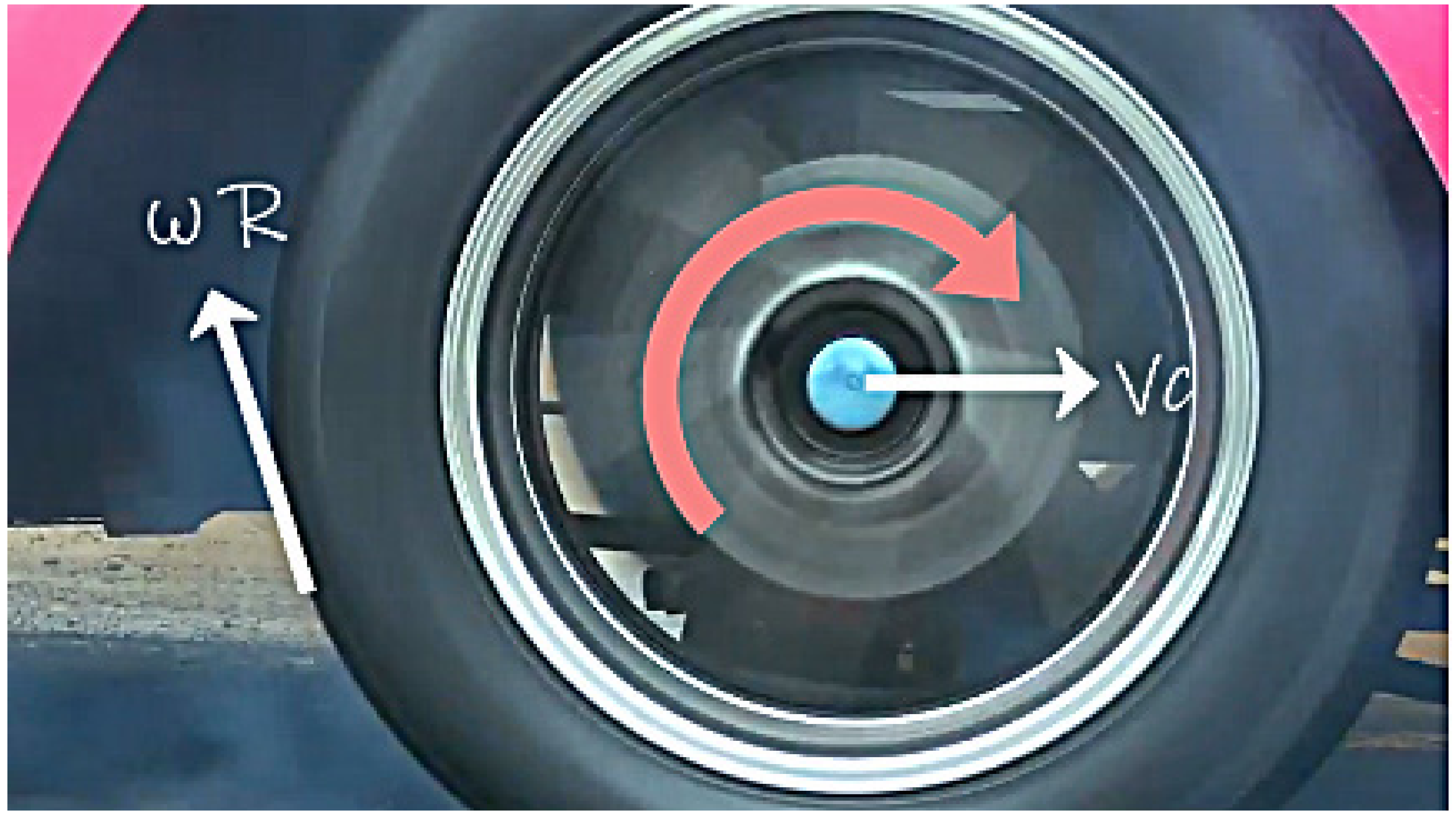
$$v_c = \omega R$$

is a condition that ensures rolling without slipping.

2. If this equation holds true, then:
   - There is no relative motion between the surface and the object.
   - This also means there would be
     - no slipping - tyre spins in its own place more than moves forwardor
     - no skidding - Tyre moves forward more than it turns

EXAMPLE: ROLLING WITH SLIPPING TIRES

1. When a racer accelerates suddenly, the rear tires may spin faster than the car's forward velocity (v_c). In that case:
2. The tangential velocity at the surface of the tire (ωR) exceeds v_c, causing the tires to slip.
3. Evidence: Smoke from the tires due to friction heating the rubber.

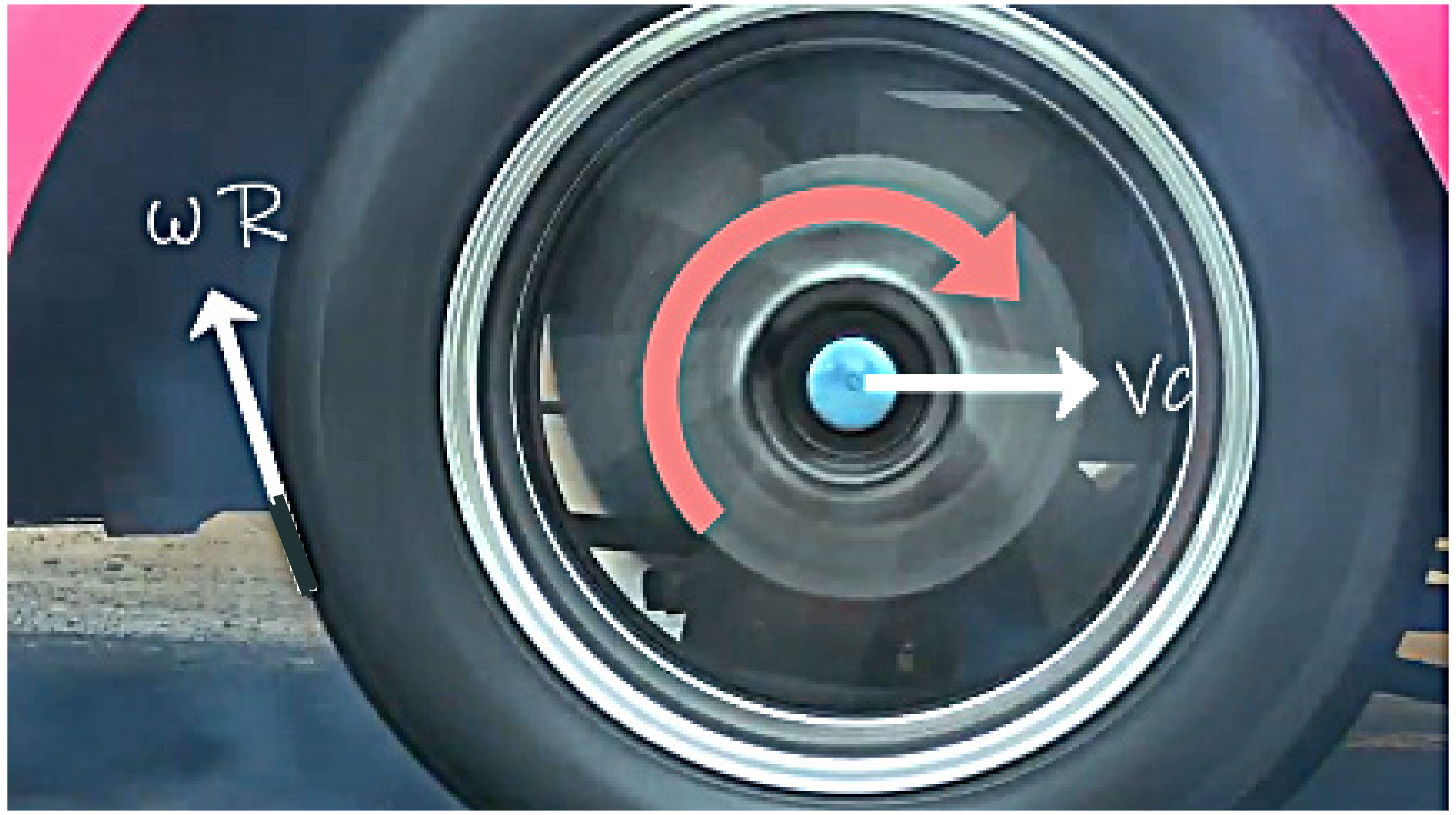


Here

$$\omega R > v_c$$

EXAMPLE: ROLLING **WITH** SKIDDING TIRES

1. If the rear tires spin slower than the car's forward velocity (v_c). In that case
2. The tangential velocity at the surface of the tire (ωR) is <u>less</u> than v_c, causing the tires to skid.
3. Evidence: Skid marks on the road.

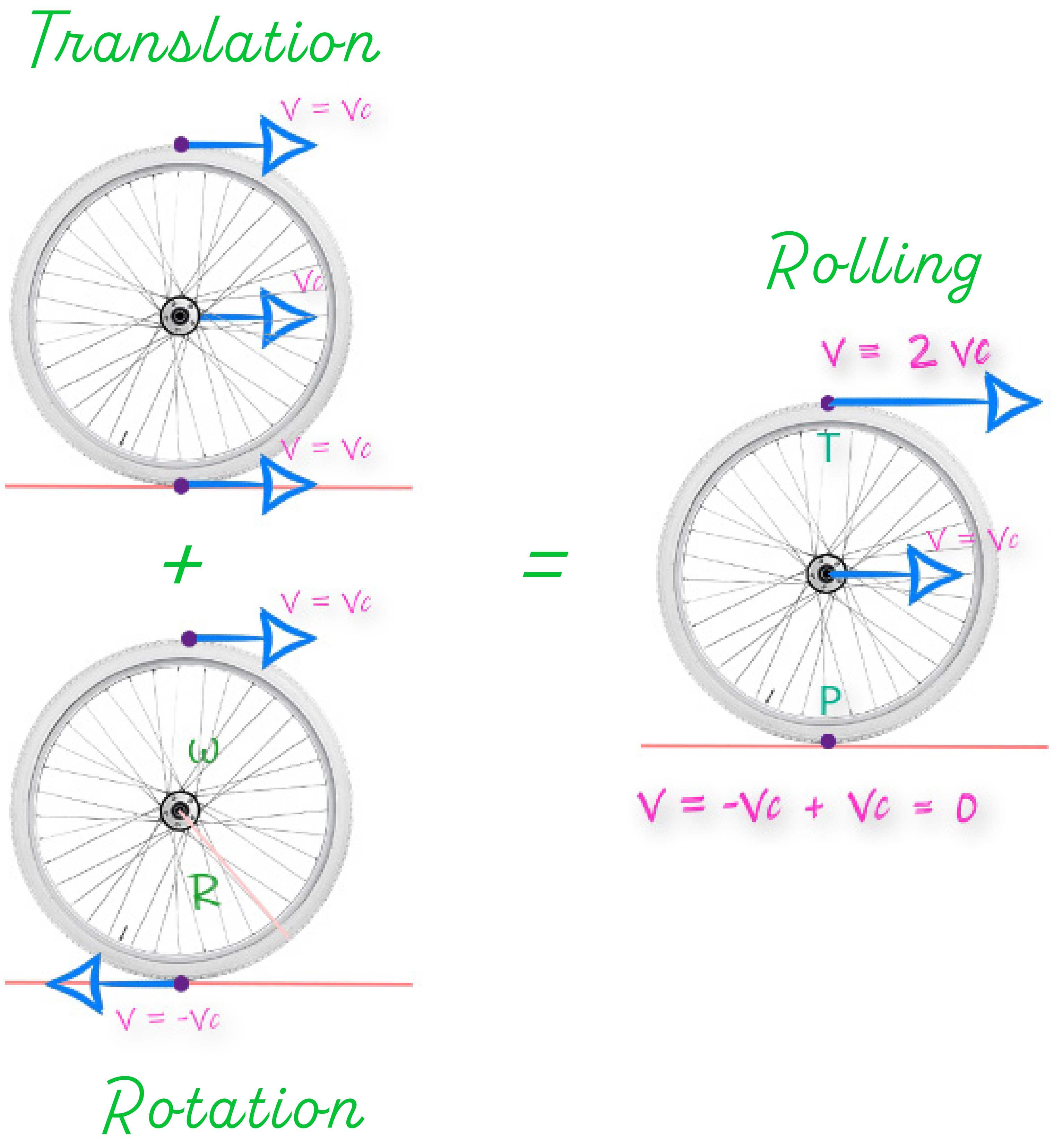


Here

$$\omega R < v_c$$

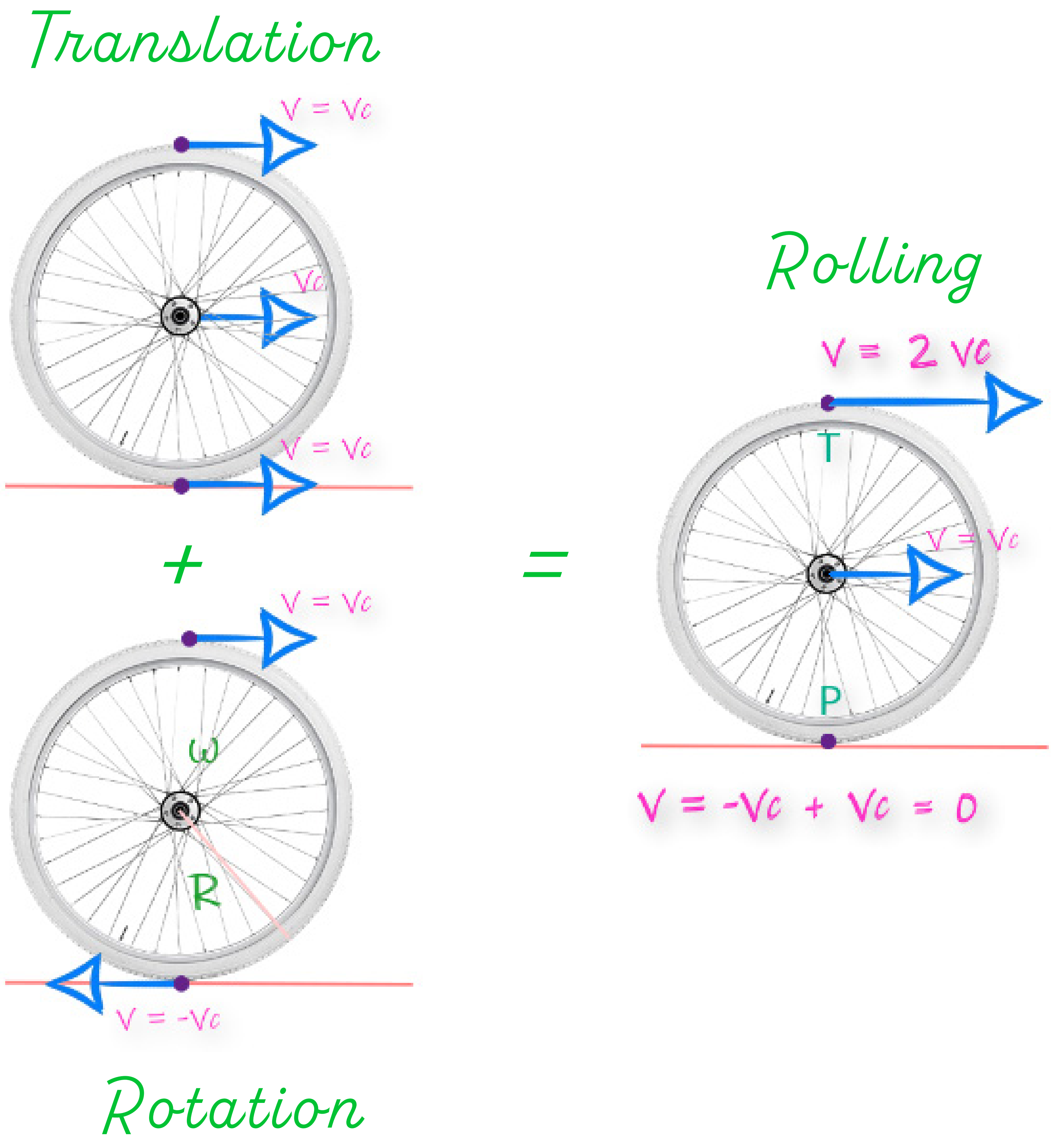
ROLLING = TRANSLATIONAL + ROTATIONAL MOTION

1. Pure Translation: Every point on the wheel moves forward with the same velocity
2. Pure Rotation: The wheel rotates about its axis, but the center does not move forward.
3. Combined Motion: When translation and rotation combine, rolling motion occurs.



VELOCITY ANALYSIS

1. Bottom of the Wheel (P):
   - Momentarily stationary as translational & rotational velocities cancel out.
   - If this point had a net velocity, the wheel would skid, breaking the "rolling without slipping" condition.
2. Top of the Wheel (T):
   - Moves at 2v_c since translational and rotational velocities add up in the same direction.



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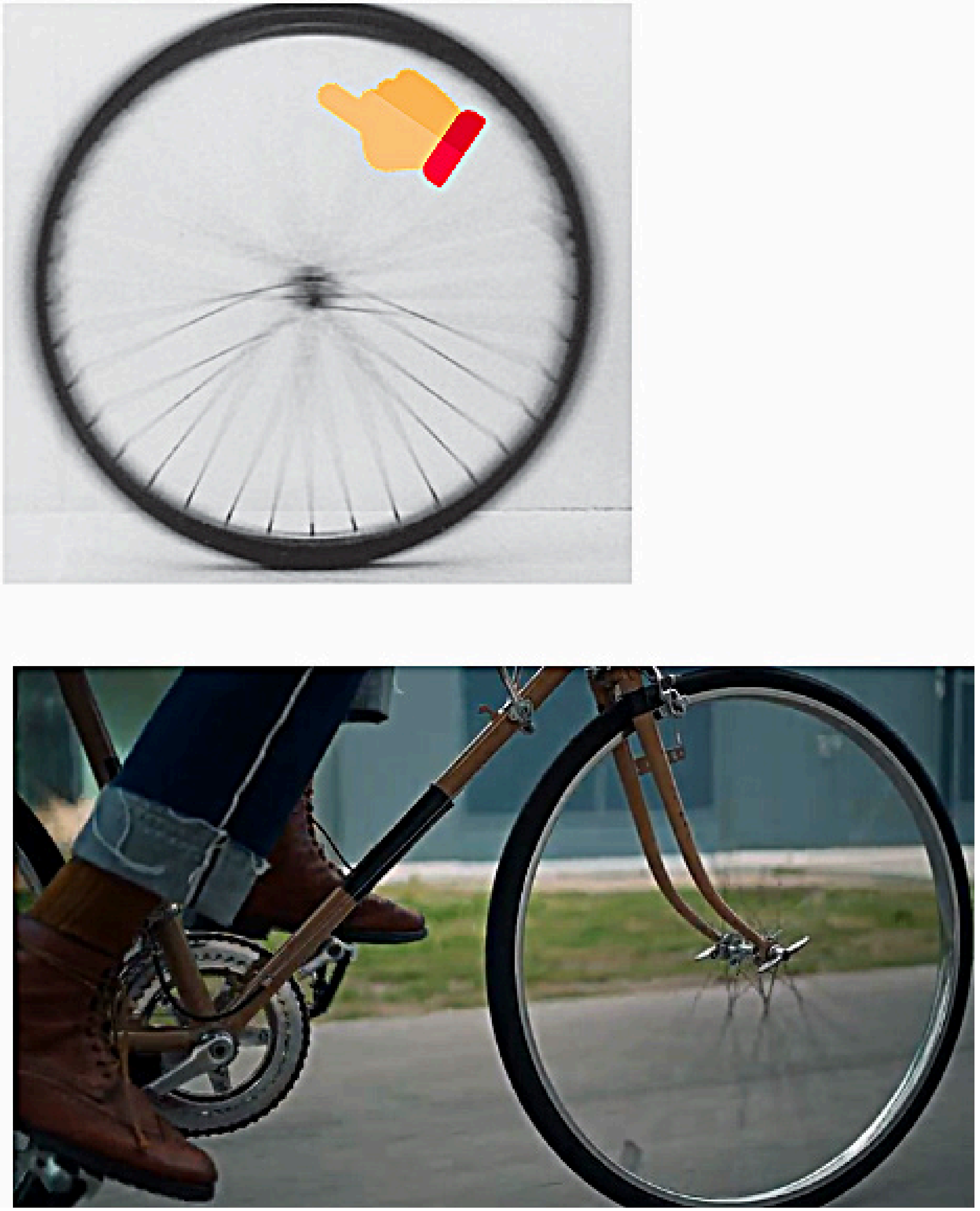
VELOCITY ANALYSIS



1. Observational Evidence: In a time-exposure photograph of a rolling wheel, the spokes appear more blurred at the top than at the bottom.
2. This shows that at the top, velocities add up, making it move at $2v_c$

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KEY FORMULAS & EQUATIONS

Formula/Equation	When to Use	Caution/Notes
$v_{com} = \omega R$	To ensure rolling without slipping.	Ensure that v_{com} and ω are for the same point and in consistent units.
$s = \theta \times R$	To calculate the distance traveled by the wheel.	θ must be in radians; R is the radius of the rolling object.
$\omega = v_{com} / R$	To find angular velocity from linear velocity.	Valid only when rolling without slipping condition is met.
$v_{bottom} = 0$	To identify the velocity at the contact point (P).	Only valid under rolling without slipping.
$v_{top} = 2v_{com}$	To calculate the velocity of the topmost point (T).	Applies when the object is rolling without slipping.

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COMMON MISTAKES AND MISCONCEPTIONS

Misunderstanding the Condition for Rolling Without Slipping:

Mistake: Believing that rolling without slipping does not require friction.

Clarification: Friction is essential to prevent slipping; it ensures no relative motion between the contact point and the surface.

Confusion Between Angular and Linear Quantities:

Mistake: Mixing up angular velocity (ω) with linear velocity (v).

Clarification: Remember the relationship $v = \omega R$. Angular velocity applies to rotation, while linear velocity is for translational motion.

COMMON MISTAKES AND MISCONCEPTIONS

Velocity at the Contact Point (P):

Mistake: Believing the contact point is stationary with respect to the wheel.

Clarification: The contact point is momentarily stationary relative to the ground but not relative to the wheel.

Top Point Velocity Misconception:

Mistake: Thinking the velocity of the topmost point (T) is equal to v_c

Clarification: At the topmost point, the velocities from rotation and translation add up, resulting in 2v_c

COMMON MISTAKES AND MISCONCEPTIONS

Ignoring Slipping Cases:

Mistake: Not analyzing what happens when the condition $v_c = \omega R$ fails.

Clarification: Slipping or skidding occurs when the tangential velocity (ωR) is not equal to the linear velocity (v_c), leading to skid marks or energy loss.