1. **Write out the following sets in list notation.**

   a. $\{x \mid 0 < x < 10 \text{ and } x \in \mathbb{Z}\}$
   b. $\{x \in \mathbb{Z} \mid 5 < x < 10 \text{ and } \frac{x}{2} < 4\}$
   c. $\{x^2 \mid x \in \mathbb{Z}\}$
   d. $\{x \in \mathbb{Z} \mid x^2 - 4 = 0\}$
   e. $\{x \mid x \text{ is even}\}$
   f. $\{x \mid x \text{ is a multiple of } 3\}$
   g. $\{x \in \mathbb{Z} \mid |2x| < 7\}$
   h. $\{3x + 2 \mid x \in \mathbb{Z}\}$

2. **Write out the following sets in set-builder notation.** These may have multiple possible solutions. Post yours in the comments to verify.

   a. $\{..., -3, -1, 1, 3, ...\}$
   b. $\{1, 8, 27, 64, ...\}$
   c. $\{-4, 4\}$
   d. $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...\}$
   e. $\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, ...\}$
   f. $\{1, -2, 3, -4, 5, -6, ...\}$
   g. $\{-2, 0, 2\}$

3. **Is each equivalency true or false?**

   a. $\{0, 2, 4, 6\} = \{x \mid x \text{ is even and } 0 < x < 6\}$
   b. $\{0, 2, 4, 6\} = \{x \mid x \text{ is even and } 0 \leq x \leq 6\}$
   c. $\{1, e, e^2, e^3, ...\} = \{e^x \mid x \in \mathbb{Z}\}$
   d. $\{..., \frac{1}{e^2}, \frac{1}{e}, 1, e, e^2, e^3, ...\} = \{e^x \mid x \in \mathbb{Z}\}$
   e. $\{3, 5, 7\} = \{|2x + 1| \mid x \in \mathbb{Z} \text{ and } -2 \leq x \leq 2\}$

4. **Modify the conditions on $\{|2x + 1| \mid x \in \mathbb{Z}$ and $-2 \leq x \leq 2\}$ to make it equivalent to $\{3, 5, 7\}$.**

Solutions are presented on the next page!

1. **Write out the following sets in list notation.**

   a. $\{x \mid 0 < x < 10 \text{ and } x \in \mathbb{Z}\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
   This says we want the set of x such that x is an integer between 0 and 10 exclusive.

   b. $\{x \in \mathbb{Z} \mid 5 < x < 10 \text{ and } \frac{x}{2} < 4\} = \{5, 6, 7\}$
   This says we want the set of x in the integers such that x is between 5 and 10 exclusive and $\frac{x}{2}$ is less than 4.

   c. $\{x^2 \mid x \in \mathbb{Z}\} = \{1, 4, 9, 16, ...\}$
   This says that for every x in the integers, we add x^2 to our set.

   d. $\{x \in \mathbb{Z} \mid x^2 - 4 = 0\} = \{-2, 2\}$
   This says that we want to include all integer solutions of $x^2 - 4 = 0$ into our set.
   $x^2 - 4 = 0$
   $x^2 = 4$
   $x = \pm 2$

   e. $\{x \mid x \text{ is even}\} = \{..., -4, -2, 0, 2, 4, ...\}$

   f. $\{x \mid x \text{ is a multiple of } 3\} = \{..., -6, -3, 0, 3, 6, ...\}$

   g. $\{x \in \mathbb{Z} \mid |2x| < 7\} = \{-3, -2, -1, 0, 1, 2, 3\}$
   This says we want to include all integers where $|2x| < 7$
   $|2x| < 7$
   $-7 < 2x < 7$
   $-\frac{7}{2} < x < \frac{7}{2}$

   h. $\{3x + 2 \mid x \in \mathbb{Z}\} = \{..., -4, -1, 2, 5, 8, 11, ...\}$
   This says we want to include all multiples of $3x + 2$ for x in the integers.

2. **Write out the following sets in set-builder notation.** These may have multiple possible solutions. Post yours in the comments to verify.

   a. $\{..., -3, -1, 1, 3, ...\} = \{x \mid x \text{ is odd}\}$ or $\{2x + 1 \mid x \in \mathbb{Z}\}$

   b. $\{1, 8, 27, 64, ...\} = \{x^3 \mid x \in \mathbb{Z} \text{ and } x > 0\}$

   c. $\{-4, 4\} = \{x \mid x^2 - 16 = 0\}$

   d. $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...\} = \{\frac{1}{x} \mid x \in \mathbb{Z} \text{ and } x > 0\}$

   e. $\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, ...\} = \{\frac{1}{x^2} \mid x \in \mathbb{Z} \text{ and } x > 0\}$

   f. $\{-1, 2, -3, 4, -5, 6, ...\} = \{x(-1)^x \mid x \in \mathbb{Z} \text{ and } x > 0\}$
   This one is tougher. To get the alternating sequence of positives and negatives we use $(-1)^x$ so that when x is odd it's negative, and when x is even it's positive. Then we multiply by x to get our increasing values.

   g. $\{-2, 0, 2\} = \{x \mid x^2 - 4 = 0 \text{ or } x = 0\}$
   There are probably elegant ways of doing this, but adding "or" allows us to specify another condition that can be satisfied.

3. **Is each equivalency true or false?**

   a. $\{0, 2, 4, 6\} = \{x \mid x \text{ is even and } 0 < x < 6\}$
   False. It should not include 0 and 6.

   b. $\{0, 2, 4, 6\} = \{x \mid x \text{ is even and } 0 \leq x \leq 6\}$
   True.

   c. $\{1, e, e^2, e^3, ...\} = \{e^x \mid x \in \mathbb{Z}\}$
   False. We did not consider the negative integers.

   d. $\{..., \frac{1}{e^2}, \frac{1}{e}, 1, e, e^2, e^3, ...\} = \{e^x \mid x \in \mathbb{Z}\}$
   True.

   e. $\{3, 5, 7\} = \{|2x + 1| \mid x \in \mathbb{Z} \text{ and } -2 \leq x \leq 2\}$
   False. This set should include the numbers $1, 3, 5$.

4. **Modify the conditions on $\{|2x + 1| \mid x \in \mathbb{Z}$ and $-2 \leq x \leq 2\}$ to make it equivalent to $\{3, 5, 7\}$.**
   $\{|2x + 1| \mid x \in \mathbb{Z} \text{ and } 1 \leq x \leq 3\}$