

Beginning and Intermediate Algebra

Student Solutions Manual

Complete worked solutions to odd problems

Solutions manual has not been cross checked for accuracy.

If you disagree with this solutions manual you should check with your instructor.

Should you find an error, please E-mail tylerw@bigbend.edu so it can be corrected.

Thank you!



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Beginning and Intermediate Algebra
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Chapter 0: Arithmetic

0.1

$$1) \quad 1 - 3 \\ 1 + (-3) = -2$$

$$3) \quad (-6) - (-8) \\ -6 + 8 = 2$$

$$5) \quad (-3) - 3 \\ (-3) + (-3) = -6$$

$$7) \quad 3 - (-5) \\ 3 + 5 = 8$$

$$9) \quad (-7) - (-5) \\ -7 + 5 = -2$$

$$11) \quad 3 - (-1) \\ 3 + 1 = 4$$

$$13) \quad 6 - 3 \\ 6 - 3 = 3$$

$$15) \quad (-5) + 3 = 2$$

$$17) \quad 2 - 3 \\ 2 + (-3) = -1$$

$$19) \quad (-8) - (-5) \\ -8 + 5 = -3$$

$$21) \quad (-2) + (-5) = -7$$

$$23) \quad 5 - (-6) \\ 5 + 6 = 11$$

$$25) \quad -6 + 3 = -3$$

$$27) \quad 4 - 7 \\ 4 + (-7) = -3$$

$$29) \quad (-7) + 7 = 0$$

$$31) \quad (4)(-1) = -4$$

$$33) \quad (10)(-8) = -80$$

$$35) \quad (-4)(-2) = 8$$

$$37) \quad (-7)(8) = -56$$

$$39) \quad (9)(-4) = -36$$

$$41) \quad (-5)(2) = -10$$

$$43) \quad (-5)(4) = -20$$

$$45) \quad (4)(-6) = -24$$

$$47) \quad \frac{-49}{-7} = 7$$

$$49) \quad \frac{-2}{-1} = 2$$

$$51) \quad \frac{20}{10} = 2$$

$$53) \quad \frac{-35}{-5} = 7$$

$$55) \quad \frac{-8}{-2} = 4$$

$$57) \quad \frac{-16}{2} = -8$$

$$59) \quad \frac{60}{-10} = -6$$

0.2

$$1) \quad \frac{42}{12} = \frac{21}{6} = \frac{7}{2}$$

$$3) \quad \frac{35}{25} = \frac{7}{5}$$

$$5) \quad \frac{54}{36} = \frac{27}{18} = \frac{9}{6} = \frac{3}{2}$$

$$7) \quad \frac{45}{36} = \frac{15}{12} = \frac{5}{4}$$

$$9) \quad \frac{27}{18} = \frac{9}{6} = \frac{3}{2}$$

$$11) \quad \frac{40}{16} = \frac{20}{8} = \frac{10}{4} = \frac{5}{2}$$

$$13) \quad \frac{63}{18} = \frac{21}{6} = \frac{7}{2}$$

$$15) \quad \frac{80}{60} = \frac{40}{30} = \frac{20}{15} = \frac{4}{3}$$

$$17) \quad \frac{72}{60} = \frac{36}{30} = \frac{18}{15} = \frac{6}{5}$$

$$19) \quad \frac{36}{24} = \frac{18}{12} = \frac{9}{6} = \frac{3}{2}$$

$$21) \quad \frac{1 \cancel{9}}{1} \cdot \frac{\cancel{8}}{\cancel{9}} = \frac{8}{1} = 8$$

$$23) \quad \frac{(2)}{1} \cdot -\frac{2}{9} = -\frac{4}{9}$$

$$25) \quad \frac{-1 \cancel{2}}{\cancel{2}} \cdot \frac{(13)}{\cancel{8}} = -\frac{13}{4}$$

$$27) \quad \frac{-3 \cancel{6}}{\cancel{5}} \cdot \frac{(-11)}{\cancel{8}} = \frac{33}{20}$$

$$29) \quad \frac{4 \cancel{8}}{\cancel{1}} \cdot \frac{\cancel{1}}{\cancel{2}} = \frac{4}{1} = 4$$

$$31) \frac{\cancel{2}}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{4}} = \frac{1}{2}$$

$$33) \frac{\cancel{1} \cdot \cancel{2} \cdot \cancel{3}}{\cancel{1} \cdot \cancel{2} \cdot \cancel{1}} = \frac{3}{1} = 3$$

$$35) \left(\frac{1}{2}\right) \left(-\frac{7}{5}\right) = -\frac{7}{10}$$

$$37) \frac{-2}{1} \div \frac{7}{4} \\ \left(\frac{-2}{1}\right) \left(\frac{4}{7}\right) = -\frac{8}{7}$$

$$39) \frac{-1}{9} \div \frac{-1}{2} \\ \left(\frac{-1}{9}\right) \left(\frac{2}{-1}\right) = \frac{2}{9}$$

$$41) \frac{-3}{2} \div \frac{13}{7} \\ \left(\frac{-3}{2}\right) \left(\frac{7}{13}\right) = -\frac{21}{26}$$

$$43) \frac{-1}{1} \div \frac{2}{3} \\ \left(\frac{-1}{1}\right) \left(\frac{3}{2}\right) = -\frac{3}{2}$$

$$45) \frac{8}{9} \div \frac{1}{5} \\ \left(\frac{8}{9}\right) \left(\frac{5}{1}\right) = \frac{40}{9}$$

$$47) \frac{-9}{7} \div \frac{1}{5} \\ \left(\frac{-9}{7}\right) \left(\frac{5}{1}\right) = -\frac{45}{7}$$

$$49) \frac{-2}{9} \div \frac{-3}{2} \\ \left(\frac{-2}{9}\right) \left(\frac{2}{-3}\right) = \frac{4}{27}$$

$$51) \frac{1}{10} \div \frac{3}{2} \\ 5 \left(\frac{1}{10}\right) \left(\frac{2}{3}\right) = \frac{1}{15}$$

$$53) \frac{1}{3} + \left(-\frac{4}{3}\right) = -\frac{3}{3} = -1$$

$$55) \frac{3}{7} - \frac{1}{7} = \frac{2}{7}$$

$$57) \frac{11}{6} + \frac{7}{6} = \frac{18}{6} = 3$$

$$59) \left(\frac{4}{4}\right) \frac{3}{5} + \frac{5}{4} \left(\frac{4}{4}\right) \\ \frac{12}{20} + \frac{25}{20} = \frac{37}{20}$$

$$61) \left(\frac{4}{4}\right) \frac{2}{5} + \frac{5}{4} \left(\frac{5}{5}\right) \\ \frac{8}{20} + \frac{25}{20} = \frac{33}{20}$$

$$63) \left(\frac{7}{7}\right) \frac{9}{8} + \left(\frac{-2}{7}\right) \left(\frac{8}{8}\right) \\ \frac{63}{56} + \frac{-16}{56} = \frac{47}{56}$$

$$65) \left(\frac{3}{3}\right) \frac{1}{1} + \left(-\frac{1}{3}\right) \\ \frac{3}{3} + \frac{-1}{3} = \frac{2}{3}$$

$$67) \left(\frac{-1}{2}\right) + \frac{3}{2} = \frac{2}{2} = 1$$

$$69) \left(\frac{4}{4}\right) \frac{1}{5} + \frac{3}{4} \left(\frac{5}{5}\right) \\ \frac{4}{20} + \frac{15}{20} = \frac{19}{20}$$

$$71) \left(\frac{8}{8}\right) \frac{-5}{7} - \frac{15}{8} \left(\frac{7}{7}\right) \\ \frac{-40}{56} + \frac{-105}{56} = \frac{-145}{56}$$

$$73) \left(\frac{7}{7}\right) \frac{6}{1} - \frac{8}{7} \\ \frac{42}{7} - \frac{8}{7} = \frac{34}{7}$$

$$75) \left(\frac{4}{4}\right) \frac{3}{2} - \frac{15}{8} \\ \frac{12}{8} - \frac{15}{8} = -\frac{3}{8}$$

$$77) \left(\frac{3}{3}\right) \left(\frac{-15}{8}\right) + \frac{5}{3} \left(\frac{8}{8}\right) \\ \frac{-45}{24} + \frac{40}{24} = \frac{-5}{24}$$

$$79) \left(\frac{6}{6}\right) \frac{-1}{1} - \left(-\frac{1}{6}\right) \\ \frac{-6}{6} + \frac{1}{6} = -\frac{5}{6}$$

$$81) \frac{5}{3} - \left(-\frac{1}{3}\right) \\ \frac{5}{3} + \frac{1}{3} = \frac{6}{3} = 2$$

0.3

$$1) \frac{-6 \cdot 4(-1)}{-24(-1)} \\ \frac{24}{24}$$

$$5) \frac{8 \div 4 \cdot 2}{2 \cdot 2} \\ 4$$

$$3) \frac{3 + (8) \div [4]}{3 + 8 \div (4)} \\ \frac{3 + 2}{5}$$

$$7) \frac{[-9 - (2 - 5)] \div (-6)}{[-9 - (-3)] \div (-6)} \\ \frac{[(-6)] \div (-6)}{1}$$

$$\begin{aligned}
 9) \quad & -6 + (-3-3)^2 \div |3| \\
 & -6 + (6)^2 \div 3 \\
 & -6 + 36 \div 3 \\
 & \frac{-6 + 12}{6}
 \end{aligned}$$

$$\begin{aligned}
 13) \quad & \frac{[-1 - (-5)] |3 + 2|}{\frac{[4](5)}}{20}}
 \end{aligned}$$

$$\begin{aligned}
 11) \quad & 4 - 2|3^2 - 16| \\
 & 4 - 2|9 - 16| \\
 & 4 - 2|-7| \\
 & 4 - 2(7) \\
 & \frac{4 - 14}{-10}
 \end{aligned}$$

$$\begin{aligned}
 15) \quad & \frac{2+4|7+2^2|}{4 \cdot 2 + 5 \cdot 3} \\
 & \text{Numerator: } 2 + 4|7 + 2^2| \\
 & \quad 2 + 4|7 + 4| \\
 & \quad 2 + 4|11| \\
 & \quad 2 + 4(11) \\
 & \quad \frac{2 + 44}{46} \\
 & \text{Denominator: } \frac{4 \cdot 2 + 5 \cdot 3}{\frac{8 + 15}{23}}
 \end{aligned}$$

$$\text{Fraction: } \frac{46}{23} = 2$$

$$\begin{aligned}
 17) \quad & \frac{[6 \cdot 2] + 2 - (-6)] \left(-5 + \left| \left(-\frac{18}{6} \right) \right| \right)}{\frac{[12 + 2] - (-6)](-5 + |-3|)}{[14 - (-6)](-5 + (3))}} \\
 & \frac{[20](-2)}{-40}
 \end{aligned}$$

$$\begin{aligned}
 19) \quad & \frac{-13-2}{2-(-1)^3+(-6)-[-1-(-3)]} \\
 & \text{Numerator: } \frac{-13-2}{-15} \\
 & \text{Denominator: } 2 - (-1)^3 + (-6) - [-1 - (-3)] \\
 & \quad 2 - (-1)^3 + (-6) - [2] \\
 & \quad 2 - (-1) + (-6) - [2] \\
 & \quad \frac{3 + (-6) - [2]}{-3 - [2]} \\
 & \quad \frac{-5}{-5}
 \end{aligned}$$

$$\text{Fraction: } -\frac{15}{-5} = 3$$

$$21) 6 \cdot \frac{-8-4+(-4)-[-4-(-3)]}{(4^2+3^2) \div 5}$$

$$\begin{aligned} \text{Numerator: } & -8 - 4 + (-4) - [-4 - (-3)] \\ & \underline{-8 - 4} + (-4) - [-1] \\ & \underline{-12 + (-4)} - [-1] \\ & \underline{-16 - [-1]} \\ & -15 \end{aligned}$$

$$\begin{aligned} \text{Denominator: } & (4^2 + 3^2) \div 5 \\ & (16 + 9) \div 5 \\ & \underline{(25) \div 5} \\ & 5 \end{aligned}$$

$$\text{Fraction: } 6 \cdot \underline{-\frac{15}{5}} = 6 \cdot -3 = -18$$

$$23) \frac{2^3+4}{-18-6+(-4)-[-5(-1)(-5)]}$$

$$\begin{aligned} \text{Numerator: } & 2^3 + 4 \\ & \underline{8 + 4} \\ & 12 \end{aligned}$$

$$\begin{aligned} \text{Denominator: } & -18 - 6 + (-4) - [-5(-1)(-5)] \\ & -18 - 6 + (-4) - [5(-5)] \\ & \underline{-18 - 6} + (-4) - [-25] \\ & \underline{-24 + (-4)} - [-25] \\ & \underline{-28 - [-25]} \\ & -3 \end{aligned}$$

$$\text{Fraction: } \frac{12}{-3} = -4$$

$$25) \frac{5+3^2-24 \div 6 \cdot 2}{[5+3(2^2-5)]+|2^2-5|^2}$$

$$\begin{aligned} \text{Numerator: } & 5 + 3^2 - 24 \div 6 \cdot 2 \\ & 5 + 9 - \underline{24 \div 6} \cdot 2 \\ & 5 + 9 - \underline{4 \cdot 2} \\ & \underline{5 + 9} - 8 \\ & \underline{14 - 8} \\ & 6 \end{aligned}$$

$$\begin{aligned} \text{Denominator: } & [5 + 3(2^2 - 5)] + |2^2 - 5|^2 \\ & [5 + 3(4 - 5)] + \underline{|4 - 5|^2} \\ & [5 + \underline{3(-1)}] + |-1|^2 \\ & \underline{5 - 3} + (-1)^2 \\ & \underline{2} + \underline{(-1)^2} \\ & \underline{2 + 1} \\ & 3 \end{aligned}$$

$$\text{Fraction: } \frac{6}{3} = 2$$

0.4

1) $p + 1 + q - m$, use $m = 1, p = 3, q = 4$

$$\begin{aligned} & \underline{(3) + 1} + (4) - (1) \\ & \underline{4 + (4)} - (1) \\ & \underline{8 - (1)} \\ & 7 \end{aligned}$$

3) $p - \frac{pq}{6}$, use $p = 6, q = 5$

$$\begin{aligned} & (6) - \frac{\cancel{(6)}(5)}{\cancel{6}} \\ & \underline{(6) - (5)} \\ & 1 \end{aligned}$$

5) $c^2 - (a - 1)$, use $a = 3, c = 5$

$$\begin{aligned} & (5)^2 - \underline{[(3) - 1]} \\ & \underline{(5)^2 - [2]} \\ & \underline{25 - [2]} \\ & 23 \end{aligned}$$

7) $5j + \frac{kh}{2}$, use $h = 5, j = 4, k = 2$

$$\begin{aligned} & 5(4) + \frac{\cancel{(2)}(5)}{\cancel{2}} \\ & \underline{5(4) + 5} \\ & \underline{20 + 5} \\ & 25 \end{aligned}$$

9) $\frac{4-(p-m)}{2} + q$, use $m = 4, p = 6, q = 6$

$$\begin{aligned} & \frac{4 - \underline{[(6) - (4)]}}{2} + (6) \\ & \underline{\frac{4 - [2]}{2}} + (6) \\ & \frac{2}{\underline{[2]}} + 6 \\ & 1 + 6 = 7 \end{aligned}$$

11) $m + n + m + \frac{n}{2}$, use $m = 1, n = 2$

$$\begin{aligned} & (1) + (2) + (1) + \frac{(2)}{\underline{[2]}} \\ & \underline{(1) + (2)} + (1) + 1 \\ & \underline{3 + (1)} + 1 \\ & \underline{4 + 1} \\ & 5 \end{aligned}$$

13) $q - p - (q - 1 - 3)$, use $p = 3, q = 6$

$$\begin{aligned} & (6) - (3) - \underline{[(6) - 1 - 3]} \\ & (6) - (3) - \underline{[5 - 3]} \\ & \underline{(6) - (3)} - [2] \\ & \underline{3 - [2]} \\ & 1 \end{aligned}$$

15) $y - [4 - y - (z - x)]$, use $x = 3, y = 1, z = 6$

$$\begin{aligned} & (1) - \{4 - (1) - \underline{[(6) - (3)]}\} \\ & (1) - \{4 - (1) - [3]\} \\ & (1) - \underline{\{3 - [3]\}} \\ & \underline{(1) - \{0\}} \\ & 1 \end{aligned}$$

17) $k \times 3^2 - (j + k) - 5$, use $j = 4, k = 5$

$$\begin{aligned} & (5)3^2 - \underline{[(4) + (5)]} - 5 \\ & (5)\underline{3^2} - [9] - 5 \\ & \underline{(5)9} - [9] - 5 \\ & \underline{45 - [9]} - 5 \\ & \underline{36 - 5} \\ & 31 \end{aligned}$$

19) $zx - \left(z - \frac{4+x}{6}\right)$, use $x = 2, z = 6$

$$\begin{aligned} & (6)(2) - \left[(6) - \frac{4 + \underline{(2)}}{6}\right] \\ & (6)(2) - \left[(6) - \frac{6}{\underline{[6]}}\right] \\ & (6)(2) - \underline{[(6) - 1]} \\ & \underline{(6)(2)} - [5] \\ & \underline{12 - [5]} \\ & 7 \end{aligned}$$

21) $r - 9 + 10$

$$r + 1$$

23) $n + n$

$$2n$$

25) $8v + 7v$

$$15v$$

$$27) -7x - 2x \\ -9x$$

$$29) k - 2 + 7 \\ k + 5$$

$$31) x - 10 - 6x + 1 \\ -5x - 9$$

$$33) m - 2m \\ -m$$

$$35) 9n - 1 + n + 4 \\ 10n + 3$$

$$37) -8(x - 4) \\ -8x + 32$$

$$39) 8n(n + 9) \\ 8n^2 + 72n$$

$$41) 7k(-k + 6) \\ -7k^2 + 42k$$

$$43) -6(1 + 6x) \\ -6 - 36x$$

$$45) 8m(5 - m) \\ 40m - 8m^2$$

$$47) -9x(4 - x) \\ -36x + 9x^2$$

$$49) -9b(b - 10) \\ -9b^2 + 90b$$

$$51) -8n(5 + 10n) \\ -40n - 80n^2$$

$$53) 9(b + 10) + 5b \\ 9b + 90 + 5b \\ 14b + 90$$

$$55) -3x(1 - 4x) - 4x^2 \\ -3x + 12x^2 - 4x^2 \\ 8x^2 - 3x$$

$$57) -4k^2 - 8k(8k + 1) \\ -4k^2 - 64k^2 - 8k \\ -68k^2 - 8k$$

$$59) 1 - 7(5 + 7p) \\ 1 - 35 - 49p \\ -49 - 49p$$

$$61) -10 - 4(n - 5) \\ -10 - 4n + 20 \\ -4n + 10$$

$$63) 4(x + 7) + 8(x + 4) \\ 4x + 28 + 8x + 32 \\ 12x + 60$$

$$65) -8(n + 6) - 8n(n + 8) \\ -8n - 48 - 8n^2 - 64n \\ -8n^2 - 72n - 48$$

$$67) 7(7 + 3v) + 10(3 - 10v) \\ 49 + 21v + 30 - 100v \\ -79v + 79$$

$$69) 2n(-10n + 5) - 7(6 - 10n) \\ -20n^2 + 10n - 42 + 70n \\ -20n^2 + 80n - 42$$

$$71) 5(1 - 6k) + 10(k - 8) \\ 5 - 30k + 10k - 80 \\ -20k - 75$$

$$73) (8n^2 - 3n) - (5 + 4n^2) \\ 8n^2 - 3n - 5 - 4n^2 \\ 4n^2 - 3n - 5$$

$$75) (5p - 6) + (1 - p) \\ 5p - 6 + 1 - p \\ 4p - 5$$

$$\begin{aligned} 77) (2 - 4v^2) + (3v^2 + 2v) \\ 2 - 4v^2 + 3v^2 + 2v \\ -v^2 + 2v + 2 \end{aligned}$$

$$\begin{aligned} 79) (4 - 2k^2) + (8 - 2k^2) \\ 4 - 2k^2 + 8 - 2k^2 \\ -4k^2 + 12 \end{aligned}$$

$$\begin{aligned} 81) (x^2 - 8) + (2x^2 - 7) \\ x^2 - 8 + 2x^2 - 7 \\ 3x^2 - 15 \end{aligned}$$

Chapter 1: Solving Linear Equations

1.1

$$1) \quad v + 9 = 16$$

$$\quad \underline{-9 \quad -9}$$

$$\quad \quad v = 7$$

$$3) \quad x - 11 = -16$$

$$\quad \underline{+11 \quad +11}$$

$$\quad \quad x = -5$$

$$5) \quad 30 = a + 20$$

$$\quad \underline{-20 \quad -20}$$

$$\quad \quad 10 = a$$

$$7) \quad x - 7 = -26$$

$$\quad \underline{+7 \quad +7}$$

$$\quad \quad x = -19$$

$$9) \quad 13 = n - 5$$

$$\quad \underline{+5 \quad +5}$$

$$\quad \quad 18 = n$$

$$11) \quad \frac{340}{-17} = \frac{-17x}{-17}$$

$$\quad \quad -20 = x$$

$$13) \quad (12)(-9) = \frac{n}{12}(12)$$

$$\quad \quad -108 = n$$

$$15) \quad \frac{20v}{20} = \frac{-160}{20}$$

$$\quad \quad v = -8$$

$$17) \quad \frac{340}{20} = \frac{20n}{20}$$

$$\quad \quad 17 = n$$

$$19) \quad \frac{16x}{16} = \frac{320}{16}$$

$$\quad \quad x = 20$$

$$21) \quad -16 + n = -13$$

$$\quad \underline{+16 \quad +16}$$

$$\quad \quad n = 3$$

$$23) \quad p - 8 = -21$$

$$\quad \underline{+8 \quad +8}$$

$$\quad \quad p = -13$$

$$25) \quad \frac{180}{12} = \frac{12x}{12}$$

$$\quad \quad 15 = x$$

$$27) \quad \frac{20b}{20} = \frac{-200}{20}$$

$$\quad \quad b = -10$$

$$29) \quad (14) \frac{r}{14} = \frac{5}{14}(14)$$

$$\quad \quad r = 5$$

$$31) \quad -7 = a + 4$$

$$\quad \underline{-4 \quad -4}$$

$$\quad \quad -11 = a$$

$$33) \quad 10 = x - 4$$

$$\quad \underline{+4 \quad +4}$$

$$\quad \quad 14 = x$$

$$35) \quad \frac{13a}{13} = \frac{-143}{13}$$

$$\quad \quad a = -11$$

$$37) \quad (20) \frac{p}{20} = -12(20)$$

$$\quad \quad p = -240$$

$$39) \quad 9 + m = -7$$

$$\quad \underline{-9 \quad -9}$$

$$\quad \quad m = -16$$

1.2

$$1) \quad 5 + \frac{n}{4} = 4$$

$$\quad \underline{-5 \quad -5}$$

$$\quad (4) \frac{n}{4} = -1(4)$$

$$\quad \quad n = -4$$

$$3) \quad 102 = -7r + 4$$

$$\quad \underline{-4 \quad -4}$$

$$\quad \frac{98}{-7} = (-7r)/-7$$

$$\quad -14 = r$$

$$5) \quad -8n + 3 = -77$$

$$\quad \underline{-3 \quad -3}$$

$$\quad \frac{-8n}{-8} = \frac{-80}{-8}$$

$$\quad \quad n = 10$$

$$7) \quad \frac{0}{-6} = \frac{-6v}{-6}$$

$$\quad \quad 0 = v$$

$$9) \quad -8 = \frac{x}{5} - 6$$

$$\quad \underline{+6 \quad +6}$$

$$\quad (5)(-2) = \frac{x}{5}(5)$$

$$\quad \quad -10 = x$$

$$11) \quad 0 = -7 + \frac{k}{2}$$

$$\quad \underline{+7 \quad +7}$$

$$\quad (2)7 = \frac{k}{2}(2)$$

$$\quad \quad 14 = k$$

$$13) \quad -12 + 3x = 0$$

$$\quad \underline{+12 \quad +12}$$

$$\quad \frac{3x}{3} = \frac{12}{3}$$

$$\quad \quad x = 4$$

$$\begin{array}{r}
 15) \quad 24 = 2n - 8 \\
 \quad +8 \quad +8 \\
 \hline
 \quad \frac{32}{2} = \frac{2n}{2} \\
 \quad 16 = n
 \end{array}$$

$$\begin{array}{r}
 25) \quad 56 + 8k = 64 \\
 \quad -56 \quad -56 \\
 \hline
 \quad \frac{8k}{8} = \frac{8}{8} \\
 \quad k = 1
 \end{array}$$

$$\begin{array}{r}
 35) \quad -40 = 4n - 32 \\
 \quad +32 \quad +32 \\
 \hline
 \quad \frac{-8}{4} = \frac{4n}{4} \\
 \quad -2 = n
 \end{array}$$

$$\begin{array}{r}
 17) \quad 2 = -12 + 2r \\
 \quad +12 \quad +12 \\
 \hline
 \quad \frac{14}{2} = \frac{2r}{2} \\
 \quad 7 = r
 \end{array}$$

$$\begin{array}{r}
 27) \quad -2x + 4 = 22 \\
 \quad -4 \quad -4 \\
 \hline
 \quad \frac{-2x}{-2} = \frac{18}{-2} \\
 \quad x = -9
 \end{array}$$

$$\begin{array}{r}
 37) \quad 87 = 3 - 7v \\
 \quad -3 \quad -3 \\
 \hline
 \quad \frac{84}{-7} = \frac{-7v}{-7} \\
 \quad -12 = v
 \end{array}$$

$$\begin{array}{r}
 19) \quad \frac{b}{3} + 7 = 10 \\
 \quad -7 \quad -7 \\
 \hline
 \quad (3)\frac{b}{3} = 3(3) \\
 \quad b = 9
 \end{array}$$

$$\begin{array}{r}
 29) \quad -20 = 4p + 4 \\
 \quad -4 \quad -4 \\
 \hline
 \quad \frac{-24}{4} = \frac{4p}{4} \\
 \quad -6 = p
 \end{array}$$

$$\begin{array}{r}
 39) \quad -x + 1 = -11 \\
 \quad -1 \quad -1 \\
 \hline
 \quad \frac{-x}{-1} = \frac{-12}{-1} \\
 \quad x = 12
 \end{array}$$

$$\begin{array}{r}
 21) \quad 152 = 8n + 64 \\
 \quad -64 \quad -64 \\
 \hline
 \quad \frac{88}{8} = \frac{8n}{8} \\
 \quad 11 = n
 \end{array}$$

$$\begin{array}{r}
 31) \quad -5 = 3 + \frac{n}{2} \\
 \quad -3 \quad -3 \\
 \hline
 \quad (2)(-8) = \frac{n}{2}(2) \\
 \quad -16 = n
 \end{array}$$

$$\begin{array}{r}
 23) \quad -16 = 8a + 64 \\
 \quad -64 \quad -64 \\
 \hline
 \quad \frac{-80}{8} = \frac{8a}{8} \\
 \quad -10 = a
 \end{array}$$

$$\begin{array}{r}
 33) \quad \frac{r}{8} - 6 = -5 \\
 \quad +6 \quad +6 \\
 \hline
 \quad (8)\frac{r}{8} = 1(8) \\
 \quad r = 8
 \end{array}$$

1.3

$$\begin{array}{r}
 1) \quad 2 - (-3a - 8) = 1 \\
 \quad 2 + 3a + 8 = 1 \\
 \quad 10 + 3a = 1 \\
 \quad -10 \quad -10 \\
 \hline
 \quad \frac{3a}{3} = \frac{-9}{3} \\
 \quad a = -3
 \end{array}$$

$$\begin{array}{r}
 5) \quad 66 = 6(6 + 5x) \\
 \quad 66 = 36 + 30x \\
 \quad -36 \quad -36 \\
 \hline
 \quad \frac{30}{30} = \frac{30x}{30} \\
 \quad 1 = x
 \end{array}$$

$$\begin{array}{r}
 3) \quad -5(-4 + 2v) = -50 \\
 \quad 20 - 10v = -50 \\
 \quad -20 \quad -20 \\
 \hline
 \quad -\frac{10v}{-10} = -\frac{70}{-10} \\
 \quad v = 7
 \end{array}$$

$$\begin{array}{r}
 7) \quad 0 = -8(p - 5) \\
 \quad 0 = -8p + 40 \\
 \quad -40 \quad -40 \\
 \hline
 \quad \frac{-40}{-8} = \frac{-8p}{-8} \\
 \quad 5 = p
 \end{array}$$

$$\begin{aligned}
9) \quad & -2 + 2(8x - 7) = -16 \\
& -2 + 16x - 14 = -16 \\
& 16x - 16 = -16 \\
& \quad \underline{+16 \quad +16} \\
& \quad \frac{16x}{16} = \frac{0}{16} \\
& \quad x = 0
\end{aligned}$$

$$\begin{aligned}
11) \quad & -21x + 12 = -6 - 3x \\
& \quad \underline{+21x \quad \quad +21x} \\
& 12 = -6 + 18x \\
& \quad \underline{+6 \quad +6} \\
& \quad \frac{18}{18} = \frac{18x}{18} \\
& 1 = x
\end{aligned}$$

$$\begin{aligned}
13) \quad & -1 - 7m = -8m + 7 \\
& \quad \underline{+8m \quad +8m} \\
& -1 + m = 7 \\
& \quad \underline{+1 \quad +1} \\
& \quad m = 8
\end{aligned}$$

$$\begin{aligned}
15) \quad & 1 - 12r = 29 - 8r \\
& \quad \underline{+12r \quad \quad +12r} \\
& 1 = 29 + 4r \\
& \quad \underline{-29 - 29} \\
& \quad \frac{-28}{4} = \frac{4r}{4} \\
& -7 = r
\end{aligned}$$

$$\begin{aligned}
17) \quad & 20 - 7b = -12b + 30 \\
& \quad \underline{+12b \quad +12b} \\
& 20 + 5b = 30 \\
& \quad \underline{-20 \quad -20} \\
& \quad \frac{5b}{5} = \frac{10}{5} \\
& \quad b = 2
\end{aligned}$$

$$\begin{aligned}
19) \quad & -32 - 24v = 34 - 2v \\
& \quad \underline{+24v \quad \quad +24v} \\
& -32 = 34 + 22v \\
& \quad \underline{-34 \quad -34} \\
& \quad \frac{-66}{22} = \frac{22v}{22} \\
& -3 = v
\end{aligned}$$

$$\begin{aligned}
21) \quad & -2 - 5(2 - 4m) = 33 + 5m \\
& -2 - 10 + 20m = 33 + 5m \\
& -12 + 20m = 33 + 5m \\
& \quad \quad \underline{-5m \quad \quad -5m} \\
& -12 + 15m = 33 \\
& \quad \underline{+12 \quad \quad +12} \\
& \quad \frac{15m}{15} = \frac{45}{15} \\
& \quad m = 3
\end{aligned}$$

$$\begin{aligned}
23) \quad & -4n + 11 = 2(1 - 8n) + 3n \\
& -4n + 11 = 2 - 16n + 3n \\
& -4n + 11 = 2 - 13n \\
& \quad \underline{+13n \quad \quad +13n} \\
& 9n + 11 = 2 \\
& \quad \underline{-11 \quad -11} \\
& \quad \frac{9n}{9} = \frac{-9}{9} \\
& \quad n = -1
\end{aligned}$$

$$\begin{aligned}
25) \quad & -6v - 29 = -4v - 5(v + 1) \\
& -6v - 29 = -4v - 5v - 5 \\
& -6v - 29 = -9v - 5 \\
& \quad \underline{+9v \quad \quad +9v} \\
& 3v - 29 = -5 \\
& \quad \underline{+29 \quad +29} \\
& \quad \frac{3v}{3} = \frac{24}{3} \\
& \quad v = 8
\end{aligned}$$

$$\begin{aligned}
27) \quad & 2(4x - 4) = -20 - 4x \\
& 8x - 8 = -20 - 4x \\
& \quad \underline{+4x \quad \quad +4x} \\
& 12x - 8 = -20 \\
& \quad \underline{+8 \quad +8} \\
& \quad \frac{12x}{12} = \frac{-12}{12} \\
& \quad x = -1
\end{aligned}$$

$$29) -a - 5(8a - 1) = 39 - 7a$$

$$-a - 40a + 5 = 39 - 7a$$

$$-41a + 5 = 39 - 7a$$

$$\begin{array}{r} +41a \qquad \qquad +41a \\ \hline 5 = 39 + 34a \end{array}$$

$$\begin{array}{r} -39 - 39 \\ \hline \end{array}$$

$$\frac{-34}{34} = \frac{34a}{34}$$

$$-1 = a$$

$$31) -57 = -(-p + 1) + 2(6 + 8p)$$

$$-57 = p - 1 + 12 + 16p$$

$$-57 = 17p + 11$$

$$\begin{array}{r} -11 \qquad \qquad -11 \\ \hline \end{array}$$

$$-\frac{68}{17} = \frac{17p}{17}$$

$$-4 = p$$

$$33) -2(m - 2) + 7(m - 8) = -67$$

$$-2m + 4 + 7m - 56 = -67$$

$$5m - 52 = -67$$

$$\begin{array}{r} +52 \qquad +52 \\ \hline \end{array}$$

$$\frac{5m}{5} = \frac{-15}{5}$$

$$m = -3$$

$$35) 50 = 8(7 + 7r) - (4r + 6)$$

$$50 = 56 + 56r - 4r - 6$$

$$50 = 52r + 50$$

$$\begin{array}{r} -50 \qquad \qquad -50 \\ \hline \end{array}$$

$$\frac{0}{52} = \frac{52r}{52}$$

$$0 = r$$

$$37) -8(n - 7) + 3(3n - 3) = 41$$

$$-8n + 56 + 9n - 27 = 41$$

$$n + 47 = 41$$

$$\begin{array}{r} -47 \qquad -47 \\ \hline \end{array}$$

$$n = -6$$

$$39) -61 = -5(5r - 4) + 4(3r - 4)$$

$$-61 = -25r + 20 + 12r - 16$$

$$-61 = -13r + 4$$

$$\begin{array}{r} -4 \qquad \qquad -4 \\ \hline \end{array}$$

$$\frac{-65}{-13} = \frac{-13r}{-13}$$

$$5 = r$$

$$41) -2(8n - 4) = 8(1 - n)$$

$$-16n + 8 = 8 - 8n$$

$$\begin{array}{r} +16n \qquad \qquad +16n \\ \hline \end{array}$$

$$8 = 8 + 8n$$

$$-8 \qquad -8$$

$$\frac{0}{8} = \frac{8n}{8}$$

$$0 = n$$

$$43) -3(-7v + 3) + 8v = 5v - 4(1 - 6v)$$

$$21v - 9 + 8v = 5v - 4 + 24v$$

$$29v - 9 = 29v - 4$$

$$\begin{array}{r} -29v \qquad -29v \\ \hline \end{array}$$

$$-9 = -4$$

false, No Solution \emptyset

$$45) -7(x - 2) = -4 - 6(x - 1)$$

$$-7x + 14 = -4 - 6x + 6$$

$$-7x + 14 = 2 - 6x$$

$$\begin{array}{r} +7x \qquad \qquad +7x \\ \hline \end{array}$$

$$14 = 2 + x$$

$$\begin{array}{r} -2 \qquad -2 \\ \hline \end{array}$$

$$12 = x$$

$$47) -6(8k + 4) = -8(6k + 3) - 2$$

$$-48k - 24 = -48k - 24 - 2$$

$$-48k - 24 = -48k - 26$$

$$\begin{array}{r} +48k \qquad \qquad +48k \\ \hline \end{array}$$

$$-24 = -26$$

false

No Solution \emptyset

$$49) -2(1 - 7p) = 8(p - 7)$$

$$-2 + 14p = 8p - 56$$

$$\begin{array}{r} -8p \qquad -8p \\ \hline \end{array}$$

$$-2 + 6p = -56$$

$$\begin{array}{r} +2 \qquad \qquad +2 \\ \hline \end{array}$$

$$\frac{6p}{6} = \frac{-54}{6}$$

$$p = -9$$

1.4

$$\begin{aligned}
 1) \quad \frac{3}{5}(1+p) &= \frac{21}{20} \\
 4) \quad \cancel{(20)} \frac{3}{5} + \cancel{4} \cancel{(20)} \frac{3}{5} p &= \frac{21}{\cancel{20}} \cancel{(20)} \\
 12 + 12p &= 21 \\
 \underline{-12} \quad \quad \underline{-12} \\
 \frac{12p}{12} &= \frac{9}{12} \\
 p &= \frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 11) \quad \frac{3}{2} \left(\frac{7}{3}n + 1 \right) &= \frac{3}{2} \\
 \cancel{(2)} \frac{7}{\cancel{2}} n + \cancel{(2)} \frac{3}{\cancel{2}} &= \frac{3}{\cancel{2}} \cancel{(2)} \\
 7n + 3 &= 3 \\
 \underline{-3} \quad \underline{-3} \\
 \frac{7n}{7} &= \frac{0}{7} \\
 n &= 0
 \end{aligned}$$

$$\begin{aligned}
 3) \quad 0 &= -\frac{5}{4}x - \frac{6}{5} \\
 (4) 0 &= \cancel{(4)} \left(-\frac{5}{\cancel{4}}x \right) + \frac{3}{\cancel{2}} \cancel{(4)}^2 \\
 0 &= -5x + 6 \\
 \underline{-6} \quad \quad \underline{-6} \\
 \frac{-6}{-5} &= \frac{-5x}{-5} \\
 \frac{6}{5} &= x
 \end{aligned}$$

$$\begin{aligned}
 13) \quad -a - \frac{5}{4} \left(-\frac{8}{3}a + 1 \right) &= -\frac{19}{4} \\
 -a(12) + \cancel{(12)} \frac{10}{\cancel{3}} a - \cancel{(12)} \frac{5}{\cancel{4}} &= -\frac{19}{\cancel{4}} \cancel{(12)}^3 \\
 -12a + 40a - 15 &= -57 \\
 28a - 15 &= -57 \\
 \underline{+15} \quad \underline{+15} \\
 \frac{28a}{28} &= \frac{-42}{28} \\
 a &= -\frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 5) \quad \cancel{6} \cancel{(24)} \frac{3}{\cancel{4}} - \cancel{6} \cancel{(24)} \frac{5}{\cancel{4}} m &= \frac{113}{\cancel{24}} \cancel{(24)} \\
 18 - 30m &= 113 \\
 \underline{-18} \quad \quad \underline{-18} \\
 -\frac{30m}{-30} &= \frac{95}{-30} \\
 m &= -\frac{19}{6}
 \end{aligned}$$

$$\begin{aligned}
 15) \quad \frac{55}{6} &= -\frac{5}{2} \left(\frac{3}{2}p - \frac{5}{3} \right) \\
 \cancel{(12)} \frac{55}{\cancel{6}} &= \cancel{(12)} - \frac{15}{\cancel{4}} p + \frac{25}{\cancel{6}} \cancel{(12)}^2 \\
 110 &= -45p + 50 \\
 \underline{-50} \quad \quad \underline{-50} \\
 \frac{60}{-45} &= \frac{-45p}{-45} \\
 -\frac{4}{3} &= p
 \end{aligned}$$

$$\begin{aligned}
 7) \quad \frac{635}{72} &= -\frac{5}{2} \left(-\frac{11}{4} + x \right) \\
 \cancel{(72)} \frac{635}{\cancel{72}} &= \cancel{(72)} \frac{55}{\cancel{8}} - \frac{5}{\cancel{2}} x \cancel{(72)}^3 \\
 635 &= 495 - 180x \\
 \underline{-495} \quad \underline{-495} \\
 \frac{140}{-180} &= \frac{-180x}{-180} \\
 -\frac{7}{9} &= x
 \end{aligned}$$

$$\begin{aligned}
 17) \quad \frac{16}{9} &= -\frac{4}{3} \left(-\frac{4}{3}n - \frac{4}{3} \right) \\
 \cancel{(9)} \frac{16}{\cancel{9}} &= \cancel{(9)} \frac{16}{\cancel{9}} n + \frac{16}{\cancel{9}} \cancel{(9)} \\
 16 &= 16n + 16 \\
 \underline{-16} \quad \underline{-16} \\
 \frac{0}{16} &= \frac{16n}{16} \\
 0 &= n
 \end{aligned}$$

$$\begin{aligned}
 9) \quad (5) 2b + \cancel{(5)} \frac{9}{\cancel{5}} &= -\frac{11}{\cancel{5}} \cancel{(5)} \\
 10b + 9 &= -11 \\
 \underline{-9} \quad \underline{-9} \\
 \frac{10b}{10} &= \frac{-20}{10} \\
 b &= -2
 \end{aligned}$$

$$\begin{aligned}
 19) \quad -\frac{5}{8} &= \frac{5}{4} \left(r - \frac{3}{2} \right) \\
 -\frac{5}{8}(\cancel{8}) &= (\cancel{8})\frac{5}{4}r - \frac{15}{8}(\cancel{8}) \\
 -5 &= 10r - 15 \\
 +15 &\quad +15 \\
 \hline
 \frac{10}{10} &= \frac{10r}{10} \\
 1 &= r
 \end{aligned}$$

$$\begin{aligned}
 21) \quad -\frac{11}{3} + \frac{3}{2}b &= \frac{5}{2} \left(b - \frac{5}{3} \right) \\
 -\frac{11}{3}(\cancel{6}) + (\cancel{6})\frac{3}{2}b &= (\cancel{6})\frac{5}{2}b - \frac{25}{6}(\cancel{6}) \\
 -22 + 9b &= 15b - 25 \\
 -9b &\quad -9b \\
 \hline
 -22 &= 6b - 25 \\
 +25 &\quad +25 \\
 \hline
 \frac{3}{6} &= \frac{6b}{6} \\
 \frac{1}{2} &= b
 \end{aligned}$$

$$\begin{aligned}
 23) \quad -\left(-\frac{5}{2}x - \frac{3}{2}\right) &= -\frac{3}{2} + x \\
 (\cancel{2})\frac{5}{2}x + (\cancel{2})\frac{3}{2} &= -\frac{3}{\cancel{2}}(\cancel{2}) + (2)x \\
 5x + 3 &= -3 + 2x \\
 -2x &\quad -2x \\
 \hline
 3x + 3 &= -3 \\
 -3 &\quad -3 \\
 \hline
 \frac{3x}{3} &= \frac{-6}{3} \\
 x &= -2
 \end{aligned}$$

$$\begin{aligned}
 25) \quad (\cancel{16})\frac{45}{16} + (\cancel{16})\frac{3}{2}n &= (\cancel{16})\frac{7}{4}n - \frac{19}{\cancel{16}}(\cancel{16}) \\
 45 + 24n &= 28n - 19 \\
 -24n &\quad -24n \\
 \hline
 45 &= 4n - 19 \\
 +19 &\quad +19 \\
 \hline
 \frac{64}{4} &= \frac{4n}{4} \\
 16 &= n
 \end{aligned}$$

$$\begin{aligned}
 27) \quad \frac{3}{2} \left(v + \frac{3}{2} \right) &= -\frac{7}{4}v - \frac{19}{6} \\
 \frac{3}{2}(\cancel{12})\frac{3}{2}v + \frac{3}{2}(\cancel{12})\frac{3}{2} &= -\frac{7}{4}v(\cancel{12}) - \frac{19}{6}(\cancel{12})^2 \\
 18v + 27 &= -21v - 38 \\
 +21v &\quad +21v \\
 \hline
 39v + 27 &= -38 \\
 -27 &\quad -27 \\
 \hline
 \frac{39v}{39} &= -\frac{65}{39} \\
 v &= -\frac{5}{3}
 \end{aligned}$$

$$\begin{aligned}
 29) \quad \frac{47}{9} + \frac{3}{2}x &= \frac{5}{3} \left(\frac{5}{2}x + 1 \right) \\
 (\cancel{18})\frac{47}{9} + (\cancel{18})\frac{3}{2}x &= (\cancel{18})\frac{25}{6}x + (\cancel{18})\frac{5}{3} \\
 94 + 27x &= 75x + 30 \\
 -27x &\quad -27x \\
 \hline
 94 &= 48x + 30 \\
 -30 &\quad -30 \\
 \hline
 \frac{64}{48} &= \frac{48x}{48} \\
 \frac{4}{3} &= x
 \end{aligned}$$

1.5

$$\begin{aligned}
 1) \quad \frac{ab}{a} &= \frac{c}{a} \text{ for } b \\
 b &= \frac{c}{a}
 \end{aligned}$$

$$\begin{aligned}
 3) \quad (g)\frac{f}{g}x &= b(g) \text{ for } x \\
 \frac{fx}{f} &= \frac{bg}{f} \\
 x &= \frac{bg}{f}
 \end{aligned}$$

$$\begin{aligned}
 5) \quad (b)3x &= \frac{a}{b}(b) \text{ for } x \\
 \frac{3bx}{3b} &= \frac{a}{3b} \\
 x &= \frac{a}{3b}
 \end{aligned}$$

$$\begin{aligned}
 7) \quad \frac{E}{c^2} &= \frac{mc^2}{c^2} \text{ for } m \\
 \frac{E}{c^2} &= m
 \end{aligned}$$

$$9) (3)V = (3)\frac{4}{3}\pi r^3 \text{ for } \pi$$

$$\frac{3v}{4r^3} = \frac{4\pi r^3}{4r^3}$$

$$\frac{3v}{4r^3} = \pi$$

$$11) a + c = b \text{ for } c$$

$$\frac{-a}{c} = \frac{-a}{b-a}$$

$$13) c(m+n) = \frac{4y}{m+n} (m+n) \text{ for } y$$

$$\frac{c(m+n)}{4} = \frac{4y}{4}$$

$$\frac{c(m+n)}{4} = y$$

$$15) (12)V = \frac{\pi Dn}{12} (12) \text{ for } D$$

$$\frac{12v}{\pi n} = \frac{\pi Dn}{\pi n}$$

$$\frac{12v}{\pi n} = D$$

$$17) \frac{p}{p-c} = \frac{n(p-c)}{p-c} \text{ for } n$$

$$\frac{p}{p-c} = n$$

$$19) (L)T = \frac{D-d}{L} (L) \text{ for } D$$

$$LT = D - d$$

$$\frac{+d}{LT+d} = \frac{+d}{D}$$

$$21) \frac{L}{1+at} = \frac{L_0(1+at)}{1+at}$$

$$\frac{L}{1+at} = L_0$$

$$23) 2m + p = 4m + q \text{ for } m$$

$$\frac{-2m}{p} = \frac{2m}{2m+q}$$

$$\frac{-q}{p-q} = \frac{-q}{2m+q}$$

$$\frac{p-q}{2} = \frac{2m}{2}$$

$$\frac{p-q}{2} = m$$

$$25) (r)\frac{k-m}{r} = q(r) \text{ for } k$$

$$k - m = qr$$

$$\frac{+m}{k} = \frac{+m}{qr+m}$$

$$27) h = vt - 16t^2 \text{ for } v$$

$$\frac{+16t^2}{h+16t^2} = \frac{+16t^2}{h+16t^2}$$

$$\frac{h+16t^2}{t} = \frac{vt}{t}$$

$$\frac{h+16t^2}{2} = v$$

$$29) Q_1 = P(Q_2 - Q_1) \text{ for } Q_2$$

$$Q_1 = PQ_2 - PQ_1$$

$$\frac{+PQ_1}{Q_1+PQ_1} = \frac{+PQ_1}{PQ_2}$$

$$\frac{Q_1+PQ_1}{P} = \frac{PQ_2}{P}$$

$$\frac{Q_1+PQ_1}{P} = Q_2$$

$$31) (d)R = \frac{kA(T_1+T_2)}{d} (d) \text{ for } T_1$$

$$dR = kA(T_1 + T_2)$$

$$dR = kAT_1 + kAT_2$$

$$\frac{-kAT_2}{dR-kAT_2} = \frac{-kAT_2}{kAT_1}$$

$$\frac{dR-kAT_2}{kA} = \frac{kAT_1}{kA}$$

$$\frac{dR-kAT_2}{kA} = T_1$$

$$33) ax + b = c \text{ for } a$$

$$\frac{-b}{ax} = \frac{-b}{c-b}$$

$$\frac{ax}{x} = \frac{c-b}{x}$$

$$a = \frac{c-b}{x}$$

$$35) \frac{twh}{lh} = \frac{v}{lh} \text{ for } w$$

$$w = \frac{v}{lh}$$

$$37) \frac{1}{a}(a) + b(a) = \frac{c}{a}(a) \text{ for } a$$

$$1 + ba = c$$

$$\frac{-1}{ba} = \frac{-1}{c-1}$$

$$\frac{ba}{b} = \frac{c-1}{b}$$

$$a = \frac{c-1}{b}$$

$$39) at - bw = 5 \text{ for } t$$

$$\begin{array}{r} +bw \quad +bw \\ \hline at = \frac{5+bw}{a} \\ t = \frac{5+bw}{a} \end{array}$$

$$41) ax + bx = c \text{ for } a$$

$$\begin{array}{r} -bx \quad -bx \\ \hline ax = \frac{c-bx}{x} \\ a = \frac{c-bx}{x} \end{array}$$

$$43) x + 5y = 3 \text{ for } y$$

$$\begin{array}{r} -x \quad -x \\ \hline 5y = \frac{3-x}{5} \\ y = \frac{3-x}{5} \end{array}$$

$$45) 3x + 2y = 7 \text{ for } y$$

$$\begin{array}{r} -3x \quad -3x \\ \hline 2y = \frac{7-3x}{2} \\ y = \frac{7-3x}{2} \end{array}$$

$$47) 5a - 7b = 4 \text{ for } b$$

$$\begin{array}{r} -5a \quad -5a \\ \hline -7b = \frac{4-5a}{-7} \\ b = \frac{4-5a}{-7} \end{array}$$

$$49) 4x - 5y = 8 \text{ for } y$$

$$\begin{array}{r} -4x \quad -4x \\ \hline -5y = \frac{8-4x}{-5} \\ y = \frac{8-4x}{-5} \end{array}$$

1.6

$$1) |m| = -6$$

false

No Solution∅

$$3) |n| = 4$$

$$n = 4, -4$$

$$5) |b| = 7$$

$$b = 7, -7$$

$$7) (7) \frac{|x|}{7} = 5(7)$$

$$|x| = 35$$

$$x = 35, -35$$

$$9) -10 + |k| = -15$$

$$\begin{array}{r} +10 \quad +10 \\ \hline |k| = -5 \end{array}$$

false

No Solution∅

$$11) 10|x| + 7 = 57$$

$$\begin{array}{r} -7 \quad -7 \\ \hline 10|x| = \frac{50}{10} \end{array}$$

$$|x| = 5$$

$$x = 5, -5$$

$$13) 10 - 5|m| = 70$$

$$\begin{array}{r} -10 \quad -10 \\ \hline -5|m| = \frac{60}{-5} \end{array}$$

$$|m| = -12$$

false

No Solution∅

$$15) 9|x| - 4 = 5$$

$$\begin{array}{r} +4 \quad +4 \\ \hline 9|x| = \frac{9}{9} \end{array}$$

$$|x| = 1$$

$$x = 1, -1$$

$$17) \left| \frac{n}{10} \right| = 1$$

$$(10) \frac{n}{10} = 1(10), -1(10)$$

$$n = 10, -10$$

$$19) |v + 10| = 2$$

$$v + 10 = 2, -2$$

$$\frac{-10 \quad -10 \quad -10}{v = -8, -12}$$

$$21) -4 - |a - 5| = -13$$

$$\frac{+4 \quad \quad \quad +4}{-|a-5| = \frac{-9}{-1}}$$

$$|a - 5| = 9$$

$$a - 5 = 9, -9$$

$$\frac{+5 \quad +5 \quad +5}{a = 14, -4}$$

$$23) \frac{10|-6x|}{10} = \frac{60}{10}$$

$$|-6x| = 6$$

$$-\frac{6x}{-6} = \frac{6}{-6}, -\frac{6}{-6}$$

$$x = -1, 1$$

$$25) \frac{-7\left|\frac{n}{7}\right|}{-7} = -\frac{2}{-7}$$

$$\left|\frac{n}{7}\right| = \frac{2}{7}$$

$$(7) \frac{n}{7} = \frac{2}{7}(7), -\frac{2}{7}(7)$$

$$n = 2, -2$$

$$27) -8|-7 + p| - 6 = -14$$

$$\frac{+6 \quad +6}{-8|-7+p| = \frac{-8}{-8}}$$

$$|-7 + p| = 1$$

$$-7 + p = 1, -1$$

$$\frac{+7 \quad +7 \quad +7}{p = 8, 6}$$

$$29) -3|7 + x| - 7 = -1$$

$$\frac{+7 \quad +7}{-3|7+x| = \frac{6}{-3}}$$

$$|7 + x| = -2$$

false

No Solution

$$31) |-7 - 5r| = 32$$

$$-7 - 5r = 32, -32$$

$$\frac{+7 \quad +7 \quad +7}{-\frac{5r}{-5} = \frac{39}{-5}, \frac{-25}{-5}}$$

$$r = -\frac{39}{5}, 5$$

$$33) |8n - 6| = 66$$

$$8n - 6 = 66, -66$$

$$\frac{+6 \quad +6 \quad +6}{\frac{8n}{8} = \frac{72}{8}, -\frac{60}{8}}$$

$$n = 9, -\frac{15}{2}$$

$$35) |2v + 7| = 11$$

$$2v + 7 = 11, -11$$

$$\frac{-7 \quad -7 \quad -7}{\frac{2v}{2} = \frac{4}{2}, -\frac{18}{2}}$$

$$v = 2, -9$$

$$37) \frac{9|10+6x|}{9} = \frac{72}{9}$$

$$|10 + 6x| = 8$$

$$10 + 6x = 8, -8$$

$$\frac{-10 \quad -10 \quad -10}{\frac{6x}{6} = -\frac{2}{6}, -\frac{18}{6}}$$

$$x = -\frac{1}{3}, -3$$

$$39) -3 + |6 + 6k| = -45$$

$$\frac{+3 \quad \quad \quad +3}{|6 + 6k| = -42}$$

false

No Solution

$$41) |2n + 5| + 5 = 0$$

$$\begin{array}{r} -5 \quad -5 \\ |2n + 5| = -5 \end{array}$$

false No Solution \emptyset

$$43) 3 - 2|5 - m| = 9$$

$$\begin{array}{r} -3 \quad -3 \\ -2|5-m| = \frac{6}{-2} \end{array}$$

$$|5 - m| = -3$$

false No Solution \emptyset

$$45) |-10x - 4| - 10 = 66$$

$$\begin{array}{r} +10 \quad +10 \\ |-10x - 4| = 76 \end{array}$$

$$-10x - 4 = 76, \quad -76$$

$$\begin{array}{r} +4 \quad +4 \quad +4 \\ \frac{-10x}{-10} = \frac{80}{-10}, \frac{-72}{-10} \end{array}$$

$$x = -8, \frac{36}{5}$$

$$47) |2 + 3x| = |4 - 2x|$$

$$\begin{array}{r} 2 + 3x = 4 - 2x, \quad 2 + 3x = -(4 - 2x) \\ +2x \quad +2x \quad \quad \quad 2 + 3x = -4 + 2x \end{array}$$

$$2 + 5x = 4$$

$$\begin{array}{r} -2 \quad -2 \\ 2 + x = -4 \end{array}$$

$$\frac{5x}{5} = \frac{2}{5}$$

$$x = \frac{2}{5}$$

$$\begin{array}{r} -2x \quad -2x \\ 2 + x = -4 \end{array}$$

$$\begin{array}{r} -2 \quad -2 \\ x = -6 \end{array}$$

$$x = \frac{2}{5}, -6$$

$$49) \left| \frac{2x-5}{3} \right| = \left| \frac{3x+4}{2} \right|$$

$$(6) \frac{2x-5}{3} = \frac{3x+4}{2} \quad (6), \quad \frac{2x-5}{3} = \frac{-(3x+4)}{2}$$

$$4x - 10 = 9x + 12 \quad (6) \quad \frac{2x-5}{3} = \frac{-3x-4}{2} \quad (6)$$

$$\begin{array}{r} -4x \quad -4x \quad \quad \quad 4x - 10 = -9x - 12 \end{array}$$

$$\begin{array}{r} -10 = 5x + 12 \quad +9x \quad +9x \end{array}$$

$$\begin{array}{r} -12 \quad -12 \quad \quad \quad 13x - 10 = -12 \end{array}$$

$$\frac{-22}{5} = \frac{5x}{5}$$

$$-\frac{22}{5} = x$$

$$\begin{array}{r} +10 \quad +10 \end{array}$$

$$\frac{13x}{13} = -\frac{2}{13}$$

$$x = -\frac{2}{13}$$

$$x = -\frac{22}{5}, -\frac{2}{13}$$

1.7

1) c varies directly as a

$$\frac{c}{a} = k$$

3) w varies inversely as x

$$wx = k$$

5) f varies jointly as x and y

$$\frac{f}{xy} = k$$

7) h is directly proportional to b

$$\frac{h}{b} = k$$

9) a is inversely proportional to b

$$ab = k$$

- 11) p is jointly proportional to q and r and p = 12 when q = 8 and r = 3

$$\frac{p}{qr} = k$$
$$\frac{12}{(8)(3)} = \frac{12}{24} = \frac{1}{2}$$

- 13) t varies directly as the square of u and t = 6 when u = 3

$$\frac{t}{u^2} = k$$
$$\frac{(6)}{(3)^2} = \frac{6}{9} = \frac{2}{3}$$

- 15) w is inversely proportional to the cube of x and w is 54 when x = 3

$$wx^3 = k$$
$$(54)(3)^3 = 54(27) = 1458$$

- 17) a is jointly proportional with the square of x and the square root of y and

$$a = 25 \text{ when } x = 5 \text{ and } y = 9$$

$$\frac{a}{x^2\sqrt{y}} = k$$
$$\frac{(25)}{(5)^2\sqrt{(9)}} = \frac{25}{25 \cdot 3} = \frac{1}{3}$$

- 19) The electrical current, in amperes, in a circuit varies directly as the voltage. When 15 volts are applied, the current is 5 amperes. What is the current when 18 volts are applied?

$$\frac{a}{v} = k$$
$$\frac{(5)}{15} = \frac{1}{3}$$
$$(18) \frac{a}{18} = \frac{1}{3} \quad (18)$$
$$a = 6 \text{ amperes}$$

- 21) Hooke's law states that the distance that a spring is stretched by hanging object varies directly as the mass of the object. If the distance is 20 cm when the mass is 3 kg, what is the distance when the mass is 5 kg?

$$\frac{d}{m} = k$$
$$\frac{20}{(3)} = \frac{20}{3}$$
$$\frac{x}{5} = \frac{20}{3}$$
$$x = \frac{100}{3} = 33.3 \text{ cm.}$$

- 23) The number of aluminum cans used each year varies directly as the number of people using the cans. If 250 people use 60,000 cans in one year, how many cans are used each year in Dallas, which has a population of 1,008,000?

$$\frac{c}{p} = k$$

$$\frac{60000}{250} = 250$$

$$(1,008,000) \frac{c}{1,000,000} = 250 (1,000,000)$$

$$c = 250,800,000 \text{ cans}$$

- 25) According to Fidelity Investment Vision Magazine, the average weekly allowance of children varies directly as their grade level. In a recent year, the average allowance of a 9th-grade student was 9.66 dollars per week. What was the average weekly allowance of a 4th-grade student?

$$\frac{a}{g} = k \quad \frac{(9.66)}{(9)} = 1.07$$

$$(4) \frac{x}{4} = 1.07(4)$$

$$x = \$4.28$$

- 27) The number of kilograms of water in a human body varies directly as the mass of the body. A 96-kg person contains 64 kg of water. How many kilo grams of water are in a 60-kg person?

$$\frac{w}{m} = k \quad \frac{(64)}{(96)} = \frac{2}{3}$$

$$(60) \frac{x}{60} = \frac{2}{3}(60)$$

$$x = 40 \text{ kg}$$

- 29) The weight of an object on Mars varies directly as its weight on Earth. A person weighs 95lb on Earth weighs 38 lb on Mars. How much would a 100-lb person weigh on Mars?

$$\frac{m}{e} = k \quad \frac{38}{95} = \frac{2}{5}$$

$$(100) \frac{m}{100} = \frac{2}{5}(100)$$

$$m = 40 \text{ lbs}$$

- 31) The time required to empty a tank varies inversely as the rate of pumping. If a pump can empty a tank in 45 min at the rate of 600 kL/min, how long will it take the pump to empty the same tank at the rate of 1000 kL/min?

$$tr = k \quad (45)(600) = 27000$$

$$\frac{t(1000)}{1000} = \frac{27000}{1000}$$

$$t = 27 \text{ min}$$

- 33) The stopping distance of a car after the brakes have been applied varies directly as the square of the speed r . If a car, traveling 60 mph can stop in 200 ft, how fast can a car go and still stop in 72 ft?

$$\begin{aligned} \frac{d}{r^2} &= k & \frac{(200)}{(60)^2} &= \frac{200}{600} = \frac{1}{18} \\ (r^2) \frac{72}{r^2} &= \frac{1}{18} r^2 \\ (18)(72) &= \frac{r^2}{18} (18) \\ \sqrt{1296} &= \sqrt{r^2} \\ 36 \text{ mph} &= r \end{aligned}$$

- 35) The intensity of a light from a light bulb varies inversely as the square of the distance from the bulb. Suppose intensity is 90 W/m^2 (watts per square meter) when the distance is 5 m. How much further would it be to a point where the intensity is 40 W/m^2 ?

$$\begin{aligned} Id^2 &= k & (90)(5)^2 &= k \\ 90(25) &= 2250 \\ \frac{40d^2}{40} &= \frac{2250}{40} \\ \sqrt{d^2} &= \sqrt{56.25} \\ d &= 7.5 \\ 7.5 - 5 &= 2.5 \text{ m further} \end{aligned}$$

- 37) The intensity of a television signal varies inversely as the square of the distance from the transmitter. If the intensity is 25 W/m^2 at a distance of 2 km, how far from the transmitter are you when the intensity is 2.56 W/m^2 ?

$$\begin{aligned} Id^2 &= k & (25)(2)^2 &= k \\ 25(4) &= 100 \\ \frac{(2.56)d^2}{2.56} &= \frac{100}{2.56} \\ \sqrt{d^2} &= \sqrt{39.0625} \\ d &= 6.25 \text{ m} \end{aligned}$$

1.8

- 1) When five is added to three more than a certain number, the result is 19. What is the number?

$$\begin{aligned} x + 3 + 5 &= 19 \\ x + 8 &= 19 \\ \underline{-8 \quad -8} & \\ x &= 11 \end{aligned}$$

- 3) When 18 is subtracted from six times a certain number, the result is -42 . What is the number?

$$\begin{array}{r}
 6x - 18 = -42 \\
 \underline{+18 \quad +18} \\
 \frac{6x}{6} = \frac{-24}{6} \\
 x = -4
 \end{array}$$

- 5) A number plus itself, plus twice itself, plus 4 times itself, is equal to -104 . What is the number?

$$\begin{array}{r}
 x + x + 2x + 4x = -104 \\
 \frac{8x}{8} = \frac{-104}{8} \\
 x = -13
 \end{array}$$

- 7) Eleven less than seven times a number is five more than six times the number. Find the number.

$$\begin{array}{r}
 7x - 11 = 6x + 5 \\
 \underline{-6x \quad -6x} \\
 x - 11 = 5 \\
 \underline{+11 \quad +11} \\
 x = 16
 \end{array}$$

- 9) The sum of three consecutive integers is 108. What are the integers?

$$\begin{array}{r}
 F: x \\
 S: x + 1 \\
 T: x + 2 \\
 \underline{3x + 3 = 108} \\
 \underline{-3 \quad -3} \\
 \frac{3x}{3} = \frac{105}{3} \\
 x = 35 \qquad 35, 36, 37
 \end{array}$$

- 11) Find three consecutive integers such that the sum of the first, twice the second, and three times the third is -76 .

$$\begin{array}{r}
 F: x \qquad \rightarrow x \\
 2S: 2(x + 1) \rightarrow 2x + 2 \\
 3T: 3(x + 2) \rightarrow 3x + 6 \\
 \underline{6x + 8 = -76} \\
 \underline{-8 \quad -8} \\
 \frac{6x}{6} = \frac{-84}{6} \\
 x = -14 \qquad -14, -13, -12
 \end{array}$$

13) The sum of three consecutive odd integers is 189. What are the integers?

$$\begin{array}{r}
 F: x \\
 S: x + 2 \\
 T: x + 4 \\
 \hline
 3x + 6 = 189 \\
 \underline{-6 \quad -6} \\
 3x = 183 \\
 \frac{3x}{3} = \frac{183}{3} \\
 x = 61 \qquad 61, 63, 65
 \end{array}$$

15) Find three consecutive odd integers such that the sum of the first, two times the second, and three times the third is 70.

$$\begin{array}{r}
 F: x \qquad \rightarrow x \\
 2S: 2(x + 2) \rightarrow 2x + 4 \\
 3T: 3(x + 4) \rightarrow 3x + 12 \\
 \hline
 6x + 16 = 70 \\
 \underline{-16 \quad -16} \\
 6x = 54 \\
 \frac{6x}{6} = \frac{54}{6} \\
 x = 9 \qquad 9, 11, 13
 \end{array}$$

17) Two angles of a triangle are the same size. The third angle is 12 degrees smaller than the first angle. Find the measure the angles.

$$\begin{array}{r}
 F: x \qquad (64) \\
 S: x \qquad (64) \\
 T: x - 12 \qquad (64 - 12 = 52) \\
 \hline
 3x - 12 = 180 \\
 \underline{+12 \quad +12} \\
 3x = 192 \\
 \frac{3x}{3} = \frac{192}{3} \\
 x = 64 \qquad 64^\circ, 64^\circ, 52^\circ
 \end{array}$$

19) The third angle of a triangle is the same size as the first. The second angle is 4 times the third. Find the measure of the angles.

$$\begin{array}{r}
 F: x \qquad (30) \\
 S: 4x \qquad (4 \cdot 30 = 120) \\
 T: x \qquad (30) \\
 \hline
 6x = 180 \\
 \frac{6x}{6} = \frac{180}{6} \\
 x = 30 \qquad 30^\circ, 120^\circ, 30^\circ
 \end{array}$$

- 21) The second angle of a triangle is twice as large as the first. The measure of the third angle is 20 degrees greater than the first. How large are the angles?

$$\begin{array}{ll}
 F: x & (40) \\
 S: 2x & (2 \cdot 40 = 80) \\
 T: \underline{x + 20} & (40 + 20 = 60) \\
 4x + 20 = 180 & \\
 \underline{-20 \quad -20} & \\
 \frac{4x}{4} = \frac{160}{4} & \\
 x = 40 & 40^\circ, 80^\circ, 60^\circ
 \end{array}$$

- 23) The second angle of a triangle is five times as large as the first. The measure of the third angle is 12 degrees greater than that of the first angle. How large are the angles?

$$\begin{array}{ll}
 F: x & (24) \\
 S: 5x & (5 \cdot 24 = 120) \\
 T: \underline{x + 12} & (24 + 12 = 36) \\
 7x + 12 = 180 & \\
 \underline{-12 \quad -12} & \\
 \frac{7x}{7} = \frac{168}{7} & \\
 x = 24 & 24^\circ, 120^\circ, 36^\circ
 \end{array}$$

- 25) The second angle of a triangle is four times the first and the third is 5 degrees more than twice the first. Find the measures of the angles.

$$\begin{array}{ll}
 F: x & (25) \\
 S: 4x & (4 \cdot 25 = 100) \\
 T: \underline{2x + 5} & (2 \cdot 25 + 4 = 50 + 5 = 55) \\
 7x + 5 = 180 & \\
 \underline{-5 \quad -5} & \\
 \frac{7x}{7} = \frac{175}{7} & \\
 x = 25 & 25^\circ, 100^\circ, 55^\circ
 \end{array}$$

- 27) The perimeter of a rectangle is 304 cm. The length is 40 cm longer than the width. Find the length and width.

$$\begin{array}{ll}
 L: x + 40 & (56 + 40 = 96) \\
 W: \underline{x} & (56) \\
 2(2x + 40) = 304 & \\
 4x + 80 = 304 & \\
 \underline{-80 \quad -80} & \\
 \frac{4x}{4} = \frac{224}{4} & \\
 x = 56 & 56 \times 96
 \end{array}$$

- 29) The perimeter of a rectangle is 280 meters. The width is 26 meters less than the length. Find the length and width.

$$\begin{array}{l}
 L: x \qquad \qquad \qquad (83) \\
 W: x - 26 \qquad \qquad (83 - 26 = 57) \\
 \hline
 2(2x - 26) = 280 \\
 4x - 52 = 280 \\
 \quad +52 \quad +52 \\
 \hline
 \quad \frac{4x}{4} = \frac{332}{4} \\
 \quad x = 83 \qquad \quad 57 \times 83
 \end{array}$$

- 31) A mountain cabin on 1 acre of land costs \$30,000. If the land cost 4 times as much as the cabin, what was the cost of each?

$$\begin{array}{l}
 C: x \qquad \qquad \qquad (6000) \\
 L: 4x \qquad \qquad \qquad (4 \cdot 6000 = 24000) \\
 \hline
 \frac{5x}{5} = \frac{30000}{5} \\
 x = 6000 \qquad \quad \text{Cabin: } \$6,000, \text{ Land: } \$24,000
 \end{array}$$

- 33) A bicycle and a bicycle helmet cost \$240. How much did each cost, if the bicycle cost 5 times as much as the helmet?

$$\begin{array}{l}
 B: 5x \qquad \qquad \qquad (5 \cdot 40 = 200) \\
 H: x \qquad \qquad \qquad (40) \\
 \hline
 \frac{6x}{6} = \frac{240}{6} \\
 x = 40 \qquad \quad \text{Bike: } \$200, \text{ Helmet: } \$40
 \end{array}$$

- 35) If Mr. Brown and his son together had \$220, and Mr. Brown had 10 times as much as his son, how much money had each?

$$\begin{array}{l}
 B: 10x \qquad \qquad \qquad (10 \cdot 20 = 200) \\
 S: x \qquad \qquad \qquad (20) \\
 \hline
 \frac{11x}{11} = \frac{220}{11} \\
 x = 20 \qquad \quad \text{Mr. Brown: } \$200, \text{ Son: } \$20
 \end{array}$$

- 37) Aaron had 7 times as many sheep as Beth, and both together had 608. How many sheep had each?

$$\begin{array}{l}
 A: 7x \qquad \qquad \qquad (7 \cdot 76 = 532) \\
 B: x \qquad \qquad \qquad (76) \\
 \hline
 \frac{8x}{8} = \frac{608}{8} \\
 x = 76 \qquad \quad \text{Aaron: } 532 \text{ Sheep, Beth: } 76 \text{ Sheep}
 \end{array}$$

39) Jamal and Moshe began a business with a capital of \$7500. If Jamal furnished half as much capital as Moshe, how much did each furnish?

$$\begin{array}{l}
 J: x \qquad (2500) \\
 \underline{M: 2x} \qquad (2 \cdot 2500 = 5000) \\
 \frac{3x}{3} = \frac{7500}{3} \\
 x = 2500 \qquad \text{Jamal: \$2500, Moshe: \$5000}
 \end{array}$$

41) A 6 ft board is cut into two pieces, one twice as long as the other. How long are the pieces?

$$\begin{array}{l}
 L: 2x \qquad (2 \cdot 2 = 4) \\
 \underline{S: x} \qquad (2) \\
 \frac{3x}{3} = \frac{6}{3} \\
 x = 2 \qquad 4ft \ \& \ 2ft
 \end{array}$$

43) An electrician cuts a 30 ft piece of wire into two pieces. One piece is 2 ft longer than the other. How long are the pieces?

$$\begin{array}{l}
 L: x + 2 \qquad (14 + 2 = 16) \\
 \underline{S: x} \qquad (14) \\
 2x + 2 = 30 \\
 -2 \quad -2 \\
 \frac{2x}{2} = \frac{28}{2} \\
 x = 14 \qquad 16ft \ \& \ 14ft
 \end{array}$$

45) The cost of a private pilot course is \$1,275. The flight portion costs \$625 more than the ground school portion. What is the cost of each?

$$\begin{array}{l}
 F: x + 625 \qquad (325 + 625 = 950) \\
 G: x \qquad (325) \\
 2x + 625 = 1275 \\
 -625 \quad -625 \\
 \frac{2x}{2} = \frac{650}{2} \\
 x = 325 \qquad \text{Flight: \$950, Ground: \$325}
 \end{array}$$

1.9

1) A boy is 10 years older than his brother. In 4 years he will be twice as old as his brother. Find the present age of each.

	Now	+4
H-boy	x+10	x+14
B-Brother	x	x+4

$$\begin{array}{l}
 H = 2B \\
 (x + 14) = 2(x + 4) \\
 x + 14 = 2x + 8 \\
 \underline{-x \qquad -x} \\
 14 = x + 8 \\
 \underline{-8 \quad -8} \\
 6 = x
 \end{array}$$

Boy: 16, Brother: 6

- 3) Pat is 20 years older than his son James. In two years Pat will be twice as old as James. How old are they now?

	Now	+2
P	x+20	x+22
J	x	x+2

$$\begin{aligned}
 P &= 2J \\
 x + 22 &= 2(x + 2) \\
 x + 22 &= 2x + 4 \\
 \begin{array}{r}
 -x \quad \quad -x \\
 \hline
 22 = x + 4 \\
 \begin{array}{r}
 -4 \quad -4 \\
 \hline
 18 = x
 \end{array}
 \end{array}
 \end{aligned}$$

Pat: 38, James: 18

- 5) Fred is 4 years older than Barney. Five years ago the sum of their ages was 48. How old are they now?

	Now	-5
F	x+4	x-1
B	x	x-5

$$\begin{aligned}
 F + B &= 48 \\
 (x - 1) + (x - 5) &= 48 \\
 2x - 6 &= 48 \\
 \begin{array}{r}
 +6 \quad +6 \\
 \hline
 \frac{2x}{2} = \frac{54}{2} \\
 x = 27
 \end{array}
 \end{aligned}$$

Fred: 31, Barney: 27

- 7) Tim is 5 years older than JoAnn. Six years from now the sum of their ages will be 79. How old are they now?

	Now	+6
T	x+5	x+11
J	x	x+6

$$\begin{aligned}
 T + J &= 79 \\
 (x + 11) + (x + 6) &= 79 \\
 2x + 17 &= 79 \\
 \begin{array}{r}
 -17 \quad -17 \\
 \hline
 \frac{2x}{2} = \frac{62}{2} \\
 x = 31
 \end{array}
 \end{aligned}$$

Tim: 36, JoAnn: 31

- 9) The sum of the ages of John and Mary is 32. Four years ago, John was twice as old as Mary. Find the present age of each.

	Now	-4
J	x	x-4
M	32-x	28-x

$$\begin{aligned}
 J &= 2m \\
 (x - 4) &= 2(28 - x) \\
 x - 4 &= 56 - 2x \\
 \begin{array}{r}
 +2x \quad \quad +2x \\
 \hline
 3x - 4 = 56 \\
 \begin{array}{r}
 +4 \quad +4 \\
 \hline
 \frac{3x}{3} = \frac{60}{3} \\
 x = 20
 \end{array}
 \end{array}
 \end{aligned}$$

John: 20, Mary: 12

- 11) The sum of the ages of a china plate and a glass plate is 16 years. Four years ago the china plate was three times the age of the glass plate. Find the present age of each plate.

	Now	-4
C	x	$x-4$
G	$16-x$	$12-x$

$$\begin{aligned}
 C &= 3G \\
 (x - 4) &= 3(12 - x) \\
 x - 4 &= 36 - 3x \\
 +3x &\quad + 3x \\
 \hline
 4x - 4 &= 36 \\
 +4 &\quad + 4 \\
 \hline
 \frac{4x}{4} &= \frac{40}{4} \\
 x &= 10
 \end{aligned}$$

China: 10, Glass: 6

- 13) A is now 34 years old, and B is 4 years old. In how many years will A be twice as old as B?

	Now	+t
A	34	$34+t$
B	4	$4+t$

$$\begin{aligned}
 A &= 2B \\
 34 + t &= 2(4 + t) \\
 34 + t &= 8 + 2t \\
 -t &\quad -t \\
 \hline
 34 &= 8 + t \\
 -8 &\quad -8 \\
 \hline
 26 &= t
 \end{aligned}$$

- 15) An Oriental rug is 52 years old and a Persian rug is 16 years old. How many years ago was the Oriental rug four times as old as the Persian Rug?

	Now	-t
O	52	$52-t$
P	16	$16-t$

$$\begin{aligned}
 O &= 4P \\
 52 - t &= 4(16 - t) \\
 52 - t &= 64 - 4t \\
 +4t &\quad + 4t \\
 \hline
 52 + 3t &= 64 \\
 -52 &\quad - 52 \\
 \hline
 \frac{3t}{3} &= \frac{12}{3} \\
 t &= 4
 \end{aligned}$$

- 17) The age of the older of two boys is twice that of the younger; 5 years ago it was three times that of the younger. Find the age of each.

	Now	-5
O	$2x$	$2x-5$
Y	x	$x-5$

$$\begin{aligned}
 O &= 3Y \\
 2x - 5 &= 3(x - 5) \\
 2x - 5 &= 3x - 15 \\
 -2x &\quad - 2x \\
 \hline
 -5 &= x - 15 \\
 +15 &\quad + 15 \\
 \hline
 10 &= x
 \end{aligned}$$

Older: 20, Younger: 10

19) Marge is twice as old as Consuelo. The sum of their ages seven years ago was 13. How old are they now?

	Now	-7
M	2x	2x-7
C	x	x-7

$$\begin{aligned}
 M + C &= 13 \\
 (2x - 7) + (x - 7) &= 13 \\
 3x - 14 &= 13 \\
 \frac{+14}{3} &= \frac{+14}{3} \\
 \frac{3x}{3} &= \frac{27}{3} \\
 x &= 9
 \end{aligned}$$

Marge: 18, Consuelo: 9

21) A silver coin is 28 years older than a bronze coin. In 6 years, the silver coin will be twice as old as the bronze coin. Find the present age of each coin.

	Now	+6
S	x+28	x+34
B	x	x+6

$$\begin{aligned}
 S &= 2B \\
 (x + 34) &= 2(x + 6) \\
 x + 34 &= 2x + 12 \\
 \frac{-x}{34} &= \frac{-x}{-x} \\
 34 &= x + 12 \\
 \frac{-12}{22} &= \frac{-12}{-12} \\
 22 &= x
 \end{aligned}$$

Silver: 50, bronze: 22

23) A limestone statue is 56 years older than a marble statue. In 12 years, the limestone will be three times as old as the marble statue. Find the present age of the statues.

	Now	+12
L	x+56	x+68
M	x	x+12

$$\begin{aligned}
 L &= 3M \\
 (x + 68) &= 3(x + 12) \\
 x + 68 &= 3x + 36 \\
 \frac{-x}{68} &= \frac{-x}{-x} \\
 68 &= 2x + 36 \\
 \frac{-36}{32} &= \frac{-36}{-36} \\
 \frac{32}{2} &= \frac{2x}{2} \\
 16 &= x
 \end{aligned}$$

Limestone: 72, Marble: 16

25) Brandon is 9 years older than Ronda. In four years the sum of their ages will be 91. How old are they now?

	Now	+4
B	x+9	x+13
R	x	x+4

$$\begin{aligned}
 B + R &= 91 \\
 (x + 13) + (x + 4) &= 91 \\
 2x + 17 &= 91 \\
 \frac{-17}{2x} &= \frac{-17}{-17} \\
 \frac{2x}{2} &= \frac{74}{2} \\
 x &= 37
 \end{aligned}$$

Brandon: 46, Ronda: 37

27) A father is three times as old as his son, and his daughter is 3 years younger than the son. If the sum of their ages 3 years ago was 63 years, find the present age of the father.

	Now	-3
F	3x	3x-3
S	x	x-3
D	x-3	x-6

$$F + S + D = 63$$

$$(3x - 3) + (x - 3) + (x - 6) = 63$$

$$5x - 12 = 63$$

$$\begin{array}{r} +12 \quad +12 \\ \hline \end{array}$$

$$\frac{5x}{5} = \frac{75}{5}$$

$$x = 15$$

Father: 45, Son = 15, Daughter: 12

29) The sum of the ages of two ships is 12 years. Two years ago, the age of the older ship was three times the age of the newer ship. Find the present age of each ship.

	Now	-2
O	x	x-2
Y	12-x	10-x

$$O = 3Y$$

$$(x - 2) = 3(10 - x)$$

$$x - 2 = 30 - x$$

$$\begin{array}{r} +x \quad \quad +x \\ \hline \end{array}$$

$$2x - 2 = 30$$

$$\begin{array}{r} +2 \quad +2 \\ \hline \end{array}$$

$$\frac{4x}{4} = \frac{32}{4}$$

$$x = 8$$

8 & 4

31) Ann is eighteen years older than her son. One year ago, she was three times as old as her son. How old are they now?

	Now	-1
A	x+18	x+17
S	x	x-1

$$A = 3S$$

$$(x + 17) = 3(x - 1)$$

$$x + 17 = 3x - 3$$

$$\begin{array}{r} -x \quad \quad -x \\ \hline \end{array}$$

$$17 = 2x - 3$$

$$\begin{array}{r} +3 \quad \quad +3 \\ \hline \end{array}$$

$$\frac{20}{2} = \frac{2x}{2}$$

$$10 = x$$

Ann: 28, Son: 10

33) A mosaic is 74 years older than the engraving. Thirty years ago, the mosaic was three times as old as the engraving. Find the present age of each.

	Now	-30
M	$x+74$	$x+44$
E	x	$x-30$

$$\begin{aligned}
 M &= 3E \\
 (x + 44) &= 3(x - 30) \\
 x + 44 &= 3x - 90 \\
 \begin{array}{r}
 -x \qquad -x \\
 \hline
 44 = 2x - 90 \\
 +90 \qquad +90 \\
 \hline
 \frac{134}{2} = \frac{2x}{2} \\
 67 = x
 \end{array}
 \end{aligned}$$

Mosaic: 141, Engraving: 67

35) A wool tapestry is 32 years older than a linen tapestry. Twenty years ago, the wool tapestry was twice as old as the linen tapestry. Find the present age of each.

	Now	-20
W	$x+32$	$x+12$
L	x	$x-20$

$$\begin{aligned}
 W &= 2L \\
 (x + 12) &= 2(x - 20) \\
 x + 12 &= 2x - 20 \\
 \begin{array}{r}
 -x \qquad -x \\
 \hline
 12 = x - 20 \\
 +20 \qquad +20 \\
 \hline
 32 = x
 \end{array}
 \end{aligned}$$

Wool: 84, Linen: 52

37) Nicole is 26 years old. Emma is 2 years old. In how many years will Nicole be triple Emma's age?

	Now	+t
N	26	$26+t$
E	2	$2+t$

$$\begin{aligned}
 N &= 3E \\
 (26 + t) &= 3(2 + t) \\
 26 + t &= 6 + 3t \\
 \begin{array}{r}
 -t \qquad -t \\
 \hline
 26 = 6 + 2t \\
 -6 \quad -6 \\
 \hline
 \frac{20}{2} = \frac{2t}{2} \\
 10 = t
 \end{array}
 \end{aligned}$$

39) Mike is 4 years older than Ron. In two years, the sum of their ages will be 84. How old are they now?

	Now	+2
M	$x+4$	$x+6$
R	x	$x+2$

$$\begin{aligned}
 M + R &= 84 \\
 (x + 6) + (x + 2) &= 84 \\
 2x + 8 &= 84 \\
 \begin{array}{r}
 -8 \quad -8 \\
 \hline
 \frac{2x}{2} = \frac{76}{2} \\
 x = 38
 \end{array}
 \end{aligned}$$

Mike: 42, Ron: 38

1.10

- 1) A is 60 miles from B. An automobile at A starts for B at the rate of 20 miles an hour at the same time that an automobile at B starts for A at the rate of 25 miles an hour. How long will it be before the automobiles meet?

	r	t	d
A	20	t	20t
B	25	t	25t

$$20t + 25t = 60$$

$$\frac{45t}{45} = \frac{60}{45}$$

$$t = 1.33 \text{ hr}$$

- 3) Two trains travel toward each other from points which are 195 miles apart. They travel at rate of 25 and 40 miles an hour respectively. If they start at the same time, how soon will they meet?

	r	t	d
	25	t	25t
	40	t	40t

$$25t + 40t = 195$$

$$\frac{65t}{65} = \frac{195}{65}$$

$$t = 3 \text{ hr}$$

- 5) A passenger and a freight train start toward each other at the same time from two points 300 miles apart. If the rate of the passenger train exceeds the rate of the freight train by 15 miles per hour, and they meet after 4 hours, what must the rate of each be?

	r	t	d
P	r+15	4	4r+60
F	r	4	4r

$$4r + 4r + 60 = 300$$

$$8r + 60 = 300$$

$$\begin{array}{r} -60 \quad -60 \\ \hline 8r = 240 \\ \frac{8r}{8} = \frac{240}{8} \\ r = 30 \end{array}$$

Passenger: 45 mph, Freight: 30 mph

- 7) A man having ten hours at his disposal made an excursion, riding out at the rate of 10 miles an hour and returning on foot, at the rate of 3 miles an hour. Find the distance he rode.

	r	t	D
r	10	t	10t
w	3	10-t	30-3t

$$10t = 30 - 3t$$

$$\begin{array}{r} +3t \quad \quad +3t \\ \hline 13t = 30 \\ \frac{13t}{13} = \frac{30}{13} \\ t = \frac{30}{13} \\ d = 10\left(\frac{30}{13}\right) = \frac{300}{13} \text{ mi} \end{array}$$

- 9) A boy rides away from home in an automobile at the rate of 28 miles an hour and walks back at the rate of 4 miles an hour. The round trip requires 2 hours. How far does he ride?

	r	t	d
r	28	t	28t
w	4	2-t	8-4t

$$28t = 8 - 4t$$

$$\begin{array}{r} +4t \quad \quad +4t \\ \hline 32t = 8 \\ \frac{32t}{32} = \frac{8}{32} \\ t = .25 \\ d = 28(.25) = 7 \text{ mi.} \end{array}$$

- 11) A family drove to a resort at an average speed of 30 mph and later returned over the same road at an average speed of 50 mph. Find the distance to the resort if the total driving time was 8 hours.

	r	t	d
T	30	t	30t
R	50	8-t	400-50t

$$\begin{array}{r} 30t = 400 - 50t \\ +50t \quad \quad + 50t \\ \hline \end{array}$$

$$\frac{80t}{80} = \frac{400}{80}$$

$$t = 5$$

$$d = 30(5) = 150 \text{ mi.}$$

- 13) A, who travels 4 miles an hour starts from a certain place 2 hours in advance of B, who travels 5 miles an hour in the same direction. How many hours must B travel to overtake A?

	r	t	D
A	4	t+2	4t+8
B	5	t	5t

$$4t + 8 = 5t$$

$$\begin{array}{r} -4t \quad \quad - 4t \\ \hline \end{array}$$

$$8hr = t$$

- 15) A motorboat leaves a harbor and travels at an average speed of 8 mph toward a small island. Two hours later a cabin cruiser leaves the same harbor and travels at an average speed of 16 mph toward the same island. In how many hours after the cabin cruiser leaves will the cabin cruiser be alongside the motorboat?

	r	t	d
M	8	t+2	8t+16
C	16	t	16t

$$8t + 16 = 16t$$

$$\begin{array}{r} -8t \quad \quad - 8t \\ \hline \end{array}$$

$$\frac{16}{8} = \frac{8t}{8}$$

$$2hr = t$$

- 17) A car traveling at 48 mph overtakes a cyclist who, riding at 12 mph, has had a 3 hour head start. How far from the starting point does the car overtake the cyclist?

	r	t	d
Car	48	t	48t
Cy	12	t+3	12t+36

$$48t = 12t + 36$$

$$\begin{array}{r} -12t - 12t \\ \hline \end{array}$$

$$\frac{36t}{36} = \frac{36}{36}$$

$$t = 1$$

$$d = 48(1) = 48 \text{ mi.}$$

- 19) Two men are traveling in opposite directions at the rate of 20 and 30 miles an hour at the same time and from the same place. In how many hours will they be 300 miles apart?

	r	t	d
	20	t	20t
	30	t	30t
			300

$$20t + 30t = 300$$

$$\frac{50t}{50} = \frac{300}{50}$$

$$t = 6 \text{ hr}$$

- 21) A motorboat leaves a harbor and travels at an average speed of 18 mph to an island. The average speed on the return trip was 12 mph. How far was the island from the harbor if the total trip took 5 h?

	r	t	d
T	18	t	18t
R	12	5-t	60-12t

$$18t = 60 - 12t$$

$$\frac{+12t}{30} = \frac{+12t}{30}$$

$$\frac{30t}{30} = \frac{60}{30}$$

$$t = 2$$

$$d = 18(2) = 36 \text{ mi}$$

- 23) A jet plane traveling at 570 mph overtakes a propeller-driven plane that has had a 2 h head start. The propeller-driven plane is traveling at 190 mph. How far from the starting point does the jet overtake the propeller-driven plane?

	r	t	d
J	570	t	570t
P	190	t+2	190t+380

$$570t = 190t + 380$$

$$\frac{-190t}{380} = \frac{-190t}{380}$$

$$\frac{380t}{380} = \frac{380}{380}$$

$$t = 1$$

$$d = 570(1) = 570 \text{ mi.}$$

- 25) As part of flight training, a student pilot was required to fly to an airport and then return. The average speed on the way to the airport was 100 mph, and the average speed returning was 150 mph. Find the distance between the two airports if the total flight time was 5 h.

	r	t	d
T	100	t	100t
R	150	5-t	750-150t

$$100t = 750 - 150t$$

$$\frac{+150t}{250} = \frac{+150t}{250}$$

$$\frac{250t}{250} = \frac{750}{250}$$

$$t = 3$$

$$d = 100(3) = 300 \text{ mi.}$$

- 27) A car traveling at 56 mph overtakes a cyclist who, riding at 14 mph, has had a 3 h head start. How far from the starting point does the car overtake the cyclist?

	r	t	d
Car	56	t	56t
Cy	14	t+3	14t+42

$$56t = 14t + 42$$

$$\frac{-14t}{42} = \frac{-14t}{42}$$

$$\frac{42t}{42} = \frac{42}{42}$$

$$t = 1$$

$$d = 56(1) = 56 \text{ mi}$$

- 29) A bus traveling at a rate of 60 mph overtakes a car traveling at a rate of 45 mph. If the car had a 1 h head start, how far from the starting point does the bus overtake the car?

	r	t	d
B	60	t	60t
C	45	t+1	45t+45

$$60t = 45t + 45$$

$$\frac{-45t - 45t}{15} = \frac{45}{15}$$

$$\frac{15t}{15} = \frac{45}{15}$$

$$t = 3$$

$$d = 60(3) = 180 \text{ mi.}$$

- 31) A truck leaves a depot at 11 A.M. and travels at a speed of 45 mph. At noon, a van leaves the same place and travels the same route at a speed of 65 mph. At what time does the van overtake the truck?

	r	t	d
T	45	t+1	45t+45
V	65	t	65t

$$45t + 45 = 65t$$

$$\frac{-45t}{20} = \frac{-45t}{20}$$

$$\frac{45}{20} = \frac{20t}{20}$$

$$2.25 = t$$

$$2.25 = 2 \text{ hr } 15 \text{ min.}$$

$$12:00 \text{ pm} + 2:15 = 2:15 \text{ pm}$$

- 33) Three campers left their campsite by canoe and paddled downstream at an average rate of 10 mph. They then turned around and paddled back upstream at an average rate of 5 mph to return to their campsite. How long did it take the campers to canoe downstream if the total trip took 1 hr?

	r	t	d
d	10	t	10t
v	5	1-t	5-5t

$$10t = 5 - 5t$$

$$\frac{+5t}{15} = \frac{+5t}{15}$$

$$\frac{15t}{15} = \frac{5}{15}$$

$$t = \frac{1}{3} \text{ hr} = 20 \text{ min.}$$

- 35) A student walks and jogs to college each day. The student averages 5 km/hr walking and 9 km/hr jogging. The distance from home to college is 8 km, and the student makes the trip in one hour. How far does the student jog?

	r	t	d
W	5	t	5t
J	9	1-t	9-9t
			8

$$5t + 9 - 9t = 8$$

$$-4t + 9 = 8$$

$$\frac{-9}{-4} = \frac{-9}{-4}$$

$$-\frac{4t}{-4} = -\frac{1}{-4}$$

$$t = .25$$

$$d = 9 - 9(.25) = 9 - 2.25 = 6.75 \text{ mi}$$

37) On a 220 mi trip, a car traveled at an average speed of 50 mph and then reduced its average speed to 35 mph for the remainder of the trip. The trip took a total of 5 h. How long did the car travel at each speed?

	r	t	d
F	50	t	50t
S	35	5-t	175-35t
			220

$$50t + 175 - 35t = 220$$

$$15t + 175 = 220$$

$$\underline{-175 \quad -175}$$

$$\frac{15t}{15} = \frac{45}{15}$$

$$t = 3$$

3hr @ 50 mph, 2 hr @ 35 mph

Chapter 2: Graphing

2.1

- 1) $B(4, -3), C(1, 2), D(-1, 4), E(-5, 0), F(2, -3), G(1, 3), H(-1, -4), I(-2, -1), J(0, 2), K(-4, 3)$

3) $y = -\frac{1}{4}x - 3$

let $x = -4$

$y = -\frac{1}{4}(-4) - 3$

$= 1 - 3$

$= -2$

let $x = 0$

$y = -\frac{1}{4}(0) - 3$

$= 0 - 3$

$= -3$

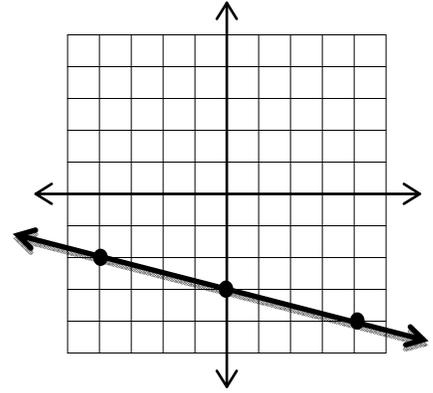
let $x = 4$

$y = -\frac{1}{4}(4) - 3$

$= -1 - 3$

$= -4$

x	y
-4	-2
0	-3
4	-4



5) $y = -\frac{5}{4}x - 4$

let $x = -4$

$y = -\frac{5}{4}(-4) - 4$

$= 5 - 4$

$= 1$

let $x = 0$

$y = -\frac{5}{4}(0) - 4$

$= 0 - 4$

$= -4$

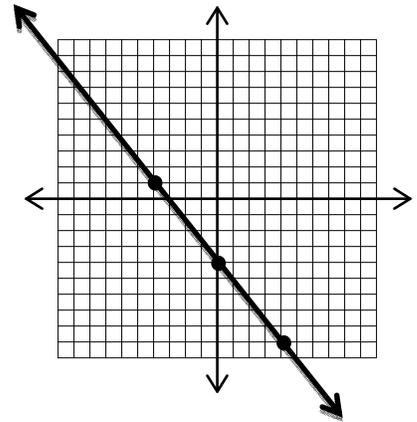
let $x = 4$

$y = -\frac{5}{4}(4) - 4$

$= -5 - 4$

$= -9$

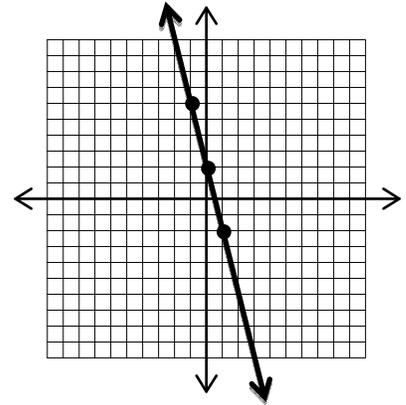
x	y
-4	1
0	-4
4	-9



7) $y = -4x + 2$

$$\begin{aligned} \text{let } x &= -1 \\ y &= -4(-1) + 2 \\ &= 4 + 2 \\ &= 6 \\ \text{let } x &= 0 \\ y &= -4(0) + 2 \\ &= 0 + 2 \\ &= 2 \\ \text{let } x &= 1 \\ y &= -4(1) + 2 \\ &= -4 + 2 \\ &= -2 \end{aligned}$$

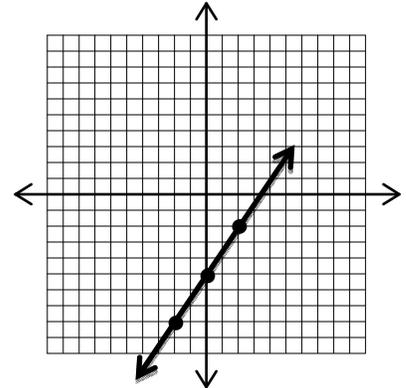
x	y
-1	6
0	2
1	-2



9) $y = \frac{3}{2}x - 5$

$$\begin{aligned} \text{let } x &= -2 \\ y &= \frac{3}{2}(-2) - 5 \\ &= -3 - 5 \\ &= -8 \\ \text{let } x &= 0 \\ y &= \frac{3}{2}(0) - 5 \\ &= 0 - 5 \\ &= -5 \\ \text{let } x &= 2 \\ y &= \frac{3}{2}(2) - 5 \\ &= 3 - 5 \\ &= -2 \end{aligned}$$

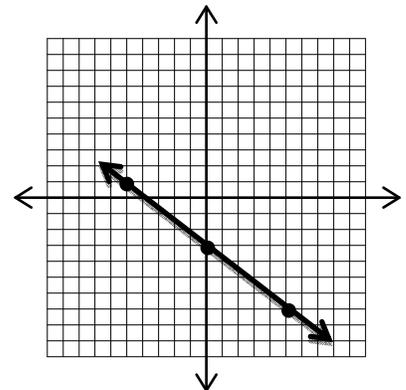
x	y
-2	-8
0	-5
2	-2



11) $y = -\frac{4}{5}x - 3$

$$\begin{aligned} \text{let } x &= -5 \\ y &= -\frac{4}{5}(-5) - 3 \\ &= 4 - 3 \\ &= 1 \\ \text{let } x &= 0 \\ y &= -\frac{4}{5}(0) - 3 \\ &= 0 - 3 \\ &= -3 \\ \text{let } x &= 5 \\ y &= -\frac{4}{5}(5) - 3 \\ &= -4 - 3 \\ &= -7 \end{aligned}$$

x	y
-5	1
0	-3
5	-7



$$13) \quad x + 5y = -15$$

$$\begin{array}{r} -x \qquad -x \\ \hline \frac{5y}{5} = \frac{-x}{5} - \frac{15}{5} \\ y = -\frac{1}{5}x - 3 \end{array}$$

$$\text{let } x = -5$$

$$\begin{aligned} y &= -\frac{1}{5}(-5) - 3 \\ &= 1 - 3 \\ &= -2 \end{aligned}$$

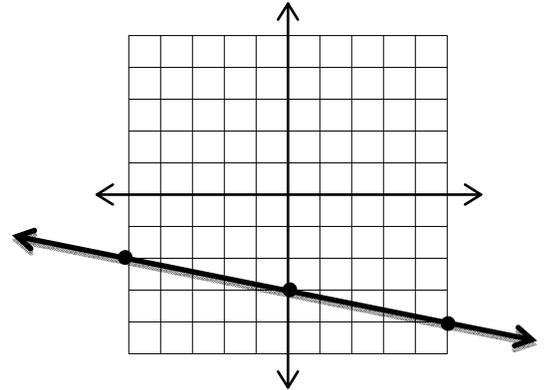
$$\text{let } x = 0$$

$$\begin{aligned} y &= -\frac{1}{5}(0) - 3 \\ &= 0 - 3 \\ &= -3 \end{aligned}$$

$$\text{let } x = 5$$

$$\begin{aligned} y &= -\frac{1}{5}(5) - 3 \\ &= -1 - 3 \\ &= -4 \end{aligned}$$

x	y
-5	-2
0	-3
5	-4



$$15) \quad 4x + y = 5$$

$$\begin{array}{r} -4x \qquad -4x \\ \hline y = -4x + 5 \end{array}$$

$$\text{let } x = -1$$

$$\begin{aligned} y &= -4(-1) + 5 \\ &= 4 + 5 \\ &= 9 \end{aligned}$$

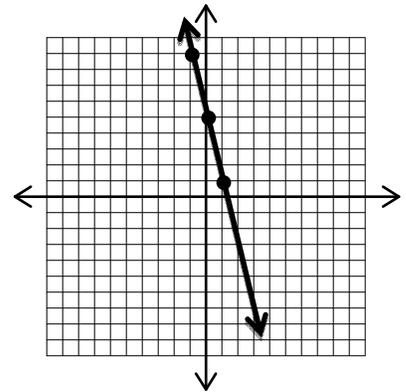
$$\text{let } x = 0$$

$$\begin{aligned} y &= -4(0) + 5 \\ &= 0 + 5 \\ &= 5 \end{aligned}$$

$$\text{let } x = 1$$

$$\begin{aligned} y &= -4(1) + 5 \\ &= -4 + 5 \\ &= 1 \end{aligned}$$

x	y
-1	9
0	5
1	1



$$17) \quad 2x - y = 2$$

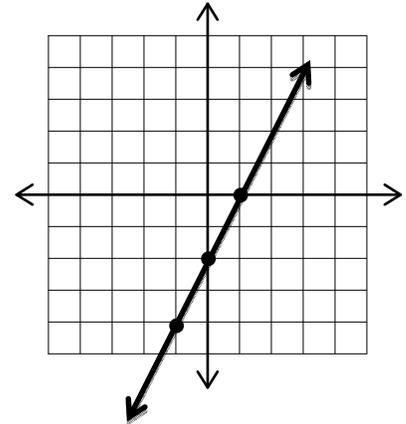
$$\begin{array}{r} -2x \quad -2x \\ \hline \frac{-y}{-1} = \frac{-2x}{-1} + \frac{2}{-1} \\ y = 2x - 2 \end{array}$$

$$\begin{array}{l} \text{let } x = -1 \\ y = 2(-1) - 2 \\ \quad = -2 - 2 \\ \quad = -4 \end{array}$$

$$\begin{array}{l} \text{let } x = 0 \\ y = 2(0) - 2 \\ \quad = 0 - 2 \\ \quad = -2 \end{array}$$

$$\begin{array}{l} \text{let } x = 1 \\ y = 2(1) - 2 \\ \quad = 2 - 2 \\ \quad = 0 \end{array}$$

x	y
-1	-4
0	-2
1	0



$$19) \quad x + y = -1$$

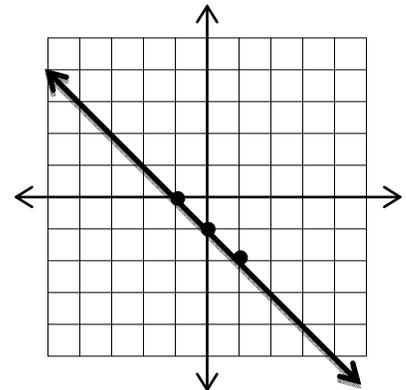
$$\begin{array}{r} -x \quad -x \\ \hline y = -x - 1 \end{array}$$

$$\begin{array}{l} \text{let } x = -1 \\ y = -(-1) - 1 \\ \quad = 1 - 1 \\ \quad = 0 \end{array}$$

$$\begin{array}{l} \text{let } x = 0 \\ y = -(0) - 1 \\ \quad = 0 - 1 \\ \quad = -1 \end{array}$$

$$\begin{array}{l} \text{let } x = 1 \\ y = -(1) - 1 \\ \quad = -1 - 1 \\ \quad = -2 \end{array}$$

x	y
-1	0
0	-1
1	-2

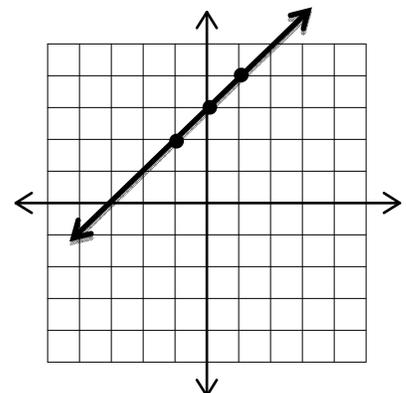


$$21) \quad x - y = -3$$

$$\begin{array}{r} -x \quad -x \\ \hline \frac{-y}{-1} = \frac{-x}{-1} - \frac{3}{-1} \\ y = x + 3 \end{array}$$

$$\begin{array}{l} \text{let } x = -1 \\ y = (-1) + 3 \\ \quad = -1 + 3 \end{array}$$

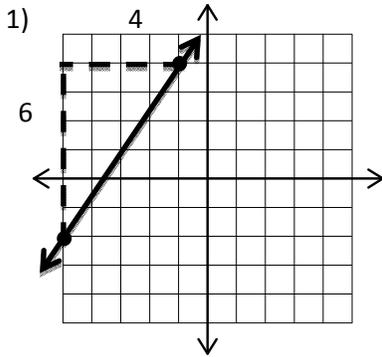
x	y
-1	2



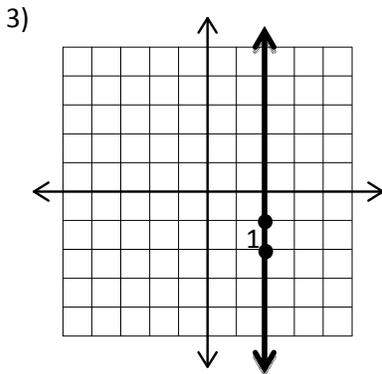
$$\begin{aligned}
 &= 2 \\
 \text{let } x &= 0 \\
 y &= (0) + 3 \\
 &= 3 \\
 \text{let } x &= 1 \\
 y &= (1) + 3 \\
 &= 4
 \end{aligned}$$

$$\begin{array}{cc}
 0 & 3 \\
 1 & 4
 \end{array}$$

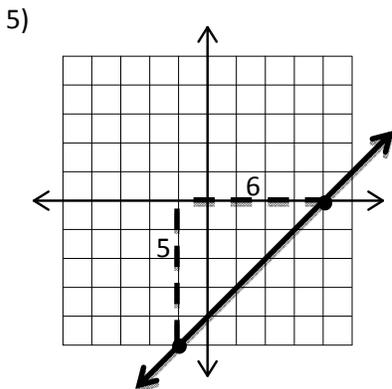
2.2



$$m = \frac{6}{4} = \frac{3}{2}$$



$$m = \frac{1}{0} = \text{undefined}$$



$$m = \frac{5}{6}$$

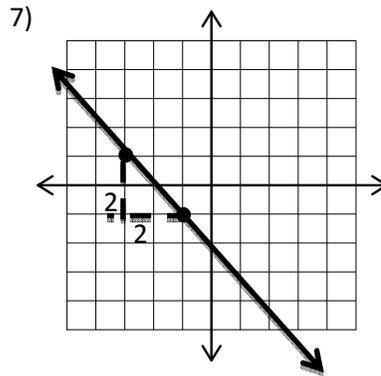
undefined

13) $(-15, 10), (16, -7)$

$$m = \frac{-7-10}{16-(-15)} = \frac{-17}{31}$$

15) $(10, 18), (-11, -10)$

$$m = \frac{-10-18}{-11-10} = \frac{-28}{-21} = \frac{4}{3}$$



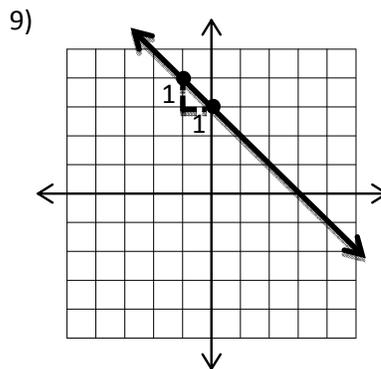
$$m = -\frac{2}{2} = -1$$

17) $(-16, -14), (11, -14)$

$$m = \frac{-14-(-14)}{11-(-16)} = \frac{0}{27} = 0$$

19) $(-4, 14), (-16, 8)$

$$m = \frac{8-14}{-16-(-4)} = \frac{-6}{-12} = \frac{1}{2}$$



$$m = -\frac{1}{1} = -1$$

21) $(12, -19), (6, 14)$

$$m = \frac{14-(-19)}{6-12} = \frac{33}{-6} = -\frac{11}{2}$$

23) $(-5, -10), (-5, 20)$

$$m = \frac{20-(-10)}{-5-(-5)} = \frac{30}{0} = \text{undefined}$$

25) $(-17, 19), (10, -7)$

$$m = \frac{-7-19}{10-(-17)} = -\frac{26}{27}$$

27) $(7, -14), (-8, -9)$

$$m = \frac{-9-(-14)}{-8-7} = \frac{5}{-15} = -\frac{1}{3}$$

11) $(-2, 10), (-2, -15)$

$$m = \frac{-15-10}{-2-(-2)} = \frac{-25}{0} = \text{undefined}$$

$$29) (-5, 7), (-18, 14)$$

$$m = \frac{14-7}{-18-(-5)} = \frac{7}{-13}$$

$$31) (2, 6) \text{ and } (x, 2); \text{ slope } \frac{4}{7}$$

$$\frac{4}{7} = \frac{2-6}{x-2}$$

$$\frac{(x-2)4}{1 \cdot 7} = \frac{-4(x-2)}{x-2 \cdot 1}$$

$$33) (-3, -2) \text{ and } (x, 6); \text{ slope} = -\frac{8}{5}$$

$$-\frac{8}{5} = \frac{6-(-2)}{x-(-3)}$$

$$(x+3) \frac{-8}{5} = \frac{8}{x+3} (x+3)$$

$$-\frac{8}{5}(x+3) = 8$$

$$5 \left(-\frac{8}{5}x - \frac{24}{5} \right) = 8(5)$$

$$-8x - 24 = 40$$

$$\frac{+24 \quad +24}{-8x \quad = \quad 64}$$

$$-\frac{8x}{-8} = \frac{64}{-8}$$

$$x = -8$$

$$35) (-8, y) \text{ and } (-1, 1); \text{ slope} = \frac{6}{7}$$

$$\frac{6}{7} = \frac{1-y}{-1-(-8)}$$

$$(7) \frac{6}{7} = \frac{1-y}{7} (7)$$

$$6 = 1 - y$$

$$\frac{-1-1}{5} = \frac{-y}{-1}$$

$$-5 = y$$

$$\frac{4}{7}(x-2) = -4$$

$$(7) \left(\frac{4}{7}x - \frac{8}{7} \right) = -4(7)$$

$$4x - 8 = -28$$

$$\frac{+8 \quad +8}{4x \quad = \quad -20}$$

$$\frac{4x}{4} = \frac{-20}{4}$$

$$x = -5$$

$$37) (x, -7) \text{ and } (-9, -9); \text{ slope} = \frac{2}{5}$$

$$\frac{2}{5} = \frac{-9-(-7)}{-9-x}$$

$$(-9-x) \frac{2}{5} = \frac{-2}{-9-x} (-9-x)$$

$$\frac{2}{5}(-9-x) = -2$$

$$5 \left(-\frac{18}{5} - \frac{2}{5}x \right) = (-2)5$$

$$-18 - 2x = -10$$

$$\frac{+18 \quad +18}{-2x \quad = \quad 8}$$

$$\frac{-2x}{-2} = \frac{8}{-2}$$

$$x = -4$$

$$39) (x, 5) \text{ and } (8, 0); \text{ slope} = -\frac{5}{6}$$

$$-\frac{5}{6} = \frac{0-5}{8-x}$$

$$(8-x) - \frac{5}{6} = \frac{-5}{8-x} (8-x)$$

$$-\frac{5}{6}(8-x) = -5$$

$$6 \left(-\frac{20}{3} + \frac{5}{6}x \right) = (-5)6$$

$$-40 + 5x = -30$$

$$\frac{+40 \quad +40}{5x \quad = \quad 10}$$

$$\frac{5x}{5} = \frac{10}{5}$$

$$x = 2$$

2.3

$$1) \text{ slope} = 2 \quad y - \text{intercept} = 5$$

$$y = mx + b$$

$$y = 2x + 5$$

$$y = mx + b$$

$$y = -\frac{3}{4}x - 1$$

$$3) \text{ slope} = 1 \quad y - \text{intercept} = -4$$

$$y = mx + b$$

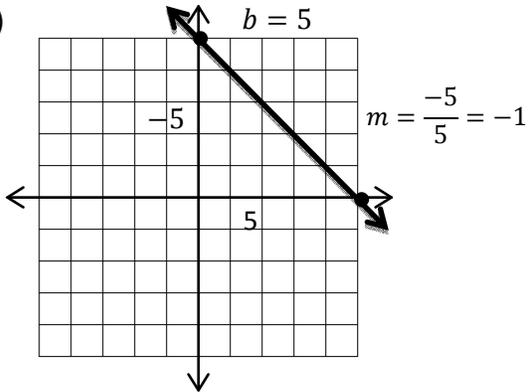
$$y = x - 4$$

$$7) \text{ slope} = \frac{1}{3} \quad y - \text{intercept} = 1$$

$$y = \frac{1}{3}x + 1$$

$$5) \text{ slope} = -\frac{3}{4} \quad y - \text{intercept} = -1$$

9)



$$y = mx + b$$

$$y = -x + 5$$

$$y = mx + b$$

$$y = -4x$$

$$15) \quad x + 10y = -37$$

$$\frac{-x}{-x} \quad \frac{-x}{-x}$$

$$\frac{10y}{10} = \frac{-x}{10} - \frac{37}{10}$$

$$y = -\frac{1}{10}x - \frac{37}{10}$$

$$17) \quad 2x + y = -1$$

$$\frac{-2x}{-2x} \quad \frac{-2x}{-2x}$$

$$y = -2x - 1$$

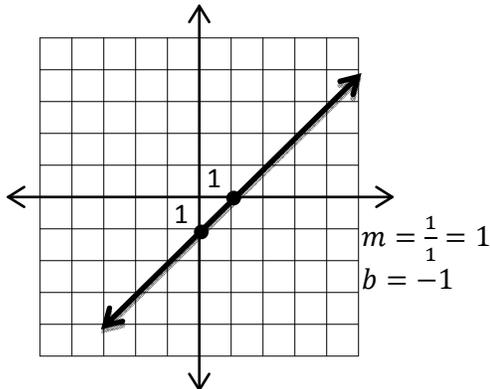
$$19) \quad 7x - 3y = 24$$

$$\frac{-7x}{-7x} \quad \frac{-7x}{-7x}$$

$$\frac{-3y}{-3} = \frac{-7x}{-3} + \frac{24}{-3}$$

$$y = \frac{7}{3}x - 8$$

11)



$$y = mx + b$$

$$y = x - 1$$

$$21) \quad x = -8$$

$$23) \quad y - 4 = -(x + 5)$$

$$y - 4 = -x - 5$$

$$\frac{+4}{+4} \quad \frac{+4}{+4}$$

$$y = -x - 1$$

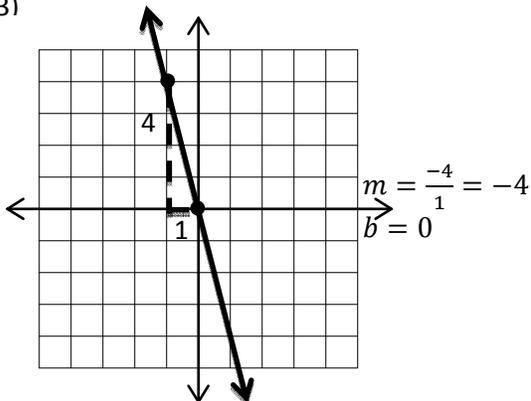
$$25) \quad y - 4 = 4(x - 1)$$

$$y - 4 = 4x - 4$$

$$\frac{+4}{+4} \quad \frac{+4}{+4}$$

$$y = 4x$$

13)



$$27) \quad y + 5 = -4(x - 2)$$

$$y + 5 = -4x + 8$$

$$\frac{-5}{-5} \quad \frac{-5}{-5}$$

$$y = -4x + 3$$

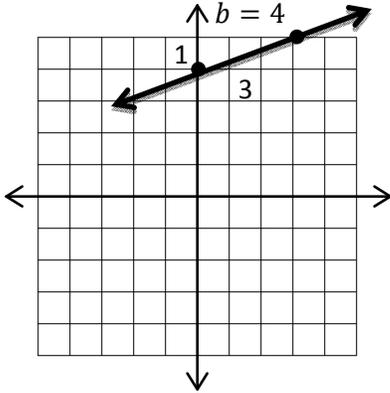
$$29) \quad y + 1 = -\frac{1}{2}(x - 4)$$

$$y + 1 = -\frac{1}{2}x + 2$$

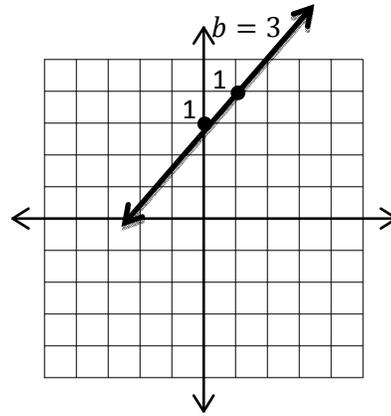
$$\frac{-1}{-1} \quad \frac{-1}{-1}$$

$$y = -\frac{1}{2}x + 1$$

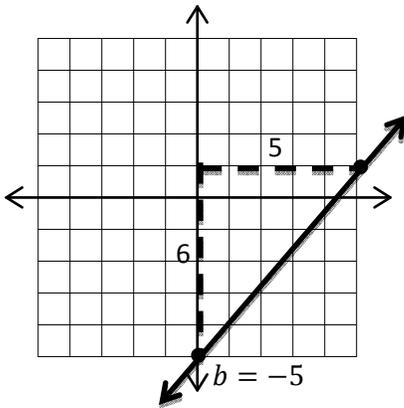
31) $y = \frac{1}{3}x + 4$
 $m = \frac{1}{3}, b = 4$



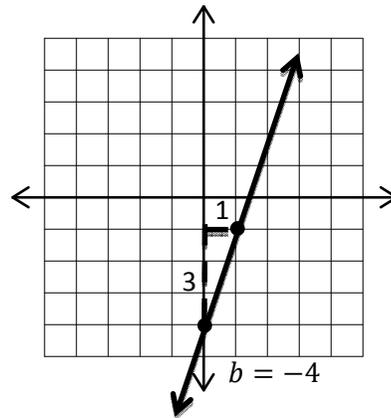
37) $x - y + 3 = 0$
 $\frac{-x}{-1} = \frac{-3}{-1} = \frac{-x-3}{-1}$
 $\frac{-y}{-1} = \frac{-x}{-1} - \frac{3}{-1}$
 $y = x + 3$
 $m = 1, b = 3$



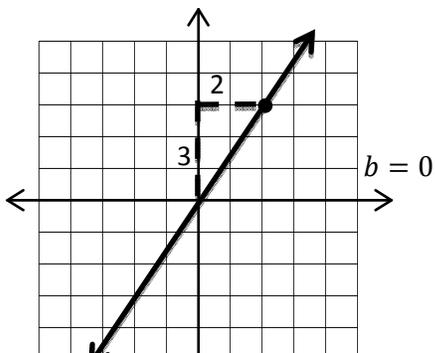
33) $y = \frac{6}{5}x - 5$
 $m = \frac{6}{5}, b = -5$



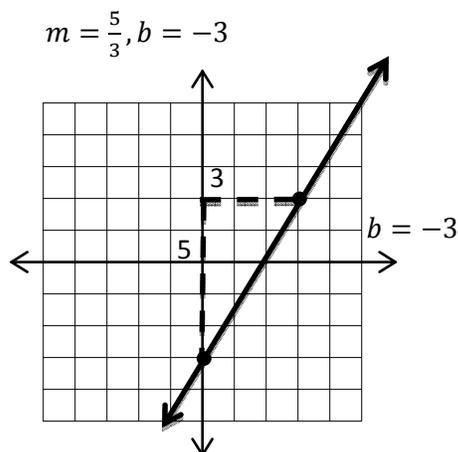
39) $-y - 4 + 3x = 0$
 $\frac{+4 - 3x}{-1} = \frac{-3x + 4}{-1}$
 $\frac{-y}{-1} = \frac{-3x}{-1} + \frac{4}{-1}$
 $y = 3x - 4$
 $m = 3, b = -4$



35) $y = \frac{3}{2}x$
 $m = \frac{3}{2}, b = 0$



41) $\frac{-3y}{-3} = \frac{-5x}{-3} + \frac{9}{-3}$
 $y = \frac{5}{3}x - 3$



2.4

1) Through $(2, 3)$, slope = undefined
 $x = 2$

3) Through $(2, 2)$, slope = $\frac{1}{2}$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{2}(x - 2)$$

5) Through $(-1, -5)$, slope = 9

$$y - y_1 = m(x - x_1)$$

$$y + 5 = 9(x + 1)$$

7) Through $(-4, 1)$, slope = $\frac{3}{4}$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{3}{4}(x + 4)$$

9) Through $(0, -2)$, slope = -3

$$y - y_1 = m(x - x_1)$$

$$y + 2 = -3(x - 0)$$

$$y + 2 = -3x$$

11) Through $(0, -5)$, slope = $-\frac{1}{4}$

$$y - y_1 = m(x - x_1)$$

$$y + 5 = -\frac{1}{4}(x - 0)$$

$$y + 5 = -\frac{1}{4}x$$

13) Through $(-5, -3)$, slope = $\frac{1}{5}$

$$y - y_1 = m(x - x_1)$$

$$y + 3 = \frac{1}{5}(x + 5)$$

15) Through $(-1, 4)$, slope = $-\frac{5}{4}$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{5}{4}(x + 1)$$

17) Through $(-1, -5)$, slope = 2

$$y - y_1 = m(x - x_1)$$

$$y + 5 = 2(x + 1)$$

$$y + 5 = 2x + 2$$

$$\frac{-5}{\quad} \quad \frac{-5}{\quad}$$

$$y = 2x - 3$$

19) Through $(5, -1)$, slope = $-\frac{3}{5}$

$$y - y_1 = m(x - x_1)$$

$$y + 1 = -\frac{3}{5}(x - 5)$$

$$y + 1 = -\frac{3}{5}x + 3$$

$$\frac{-1}{\quad} \quad \frac{-1}{\quad}$$

$$y = -\frac{3}{5}x + 2$$

21) Through $(-4, 1)$, slope = $\frac{1}{2}$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{2}(x + 4)$$

$$y - 1 = \frac{1}{2}x + 2$$

$$\frac{+1}{-2} \quad \frac{+1}{-2}$$

$$y = \frac{1}{2}x + 3$$

23) Through $(4, -2)$, slope = $-\frac{3}{2}$

$$y - y_1 = m(x - x_1)$$

$$y + 2 = -\frac{3}{2}(x - 4)$$

$$y + 2 = -\frac{3}{2}x + 6$$

$$\frac{-2}{-2} \quad \frac{-2}{-2}$$

$$y = -\frac{3}{2}x + 4$$

25) Through $(-5, -3)$, slope = $-\frac{2}{5}$

$$y - y_1 = m(x - x_1)$$

$$y + 3 = -\frac{2}{5}(x + 5)$$

$$y + 3 = -\frac{2}{5}x - 2$$

$$\frac{-3}{-2} \quad \frac{-3}{-2}$$

$$y = -\frac{2}{5}x - 5$$

27) Through $(2, -2)$, slope = 1

$$y - y_1 = m(x - x_1)$$

$$y + 2 = 1(x - 2)$$

$$y + 2 = x - 2$$

$$\frac{-2}{-2} \quad \frac{-2}{-2}$$

$$y = x - 4$$

29) Through $(-3, 4)$, slope = undefined

$$x = -3$$

31) Through $(-4, 2)$, slope = $-\frac{1}{2}$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{1}{2}(x + 4)$$

$$y - 2 = -\frac{1}{2}x - 2$$

$$\frac{+2}{+2} \quad \frac{+2}{+2}$$

$$y = -\frac{1}{2}x$$

33) Through $(-4, 3)$ & $(-3, 1)$

$$m = \frac{1-3}{-3-(-4)} = \frac{-2}{1} = -2$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -2(x + 4)$$

35) Through $(5, 1)$ & $(-3, 0)$

$$m = \frac{0-1}{-3-5} = \frac{-1}{-8} = \frac{1}{8}$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{8}(x - 5)$$

37) Through $(-4, -2)$ & $(0, 4)$

$$m = \frac{4-(-2)}{0-(-4)} = \frac{6}{4} = \frac{3}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y + 2 = \frac{3}{2}(x + 4)$$

39) Through $(3, 5)$ & $(-5, 3)$

$$m = \frac{3-5}{-5-3} = \frac{-2}{-8} = \frac{1}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{1}{4}(x - 3)$$

41) Through $(3, -3)$ & $(-4, 5)$

$$m = \frac{5-(-3)}{-4-3} = \frac{8}{-7}$$

$$y - y_1 = m(x - x_1)$$

$$y + 3 = -\frac{8}{7}(x - 3)$$

43) Through $(-5, 1)$ & $(-1, -2)$

$$m = \frac{-2-1}{-1-(-5)} = \frac{-3}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{3}{4}(x + 5)$$

$$\begin{array}{r}
 y - 1 = -\frac{3}{4}x - \frac{15}{4} \\
 y - \frac{4}{4} = -\frac{3}{4}x - \frac{15}{4} \\
 \quad + \frac{4}{4} \qquad \qquad + \frac{4}{4} \\
 \hline
 y = -\frac{3}{4}x - \frac{11}{4}
 \end{array}$$

45) Through $(-5, 5)$ & $(2, -3)$

$$\begin{array}{r}
 m = \frac{-3-5}{2-(-5)} = -\frac{8}{7} \\
 y - y_1 = m(x - x_1) \\
 y - 5 = -\frac{8}{7}(x + 5) \\
 y - 5 = -\frac{8}{7}x - \frac{40}{7} \\
 y - \frac{35}{7} = -\frac{8}{7}x - \frac{40}{7} \\
 \quad + \frac{35}{7} \qquad \qquad + \frac{35}{7} \\
 \hline
 y = -\frac{8}{7}x - \frac{5}{7}
 \end{array}$$

47) Through $(4, 1)$ & $(1, 4)$

$$\begin{array}{r}
 m = \frac{4-1}{1-4} = \frac{3}{-3} = -1 \\
 y - y_1 = m(x - x_1) \\
 y - 1 = -1(x - 4) \\
 y - 1 = -x + 4 \\
 \quad + 1 \qquad \qquad + 1 \\
 \hline
 y = -x + 4
 \end{array}$$

2.5

1) $y = 2x + 4$

$$m = 2$$

$$\parallel m = 2$$

3) $y = 4x - 5$

$$m = 4$$

$$\parallel m = 4$$

5) $x - y = 4$

$$\frac{-x}{-1} = \frac{-x}{-1} + \frac{4}{-1}$$

$$y = x - 4$$

$$m = 1$$

$$\parallel m = 1$$

49) Through $(0, 2)$ & $(5, -3)$

$$m = \frac{-3-2}{5-0} = \frac{-5}{5} = -1$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -1(x - 0)$$

$$y - 2 = -x$$

$$\quad + 2 \qquad \qquad + 2$$

$$y = -x + 2$$

51) Through $(0, 3)$ & $(-1, -1)$

$$m = \frac{-1-3}{-1-0} = \frac{-4}{-1} = 4$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 4(x - 0)$$

$$y - 3 = 4x$$

$$\quad + 3 \qquad \qquad + 3$$

$$y = 4x + 3$$

$$13) \quad x - 3y = -6$$

$$\frac{-x}{-3} = \frac{-x}{-3} - \frac{6}{-3}$$

$$y = \frac{1}{3}x + 2$$

$$m = \frac{1}{3}$$

$$\perp m = 3$$

$$15) \quad x + 2y = 8$$

$$\frac{-x}{2} = \frac{-x}{2} + \frac{8}{2}$$

$$y = -\frac{1}{2}x + 4$$

$$m = -\frac{1}{2}$$

$$\perp m = 2$$

$$18) \quad \textit{Through (2, 5), par to } x = 4$$

$$m = \textit{undefined}$$

$$\parallel m = \textit{undefined}$$

$$x = 2$$

$$19) \quad \textit{Through (3, 4), par to } y = \frac{9}{2}x - 5$$

$$m = \frac{9}{2}$$

$$\parallel m = \frac{9}{2}$$

$$y - 4 = \frac{9}{2}(x - 3)$$

$$21) \quad \textit{Through (2, 3), par to } x = 0$$

$$m = \frac{7}{5}$$

$$\parallel m = \frac{7}{5}$$

$$y - 3 = \frac{7}{5}(x - 2)$$

$$23) \quad \textit{Through (4, 2), par to } x = 0$$

$$m = \textit{undefined}$$

$$\parallel m = \textit{undefined}$$

$$x = 4$$

$$25) \quad \textit{Through (1, -5), perp to } y = x + 1$$

$$m = 1$$

$$\perp m = -1$$

$$y + 5 = -1(x - 1)$$

$$27) \quad \textit{Through (5, 2) perp to } y = -5x - 3$$

$$m = -5$$

$$\perp m = \frac{1}{5}$$

$$y - 2 = \frac{1}{5}(x - 5)$$

$$29) \quad \textit{Through (4, 2) perp to } y = 4x$$

$$m = 4$$

$$\perp m = -\frac{1}{4}$$

$$y - 2 = -\frac{1}{4}(x - 4)$$

$$31) \quad \textit{Through (2, -2), perp to } y = \frac{1}{3}x$$

$$m = \frac{1}{3}$$

$$\perp m = -3$$

$$y + 2 = -3(x - 2)$$

$$33) \quad \textit{Through (4, -3), par to } y = -2x$$

$$m = -2$$

$$\parallel m = -2$$

$$y + 3 = -2(x - 4)$$

$$y + 3 = -2x + 8$$

$$\frac{-3}{-3} \quad \frac{-8}{-3}$$

$$y = -2x + 5$$

$$35) \quad \textit{Through (-3, 1), par to } y = -\frac{4}{3}x - 1$$

$$m = -\frac{4}{3}$$

$$\begin{aligned} \parallel m &= -\frac{4}{3} \\ y - 1 &= -\frac{4}{3}(x + 3) \\ y - 1 &= -\frac{4}{3}x - 4 \\ \frac{+1}{+1} & \quad \frac{+1}{+1} \\ y &= -\frac{4}{3}x - 3 \end{aligned}$$

37) Through $(-4, -1)$ par $y = -\frac{1}{2}x + 1$

$$\begin{aligned} m &= -\frac{1}{2} \\ \parallel m &= -\frac{1}{2} \\ y + 1 &= -\frac{1}{2}(x + 4) \\ y + 1 &= -\frac{1}{2}x - 2 \\ \frac{-1}{-1} & \quad \frac{-1}{-1} \\ y &= -\frac{1}{2}x - 3 \end{aligned}$$

39) Through $(-2, -1)$ par $y = -\frac{1}{2}x - 2$

$$\begin{aligned} m &= -\frac{1}{2} \\ \parallel m &= -\frac{1}{2} \\ y + 1 &= -\frac{1}{2}(x + 2) \\ y + 1 &= -\frac{1}{2}x - 1 \\ \frac{-1}{-1} & \quad \frac{-1}{-1} \\ y &= -\frac{1}{2}x - 2 \end{aligned}$$

41) Through $(4, 3)$, perp to $y = -x - 1$

$$\begin{aligned} m &= -1 \\ \perp m &= 1 \end{aligned}$$

$$\begin{aligned} y - 3 &= 1(x - 4) \\ y - 3 &= x - 4 \\ \frac{+3}{+3} & \quad \frac{+3}{+3} \\ y &= x - 1 \end{aligned}$$

43) Through $(5, 2)$, perp to $x = 0$

$$\begin{aligned} m &= \text{undefined} \\ \perp m &= 0 \\ y &= 2 \end{aligned}$$

45) Through $(-2, 5)$, perp to $y = x - 2$

$$\begin{aligned} m &= 1 \\ \perp m &= -1 \\ y - 5 &= -1(x + 2) \\ y - 5 &= -x - 2 \\ \frac{+5}{+5} & \quad \frac{+5}{+5} \\ y &= -x + 3 \end{aligned}$$

47) Through $(4, -3)$, perp to $y = \frac{1}{2}x - 3$

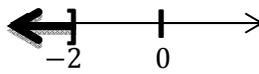
$$\begin{aligned} m &= \frac{1}{2} \\ \perp m &= -2 \\ y + 3 &= -2(x - 4) \\ y + 3 &= -2x + 8 \\ \frac{-3}{-3} & \quad \frac{-3}{-3} \\ y &= -2x + 5 \end{aligned}$$

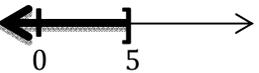
Chapter 3: Inequalities

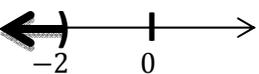
3.1

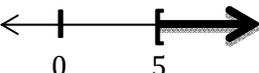
1) $n > -5$

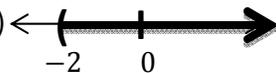
 $(-5, 0)$

3) $-2 \geq k$

 $(-\infty, -2]$

5) $5 \geq x$

 $(-\infty, 5]$

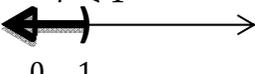
7) 
 $x < -2$

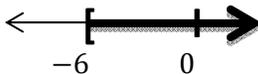
9) 
 $x \geq 5$

11) 
 $x > -2$

13) (11) $\frac{x}{11} \geq 10$ (11)
 $x \geq 110$

 $[110, \infty)$

15) $2 + r < 3$
 $\frac{-2}{-2} \quad \frac{-2}{-2}$
 $r < 1$

 $(-\infty, 1)$

17) $8 + \frac{n}{3} \geq 6$
 $\frac{-8}{-8} \quad \frac{-8}{-8}$
 $(3) \frac{n}{3} \geq -2$ (3)
 $n \geq -6$

 $[-6, \infty)$

19) (5) $2 > \frac{a-2}{5}$ (5)
 $10 > a - 2$
 $\frac{+2}{+2} \quad \frac{+2}{+2}$
 $12 > a$

 $(-\infty, 12)$

21) $-47 \geq 8 - 5x$
 $\frac{-8}{-5} \quad \frac{-8}{-5}$
 $\frac{-55}{-5} \geq \frac{-5x}{-5}$
 $11 \leq x$

 $[11, \infty)$

23) $-2(3 + k) < -44$
 $-6 - 2k < -44$
 $\frac{+6}{-2} \quad \frac{+6}{-2}$
 $\frac{-2k}{-2} < \frac{-38}{-2}$
 $k > 19$

 $(19, \infty)$

25) $18 < -2(-8 + p)$
 $18 < 16 - 2p$
 $\frac{-16}{-2} \quad \frac{-16}{-2}$
 $\frac{2}{-2} < \frac{-2p}{-2}$
 $-1 > p$

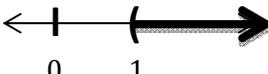
 $(-\infty, -1)$

27) $24 \geq -6(m - 6)$
 $24 \geq -6m + 36$
 $\frac{-36}{-6} \quad \frac{-36}{-6}$
 $\frac{-12}{-6} \geq \frac{-6m}{-6}$
 $2 \leq m$

 $[2, \infty)$

29) $-r - 5(r - 6) < -18$
 $-r - 5r + 30 < -18$
 $-6r + 30 < -18$
 $\frac{-30}{-6} \quad \frac{-30}{-6}$
 $\frac{-6r}{-6} < \frac{-48}{-6}$
 $r > 8$

 $(8, \infty)$

31) $24 + 4b < 4(1 + 6b)$
 $24 + 4b < 4 + 24b$
 $\frac{-4b}{24} \quad \frac{-4b}{24}$
 $24 < 4 + 20b$
 $\frac{-4}{20} \quad \frac{-4}{20}$
 $\frac{20}{20} < \frac{20b}{20}$
 $1 < b$

 $(1, \infty)$

$$33) -5v - 5 < -5(4v + 1)$$

$$-5v - 5 < -20v - 5$$

$$\frac{+20v}{15v - 5} \quad \frac{+20v}{+20v}$$

$$15v - 5 < -5$$

$$\frac{+5}{15v} \quad \frac{+5}{+5}$$

$$15v < 0$$

$$v < 0$$



$$(-\infty, 0)$$

$$35) 4 + 2(a + 5) < -2(-a - 4)$$

$$4 + 2a + 10 < 2a + 8$$

$$14 + 2a < 2a + 8$$

$$\frac{-2a - 2a}{14} < \frac{-2a - 2a}{8}$$

$$14 < 8$$

false

No solution \emptyset

$$37) -(k - 2) > -k - 20$$

$$-k + 2 > -k - 20$$

$$\frac{+k}{2} \quad \frac{+k}{+k}$$

$$2 > -20$$

true

All real numbers \mathbb{R}

3.2

$$1) (3) \frac{n}{3} \leq -3 (3) \text{ or } \frac{-5n}{-5} \leq \frac{-10}{-5}$$

$$n \leq -9 \text{ or } n \geq 2$$

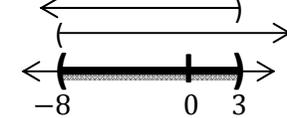


$$(-\infty, -9] \cup [2, \infty)$$

$$7) (8) \frac{v}{8} > -1(8) \text{ and } v - 2 < 1$$

$$\frac{+2}{v} \quad \frac{+2}{+2}$$

$$v > -8 \text{ and } v < 3$$

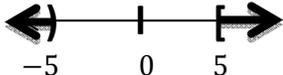


$$(-8, 3)$$

$$3) x + 7 \geq 12 \text{ or } \frac{9x}{9} < -\frac{45}{9}$$

$$\frac{-7}{x} \quad \frac{-7}{-7} \text{ or } x < -5$$

$$x \geq 5$$

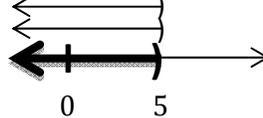


$$(-\infty, -5] \cup [5, \infty)$$

$$9) -8 + b < -3 \text{ and } \frac{4b}{4} < \frac{20}{4}$$

$$\frac{+8}{b} \quad \frac{+8}{+8}$$

$$b < 5 \text{ and } b < 5$$

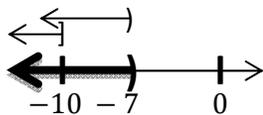


$$(-\infty, 5)$$

$$5) x - 6 < -13 \text{ or } \frac{6x}{6} \leq \frac{-60}{6}$$

$$\frac{+6}{x} \quad \frac{+6}{+6}$$

$$x < -7 \text{ or } x \leq -10$$

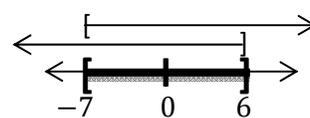


$$(-\infty, -7)$$

$$11) a + 10 \geq 3 \text{ and } \frac{8a}{8} \leq \frac{48}{8}$$

$$\frac{-10}{a} \quad \frac{-10}{-10}$$

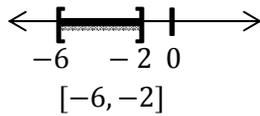
$$a \geq -7 \text{ and } a \leq 6$$



$$[-7, 6]$$

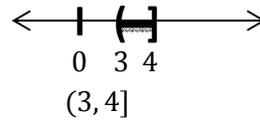
13) $3 \leq 9 + x \leq 7$

$$\frac{-9 - 9}{-6} \leq \frac{-9}{-2}$$



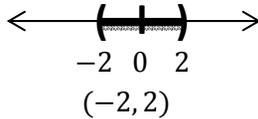
15) $11 < 8 + k \leq 12$

$$\frac{-8 - 8}{3} < \frac{-8}{4}$$



17) $-3 < x - 1 < 1$

$$\frac{-2}{-2} < \frac{0}{0} < \frac{2}{2}$$

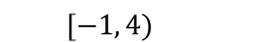


19) $-4 < 8 - 3m \leq 11$

$$\frac{-12}{-3} < \frac{-3m}{-3} \leq \frac{3}{-3}$$

$$4 > m \geq -1$$

$$-1 \leq m < 4$$



21) $-16 \leq 2n - 10 \leq -22$

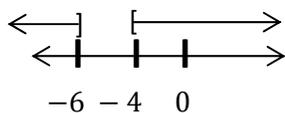
$$-16 \leq -22$$

No solution \emptyset

23) $-5b + 10 \leq 30$ and $7b + 2 \leq -40$

$$\frac{-5b}{-5} \leq \frac{20}{-5} \quad \text{and} \quad \frac{7b}{7} \leq \frac{-42}{7}$$

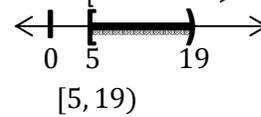
$$b \geq -4 \quad \text{and} \quad b \leq -6$$



25) $3x - 9 < 2x + 10$ and $5 + 7x \leq 10x - 10$

$$\frac{-2x - 2x}{x - 9} < \frac{-2x}{10} \quad \text{and} \quad \frac{-7x - 7x}{5} \leq \frac{-3x - 10}{3}$$

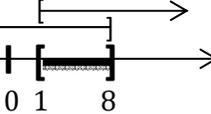
$$x < 19 \quad \text{and} \quad \frac{15}{3} \leq \frac{3x}{3}$$



27) $-8 - 6v \leq 8 - 8v$ and $7v + 9 \leq 6 + 10v$

$$\frac{-8 + 2v}{+8} \leq \frac{8 + 8v}{+8} \quad \text{and} \quad \frac{-7v}{3} \leq \frac{-7v}{3}$$

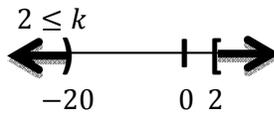
$$\frac{2v}{2} \leq \frac{16}{2} \quad \text{and} \quad \frac{3}{3} \leq \frac{3v}{3}$$



29) $1 + 5k \leq 7k - 3$ or $k - 10 > 2k + 10$

$$\frac{-5k - 5k}{1} \leq \frac{-8}{-2} \quad \text{or} \quad \frac{-k}{-10} > \frac{-k}{10}$$

$$\frac{4}{2} \leq \frac{2k}{2} \quad \text{or} \quad -20 > k$$



31) $2x + 9 \geq 10x + 1$ and $3x - 2 < 7x + 2$

$$\frac{-2x}{9} \geq \frac{-2x}{8x + 1} \quad \text{and} \quad \frac{-3x}{-2} < \frac{-3x}{4x + 2}$$

$$\frac{-1}{8} \geq \frac{-1}{8} \quad \text{and} \quad \frac{-2}{4} < \frac{-2}{4}$$

$$1 \geq x \quad \text{and} \quad -1 < x$$



3.3

1) $|n| \leq -11$

false

No Solution \emptyset

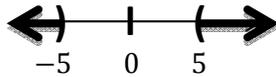
3) $|b| \leq -10$

false

No Solution \emptyset

5) $|x| > 5$

$$x > 5 \text{ or } x < -5$$

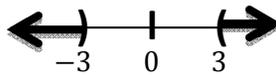


$$(-\infty, -5) \cup (5, \infty)$$

7) $\frac{10|n|}{10} > \frac{30}{10}$

$$|n| > 3$$

$$n > 3 \text{ or } n < -3$$



$$(-\infty, -3) \cup (3, \infty)$$

9) $\frac{-3|x|}{-3} < \frac{36}{-3}$

$$x > -12$$

false

No Solution \emptyset

11) $|n| + 4 > -5$

$$\frac{-4}{-4} \quad \frac{-4}{-4}$$

$$|n| > -9$$

true

All Real Numbers \mathbb{R}

13) $10 - 8|p| \geq 18$

$$\frac{-10}{-8} \quad \frac{-10}{-8}$$

$$\frac{-8|p|}{-8} \geq \frac{8}{-8}$$

$$|p| \geq -1$$

false

No solution \emptyset

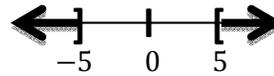
15) $9|n| - 3 \geq 42$

$$+ 3 \quad + 3$$

$$\frac{9|n|}{9} \geq \frac{45}{9}$$

$$|n| \geq 5$$

$$n \geq 5 \text{ or } n \leq -5$$



$$(-\infty, -5) \cup (5, \infty)$$

17) $\left|\frac{m}{9}\right| \geq -5$

true

All Real Numbers \mathbb{R}

19) $|9 + x| > -2$

true

All Real Numbers \mathbb{R}

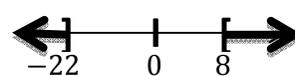
21) $\left|\frac{x+7}{3}\right| \geq 5$

$$(3) \frac{v+7}{3} \geq 5(3) \text{ or } (3) \frac{v+7}{3} \leq -5(3)$$

$$v + 7 \geq 15 \quad \text{or} \quad v + 7 \leq -15$$

$$\frac{-7}{-7} \quad \frac{-7}{-7} \quad \frac{-7}{-7} \quad \frac{-7}{-7}$$

$$v \geq 8 \quad \text{or} \quad v \leq -22$$



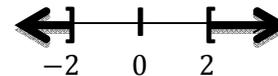
$$(-\infty, -22] \cup [8, \infty)$$

23) $\frac{7|-7x|}{7} \geq \frac{98}{7}$

$$|-7x| \geq 14$$

$$\frac{-7x}{-7} \geq \frac{14}{-7} \quad \text{or} \quad \frac{-7x}{-7} \leq \frac{-14}{-7}$$

$$x \leq -2 \quad \text{or} \quad x \geq 2$$



$$(-\infty, -2) \cup (2, \infty)$$

$$25) -5 + |-8k| \geq 51$$

$$\frac{-5}{+5} \quad \frac{+5}{+5}$$

$$|-8k| \geq 56$$

$$\frac{|-8k|}{-8} \geq \frac{56}{-8} \text{ or } \frac{|-8k|}{-8} \leq \frac{-56}{-8}$$

$$k \leq -7 \text{ or } k \geq 7$$

$$(-\infty, -7) \cup (7, \infty)$$

$$27) 8 - 4\left|\frac{x}{9}\right| > 12$$

$$\frac{-8}{-8} \quad \frac{-8}{-8}$$

$$\frac{-4\left|\frac{x}{9}\right|}{-4} > \frac{4}{-4}$$

$$\left|\frac{x}{9}\right| < -1$$

false
No Solution \emptyset

$$29) 7|-9 + m| + 3 \geq 66$$

$$\frac{-3}{-3} \quad \frac{-3}{-3}$$

$$\frac{7|-9+m|}{7} \geq \frac{63}{7}$$

$$|-9 + m| \geq 9$$

$$-9 + m \geq 9 \text{ or } -9 + m \leq -9$$

$$\frac{+9}{+9} \quad \frac{+9}{+9} \quad \frac{+9}{+9} \quad \frac{+9}{+9}$$

$$m \geq 18 \text{ or } m \leq 0$$

$$(-\infty, 0) \cup (18, \infty)$$

$$31) |3n + 10| \leq -26$$

false
No Solution \emptyset

$$33) |10b + 10| > 70$$

$$10b + 10 > 70 \text{ or } 10b + 10 < -70$$

$$\frac{-10}{-10} \quad \frac{-10}{-10} \quad \frac{-10}{-10} \quad \frac{-10}{-10}$$

$$\frac{10b}{10} > \frac{60}{10} \text{ or } \frac{10b}{10} < \frac{-80}{10}$$

$$b > 6 \text{ or } b < -8$$

$$(-\infty, -8) \cup (6, \infty)$$

$$35) |-10 + x| \geq 8$$

$$-10 + x \geq 8 \text{ or } -10 + x \leq -8$$

$$\frac{+10}{+10} \quad \frac{+10}{+10} \quad \frac{+10}{+10} \quad \frac{+10}{+10}$$

$$x \geq 18 \text{ or } x \leq 2$$

$$(-\infty, 2) \cup (18, \infty)$$

$$37) |-10 + a| - 3 \geq 7$$

$$\frac{+3}{+3} \quad \frac{+3}{+3}$$

$$|-10 + a| \geq 10$$

$$-10 + a \geq 10 \text{ or } -10 + a \leq -10$$

$$\frac{+10}{+10} \quad \frac{+10}{+10} \quad \frac{+10}{+10} \quad \frac{+10}{+10}$$

$$a \geq 20 \text{ or } a \leq 0$$

$$(-\infty, 0) \cup (20, \infty)$$

$$39) |3x - 1| - 9 \leq -8$$

$$\frac{+9}{+9} \quad \frac{+9}{+9}$$

$$|3x - 1| \leq 1$$

$$-1 \leq 3x - 1 \leq 1$$

$$\frac{+1}{+1} \quad \frac{+1}{+1} \quad \frac{+1}{+1}$$

$$\frac{0}{3} \leq \frac{3x}{3} \leq \frac{2}{3}$$

$$0 \leq x \leq \frac{2}{3}$$

$$\left[0, \frac{2}{3}\right]$$

$$41) -8|8n - 1| + 4 \geq -116$$

$$\frac{-4 \quad -4}{-8|8n-1|} \geq \frac{-120}{-8}$$

$$|8n - 1| \geq 15$$

$$-15 \leq 8n - 1 \leq 15$$

$$\frac{+1 \quad +1 \quad +1}{-14 \leq 8n \leq 16}$$

$$\frac{-14}{8} \leq \frac{8n}{8} \leq \frac{16}{8}$$

$$-\frac{7}{4} \leq n \leq 2$$



$$-\frac{7}{4} \quad 0 \quad 2$$

$$\left[-\frac{7}{4}, 2\right]$$

$$43) -10 + 9|3p - 9| < -37$$

$$\frac{+10 \quad +10}{9|3p-9|} < \frac{-27}{9}$$

$$\frac{9|3p-9|}{9} < \frac{-27}{9}$$

$$|3p - 9| < -3$$

false

No Solution \emptyset

$$45) 9|2 - 10n| - 8 > 100$$

$$\frac{+8 \quad +8}{9|2-10n|} > \frac{108}{9}$$

$$\frac{9|2-10n|}{9} > \frac{108}{9}$$

$$|2 - 10n| > 12$$

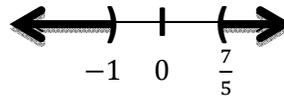
$$2 - 10n > 12 \text{ or } 2 - 10n < -12$$

$$\frac{-2 \quad -2 \quad -2}{-10n} > \frac{10}{-10} \text{ or } \frac{-10n}{-10} < \frac{-14}{-10}$$

$$\frac{-10n}{-10} > \frac{10}{-10} \text{ or } \frac{-10n}{-10} < \frac{-14}{-10}$$

$$n < -1 \text{ or } n > \frac{7}{5}$$

$$n > \frac{7}{5}$$

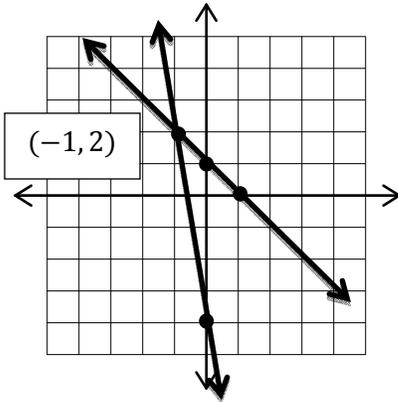


$$(-\infty, -1) \cup \left(\frac{7}{5}, \infty\right)$$

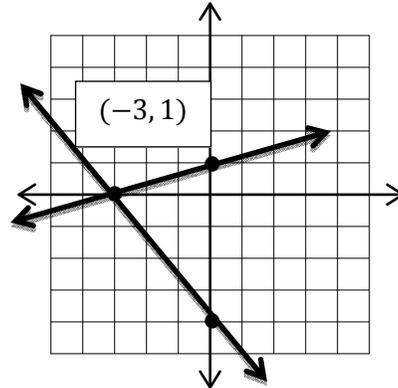
Chapter 4: Systems of Equations

4.1

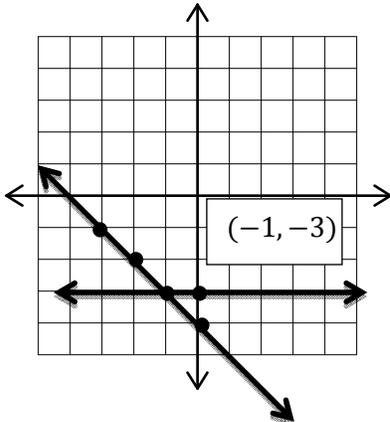
1) $y = -x + 1$
 $y = -5x - 3$



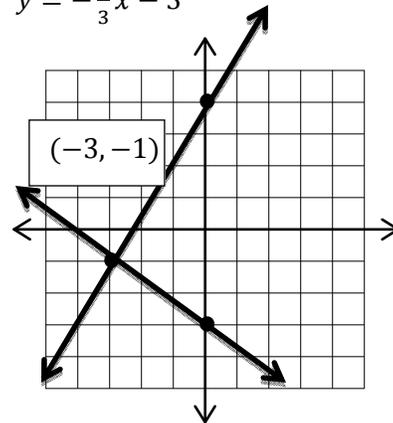
7) $y = \frac{1}{3}x + 2$
 $y = -\frac{5}{3}x - 4$



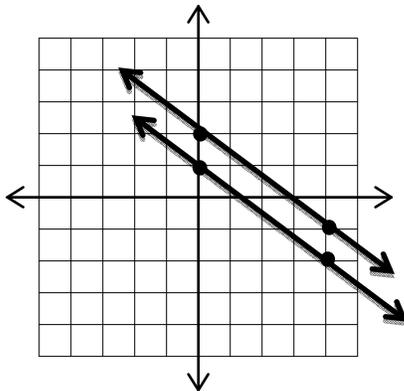
3) $y = -3$
 $y = -x - 4$



9) $y = \frac{5}{3}x + 4$
 $y = -\frac{2}{3}x - 3$



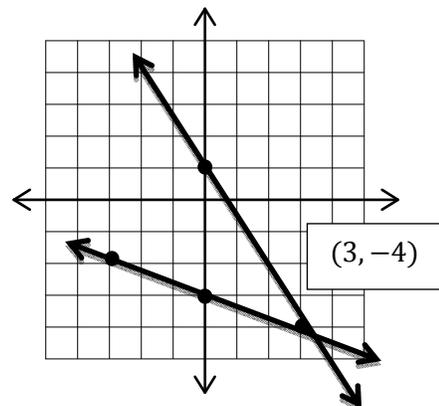
5) $y = -\frac{3}{4}x + 1$
 $y = -\frac{3}{4}x + 2$ No Solution



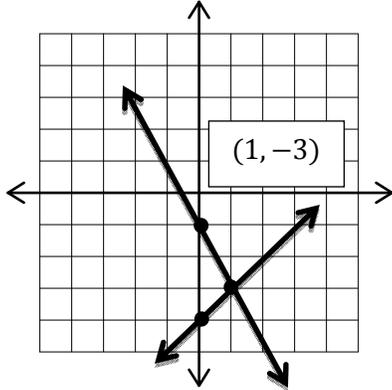
11) $x + 3y = -9$ $5x + 3y = 3$

$$\begin{array}{r} -x \\ \hline \frac{3y}{3} = -\frac{x}{3} - \frac{9}{3} \\ y = -\frac{1}{3}x - 3 \end{array}$$

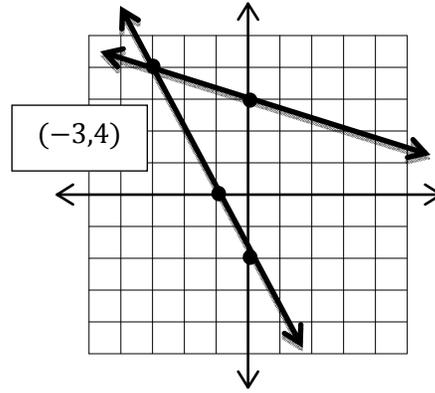
$$\begin{array}{r} -5x \\ \hline \frac{3y}{3} = -\frac{5x}{3} + \frac{3}{3} \\ y = -\frac{5}{3}x + 1 \end{array}$$



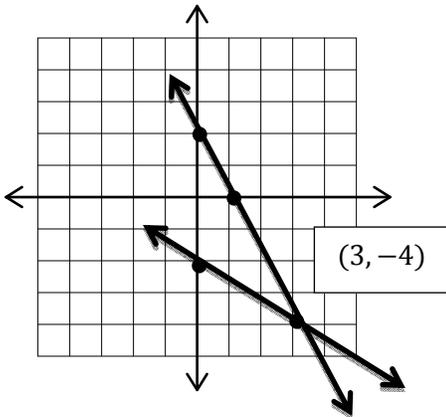
$$13) \begin{array}{r} x - y = 4 \\ -x \quad -x \\ \hline -\frac{y}{-1} = -\frac{x}{-1} + \frac{4}{-1} \\ y = x - 4 \end{array} \quad \begin{array}{r} 2x + y = -1 \\ -2x \quad -2x \\ \hline y = -2x - 1 \end{array}$$



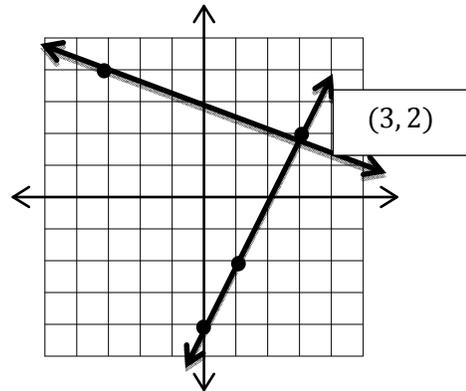
$$19) \begin{array}{r} 2x + y = 2 \\ -2x \quad -2x \\ \hline y = -2x + 2 \end{array} \quad \begin{array}{r} x + 3y = 9 \\ -x \quad -x \\ \hline \frac{3y}{3} = -\frac{x}{3} + \frac{9}{3} \\ y = -\frac{1}{3}x + 3 \end{array}$$



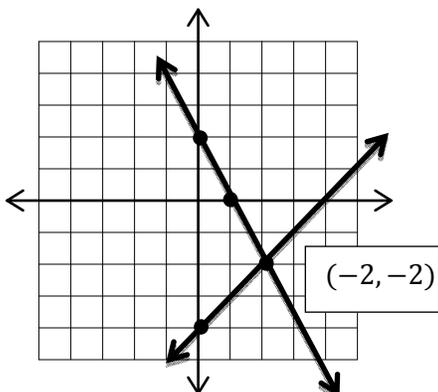
$$15) \begin{array}{r} 2x + 3y = -6 \\ -2x \quad -2x \\ \hline \frac{3y}{3} = -\frac{2x}{3} - \frac{6}{3} \\ y = -\frac{2}{3}x - 2 \end{array} \quad \begin{array}{r} 2x + y = 2 \\ -2x \quad -2x \\ \hline y = -2x + 2 \end{array}$$



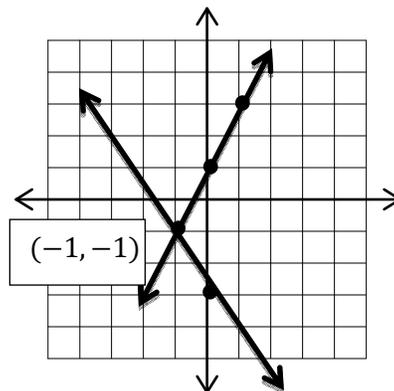
$$21) \begin{array}{r} 0 = -6x - 9y + 36 \\ +9y \quad +9y \\ \hline \frac{9y}{9} = -\frac{6x}{9} + \frac{36}{9} \\ y = -\frac{2}{3}x + 4 \end{array} \quad \begin{array}{r} 12 = 6x - 3y \\ -6x \quad -6x \\ \hline -\frac{6x}{-3} + \frac{12}{-3} = \frac{-3y}{-3} \\ 2x - 4 = y \end{array}$$



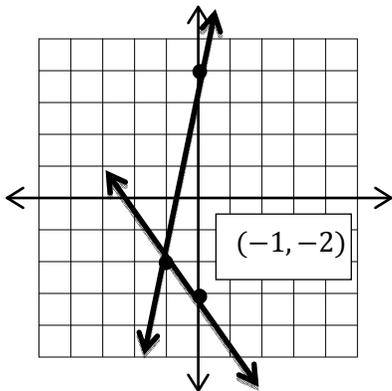
$$17) \begin{array}{r} 2x + y = 2 \\ -2x \quad -2x \\ \hline y = -2x + 2 \end{array} \quad \begin{array}{r} x - y = 4 \\ -x \quad -x \\ \hline -\frac{y}{-1} = -\frac{x}{-1} + \frac{4}{-1} \\ y = x - 4 \end{array}$$



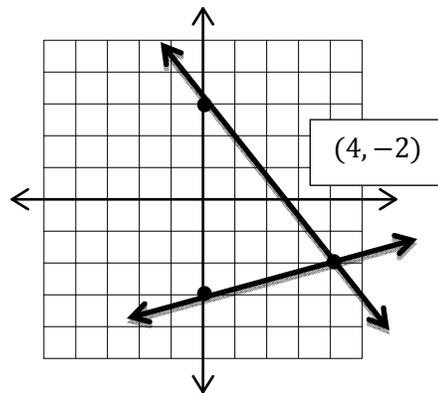
$$23) \begin{array}{r} 2x - y = -1 \\ -2x \quad -2x \\ \hline -\frac{y}{-1} = -\frac{2x}{-1} - \frac{1}{-1} \\ y = 2x + 1 \end{array} \quad \begin{array}{r} 0 = -2x - y - 3 \\ +y \quad +y \\ \hline y = -2x - 3 \end{array}$$



$$25) \begin{array}{r} 3 + y = -x \\ -3 \quad -3 \\ \hline y = -x - 3 \end{array} \quad \begin{array}{r} -\frac{4}{-1} - \frac{6x}{-1} = -\frac{y}{-1} \\ 4 + 6x = y \end{array}$$

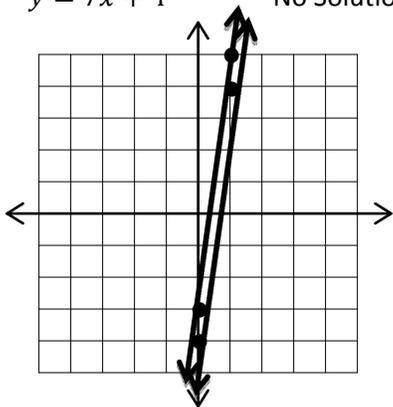


$$29) \begin{array}{r} -\frac{12}{4} + \frac{x}{4} = \frac{4y}{4} \\ -3 + \frac{1}{4}x = y \end{array} \quad \begin{array}{r} \frac{12}{4} - \frac{5x}{4} = \frac{4y}{4} \\ 3 - \frac{5}{4}x = y \end{array}$$



$$27) \begin{array}{r} -y + 7x = 4 \\ -7x \quad -7x \quad + y \quad + y \\ \hline -\frac{y}{-1} = -\frac{7x}{-1} + \frac{4}{-1} \\ y = 7x + 4 \end{array} \quad \begin{array}{r} -y - 3 + 7x = 0 \\ + y \\ \hline -3 + 7x = y \end{array}$$

No Solution



4.2

$$1) \begin{array}{r} y = -3x \\ y = 6x - 9 \\ -3x = 6x - 9 \\ -6x \quad -6x \\ \hline -\frac{9x}{-9} = -\frac{9}{-9} \\ x = 1 \\ y = -3(1) = -3 \\ (1, -3) \end{array}$$

$$3) \begin{array}{r} y = -2x - 9 \\ y = 2x - 1 \\ -2x - 9 = 2x - 1 \\ +2x \quad +2x \\ \hline -9 = 4x - 1 \\ +1 \quad +1 \\ \hline -\frac{8}{4} = \frac{4x}{4} \\ -2 = x \\ y = -2(-2) - 9 \\ y = 4 - 9 \\ y = -5 \\ (-2, -5) \end{array}$$

$$5) \begin{array}{r} y = 6x + 4 \\ y = -3x - 5 \\ 6x + 4 = -3x - 5 \\ +3x \quad +3x \\ \hline 9x + 4 = -5 \\ -4 \quad -4 \\ \hline \frac{9x}{9} = \frac{-9}{9} \\ x = -1 \\ y = 6(-1) + 4 \\ y = -6 + 4 \\ y = -2 \\ (-1, -2) \end{array}$$

$$\begin{aligned}
7) \quad y &= 3x + 2 \\
y &= -3x + 8 \\
3x + 2 &= -3x + 8 \\
+3x & \quad +3x \\
\hline
6x + 2 &= 8 \\
-2 & \quad -2 \\
\hline
\frac{6x}{6} &= \frac{6}{6} \\
x &= 1 \\
y &= 3(1) + 2 \\
y &= 3 + 2 \\
y &= 5 \\
(1, 5)
\end{aligned}$$

$$\begin{aligned}
9) \quad y &= 2x - 3 \\
y &= -2x + 9 \\
2x - 3 &= -2x + 9 \\
+2x & \quad +2x \\
\hline
4x - 3 &= 9 \\
+3 & \quad +3 \\
\hline
\frac{4x}{4} &= \frac{12}{4} \\
x &= 3 \\
y &= 2(3) - 3 \\
y &= 6 - 3 \\
y &= 3 \\
(3, 3)
\end{aligned}$$

$$\begin{aligned}
11) \quad y &= 6x - 6 \\
-3x - 3y &= -24 \\
-3x - 3(6x - 6) &= \\
-24 & \\
-3x - 18x + 18 &= -24 \\
-21x + 18 &= -24 \\
-18 & \quad -18 \\
\hline
\frac{-21x}{-21} &= \frac{-42}{-21} \\
x &= 2 \\
y &= 6(2) - 6 \\
y &= 12 - 6 \\
y &= 6 \\
(2, 6)
\end{aligned}$$

$$\begin{aligned}
13) \quad y &= -6 \\
3x - 6y &= 30 \\
3x - 6(-6) &= 30 \\
3x + 36 &= 30 \\
-36 & \quad -36 \\
\hline
\frac{3x}{3} &= \frac{-6}{3} \\
x &= -2 \\
(-2, -6)
\end{aligned}$$

$$\begin{aligned}
15) \quad y &= -5 \\
3x + 4y &= -17 \\
3x + 4(-5) &= -17 \\
3x - 20 &= -17 \\
+20 & \quad +20 \\
\hline
\frac{3x}{3} &= \frac{3}{3} \\
x &= 1 \\
(1, -5)
\end{aligned}$$

$$\begin{aligned}
17) \quad -2x + 2y &= 18 \\
y &= 7x + 15 \\
-2x + 2(7x + 15) &= 18 \\
-2x + 14x + 30 &= 18 \\
12x + 30 &= 18 \\
-30 & \quad -30 \\
\hline
\frac{12x}{12} &= \frac{-12}{12} \\
x &= -1 \\
y &= 7(-1) + 15 \\
y &= -7 + 15 \\
y &= 8 \\
(-1, 8)
\end{aligned}$$

$$\begin{aligned}
19) \quad y &= -8x + 19 \\
-x + 6y &= 16 \\
-x + 6(-8x + 19) &= \\
16 & \\
-x - 48x + 114 &= 16 \\
-49x + 114 &= 16 \\
-114 & \quad -114 \\
\hline
\frac{-49x}{-49} &= \frac{-98}{-49} \\
x &= 2 \\
y &= -8(2) + 19 \\
y &= -16 + 19 \\
y &= 3 \\
(2, 3)
\end{aligned}$$

$$\begin{aligned}
21) \quad 7x - 2y &= -7 \\
y &= 7 \\
7x - 2(7) &= -7 \\
7x - 14 &= -7 \\
+14 & \quad +14 \\
\hline
\frac{7x}{7} &= \frac{7}{7} \\
x &= 1 \\
(1, 7)
\end{aligned}$$

$$\begin{aligned}
23) \quad x - 5y &= 7 \\
2x + 7y &= -20 \\
x - 5y &= 7 \\
+5y & \quad +5y \\
\hline
x &= 5y + 7 \\
2(5y + 7) + 7y &= -20 \\
10y + 14 + 7y &= -20 \\
17y + 14 &= -20 \\
-14 & \quad -14 \\
\hline
\frac{17y}{17} &= \frac{-34}{17} \\
y &= -2 \\
x - 5(-2) &= 7 \\
x + 10 &= 7 \\
-10 & \quad -10 \\
\hline
x &= -3 \\
(-3, -2)
\end{aligned}$$

$$\begin{array}{r}
25) \quad -2x - y = -5 \\
x - 8y = -23 \\
\hline
+8y \quad +8y \\
x = 8y - 23 \\
-2(8y - 23) - y = -5 \\
-16y + 46 - y = -5 \\
-17y + 46 = -5 \\
\hline
-46 \quad -46 \\
\hline
-17y = -51 \\
-17 \quad -17 \\
\hline
y = 3 \\
x - 8(3) = -23 \\
x - 24 = -23 \\
\hline
+24 \quad +24 \\
\hline
x = 1 \\
(1, 3)
\end{array}$$

$$\begin{array}{r}
27) \quad -6x + y = 20 \\
-3x - 3y = -18 \\
-6x + y = 20 \\
\hline
+6x \quad +6x \\
y = 6x + 20 \\
-3x - 3(6x + 20) = -18 \\
-3x - 18x - 60 = -18 \\
-21x - 60 = -18 \\
\hline
+60 \quad +60 \\
\hline
-21x = 42 \\
-21 \quad -21 \\
\hline
x = -2 \\
y = 20 + 6(-2) \\
y = 20 - 12 \\
y = 8 \\
(-2, 8)
\end{array}$$

$$\begin{array}{r}
29) \quad 3x + y = 9 \\
2x + 8y = -16 \\
3x + y = 9 \\
\hline
-3x \quad -3x \\
y = -3x + 9 \\
2x + 8(-3x + 9) = \\
-16 \\
2x - 24x + 72 = -16 \\
-22x + 72 = -16 \\
\hline
-72 \quad -72 \\
\hline
-22x = -88 \\
-22 \quad -22 \\
\hline
x = 4 \\
y = -3(4) + 9 \\
y = -12 + 9 \\
y = -3 \\
(4, -3)
\end{array}$$

$$\begin{array}{r}
32) \quad 2x + y = 2 \\
3x + 7y = 14 \\
2x + y = 2 \\
\hline
-2x \quad -2x \\
y = 2 - 2x \\
3x + 7(2 - 2x) = 14 \\
3x + 14 - 14x = 14 \\
-11x + 14 = 14 \\
\hline
-14 \quad -14 \\
\hline
-11x = 0 \\
-11 \quad -11 \\
\hline
x = 0 \\
y = 2 - 2(0) \\
y = 2 - 0 \\
y = 2 \\
(0, 2)
\end{array}$$

$$\begin{array}{r}
34) \quad x + 5y = 15 \\
-3x + 2y = 6 \\
x + 5y = 15 \\
\hline
-5y \quad -5y \\
x = 15 - 5y \\
-3(15 - 5y) + 2y = 6 \\
-45 + 15y + 2y = 6 \\
-45 + 17y = 6 \\
\hline
+45 \quad +45 \\
\hline
17y = 51 \\
17 \quad 17 \\
\hline
y = 3 \\
x = 15 - 5(3) \\
x = 15 - 15 \\
x = 0 \\
(0, 3)
\end{array}$$

$$\begin{array}{r}
36) \quad -2x + 4y = -16 \\
y = -2 \\
-2x + 4(-2) = -16 \\
-2x - 8 = -16 \\
\hline
+8 \quad +8 \\
\hline
-2x = -8 \\
-2 \quad -2 \\
\hline
x = 4 \\
(4, -2)
\end{array}$$

$$\begin{array}{r}
38) \quad -6x + 6y = -12 \\
8x - 3y = 16 \\
-6x + 6y = -12 \\
\hline
+6x \quad +6x \\
\hline
\frac{6y}{6} = \frac{6x}{6} - \frac{12}{6} \\
y = x - 2 \\
8x - 3(x - 2) = 16 \\
8x - 3x + 6 = 16 \\
5x + 6 = 16 \\
\hline
-6 \quad -6 \\
\hline
5x = 10 \\
5 \quad 5 \\
\hline
x = 2 \\
y = (2) - 2 \\
y = 0 \\
(2, 0)
\end{array}$$

$$\begin{aligned}
39) \quad & 2x + 3y = 16 \\
& -7x - y = 20 \\
& \underline{+7x \quad + 7x} \\
& -\frac{y}{-1} = \frac{7x}{-1} + 20/-1 \\
& y = -7x - 20 \\
& 2x + 3(-7x - 20) = 16 \\
& 2x - 21x - 60 = 16 \\
& -19x - 60 = 16 \\
& \underline{+60 \quad + 60} \\
& \frac{-19x}{-19} = \frac{76}{-19} \\
& x = -4 \\
& y = -7(-4) - 20 \\
& y = 28 - 20 \\
& y = 8 \\
& (-4, 8)
\end{aligned}$$

4.3

$$\begin{aligned}
1) \quad & 4x + 2y = 0 \\
& -4x - 9y = -28 \\
& \underline{-\frac{7y}{-7} = -\frac{28}{-7}} \\
& y = 4 \\
& 4x + 2(4) = 0 \\
& 4x + 8 = 0 \\
& \underline{-8 \quad -8} \\
& \frac{4x}{4} = -\frac{8}{4} \\
& x = -2 \\
& (-2, 4)
\end{aligned}$$

$$\begin{aligned}
3) \quad & -9x + 5y = -22 \\
& 9x - 5y = 13 \\
& 0 = -9 \\
& \text{false} \\
& \text{No Solution } \emptyset
\end{aligned}$$

$$\begin{aligned}
5) \quad & -6x + 9y = 3 \\
& 6x - 9y = -9 \\
& 0 = -6 \\
& \text{false} \\
& \text{No Solution } \emptyset
\end{aligned}$$

$$\begin{aligned}
7) \quad & -1(4x - 6y) = (-10) - 1 \\
& 4x - 6y = -14 \\
& -4x + 6y = 10 \\
& 0 = 10 \\
& \text{false} \\
& \text{No Solution } \emptyset
\end{aligned}$$

$$\begin{aligned}
9) \quad & -1(-x - 5y) = 28(-1) \\
& -x + 4y = -17 \\
& \underline{x + 5y = -28} \\
& \frac{9y}{9} = -\frac{45}{9} \\
& y = -5 \\
& -x - 5(-5) = 28 \\
& -x + 25 = 28 \\
& \underline{-25 \quad -25} \\
& \frac{-x}{-1} = \frac{3}{-1} \\
& x = -3 \\
& (-3, -5)
\end{aligned}$$

$$\begin{aligned}
11) \quad & 2(2x - y) = (5)2 \\
& 5x + 2y = -28 \\
& \underline{4x - 2y = 10} \\
& \frac{9x}{9} = \frac{-18}{9} \\
& x = -2 \\
& 2(-2) - y = 5 \\
& -4 - y = 5 \\
& \underline{+4 \quad +4} \\
& \frac{-y}{-1} = \frac{9}{-1} \\
& y = -9 \\
& (-2, -9)
\end{aligned}$$

$$\begin{aligned}
13) \quad & 10x + 6y = 24 \\
& -6(-6x + y) = (4)(-6) \\
& 10x + 6y = 24 \\
& \underline{36x - 6y = -24} \\
& \frac{46x}{46} = \frac{0}{46} \\
& x = 0 \\
& 10(0) + 6y = 24 \\
& \frac{6y}{6} = \frac{24}{6} \\
& y = 4 \quad (0, 4)
\end{aligned}$$

$$15) 3(2x + 4y) = (24)3$$

$$4x - 12y = 8$$

$$\underline{6x + 12y = 72}$$

$$\frac{10x}{10} = \frac{80}{10}$$

$$x = 8$$

$$2(8) + 4y = 24$$

$$16 + 4y = 24$$

$$\underline{-16 \quad -16}$$

$$\frac{4y}{4} = \frac{8}{4}$$

$$y = 2$$

$$(8, 2)$$

$$17) 2(-7x + 4y) = (-4)2$$

$$10x - 8y = -8$$

$$\underline{-14x + 8y = 8}$$

$$\left(-\frac{4x}{4}\right) = -\frac{16}{-4}$$

$$x = 4$$

$$-7(4) + 4y = -4$$

$$-28 + 4y = -4$$

$$\underline{+28 \quad +28}$$

$$\frac{4y}{4} = \frac{24}{4}$$

$$y = 6$$

$$(4, 6)$$

$$19) 5x + 10y = 20$$

$$2(-6x - 5y) = (-3)2$$

$$5x + 10y = 20$$

$$\underline{-12x - 10y = -6}$$

$$\left(-\frac{7x}{7}\right) = 14/-7$$

$$x = -2$$

$$5(-2) + 10y = 20$$

$$-10 + 10y = 20$$

$$\underline{+10 \quad +10}$$

$$\frac{10y}{10} = \frac{30}{10}$$

$$y = 3$$

$$(-2, 3)$$

$$21) 5(-7x - 3y) = 12(5)$$

$$-3(-6x - 5y) = 20(-3)$$

$$-35x - 15y = 60$$

$$\underline{18x + 15y = -60}$$

$$-\frac{17x}{-17} = \frac{0}{-17}$$

$$x = 0$$

$$-7(0) - 3y = 12$$

$$\frac{-3y}{-3} = \frac{12}{-3}$$

$$y = -4$$

$$(0, -4)$$

$$23) 7(9x - 2y) = (-18)7$$

$$-2(5x - 7y) = (-10)(-2)$$

$$63x - 14y = -126$$

$$\underline{-10x + 14y = 20}$$

$$\frac{53x}{53} = \frac{-106}{53}$$

$$x = -2$$

$$9(-2) - 2y = -18$$

$$-18 - 2y = -18$$

$$\underline{+18 \quad +18}$$

$$\frac{-2y}{-2} = \frac{0}{-2}$$

$$y = 0$$

$$(-2, 0)$$

$$25) 3(9x + 6y) = (-21)3$$

$$2(-10x - 9y) = 28(2)$$

$$27x + 18y = -63$$

$$\underline{-20x - 18y = 56}$$

$$\frac{7x}{7} = \frac{-7}{7}$$

$$x = -1$$

$$9(-1) + 6y = -21$$

$$-9 + 6y = -21$$

$$\underline{+9 \quad +9}$$

$$\frac{6y}{6} = \frac{-12}{6}$$

$$y = -2$$

$$(-1, -2)$$

$$27) 3(-7x + 5y) = (-8)3$$

$$5(-3x - 3y) = 12(5)$$

$$-21x + 15 = -24$$

$$\underline{-15x - 15 = 60}$$

$$-\frac{36x}{36} = \frac{36}{-36}$$

$$x = -1$$

$$-7(-1) + 5y = -8$$

$$7 + 5y = -8$$

$$\underline{-7 \quad -7}$$

$$\frac{5y}{5} = \frac{-15}{5}$$

$$y = -3$$

$$(-1, -3)$$

$$29) 5(-8x - 8y) = (-8)5$$

$$4(10x + 9y) = (1)4$$

$$-40x - 40y = -40$$

$$\underline{40x + 36y = 4}$$

$$\frac{-4y}{-4} = \frac{-36}{-4}$$

$$y = 9$$

$$-8x - 8(9) = -8$$

$$-8x - 72 = -8$$

$$\underline{+72 \quad +72}$$

$$\frac{-8x}{8} = \frac{64}{8}$$

$$x = 8$$

$$(8, 9)$$

$$31) 9y = 7 - x$$

$$-18y + 4x = -26$$

$$9y = 7 - x$$

$$\underline{+x \quad +x}$$

$$2(9y + x) = (7)2$$

$$-18y + 4x = -26$$

$$\underline{18y + 2x = 14}$$

$$\frac{6x}{6} = \frac{-12}{6}$$

$$x = -2$$

$$9y = 7 - (-2)$$

$$\frac{9y}{9} = \frac{9}{9}$$

$$y = 1$$

$$(-2, 1)$$

$$33) 0 = 9x + 5y$$

$$(7)y = \frac{2}{7}x(7)$$

$$7y = 2x$$

$$\frac{-7y \quad -7y}{-7y \quad -7y}$$

$$(-9)0 = (2x - 7y)(-9)$$

$$2(0) = (9x + 5y)2$$

$$0 = -18x + 63y$$

$$0 = \underline{18x + 10y}$$

$$\frac{0}{73} = \frac{73y}{73}$$

$$0 = y$$

$$0 = 9x + 5(0)$$

$$\frac{0}{9} = \frac{9x}{9}$$

$$0 = x$$

$$(0,0)$$

4.4

$$1) \begin{aligned} (I) \quad a - 2b + c &= 5 \\ (II) 2a + b - c &= -1 \\ (III) 3a + 3b - 2c &= -4 \end{aligned}$$

$$A: -1(3a - b) = 4(-1)$$

$$B: 5a - b = 6$$

$$\frac{-3a + b = -4}{-3a + b = -4}$$

$$\frac{2a}{2} = \frac{2}{2}$$

$$a = 1$$

$$(1, -1, 2)$$

$$\begin{aligned} (I) \quad a - 2b + c &= 5 \\ (II) 2a + b - c &= -1 \\ A: 3a - b &= 4 \end{aligned}$$

$$A: 3(1) - b = 4$$

$$3 - b = 4$$

$$\frac{-3 \quad -3}{-3 \quad -3}$$

$$\frac{-b}{-1} = \frac{1}{-1}$$

$$b = -1$$

$$\begin{aligned} (I) 2(a - 2b + c) &= (5)2 \\ (III) 3a + 3b - 2c &= -4 \\ \underline{2a - 4b + 2c} &= 10 \\ B: 5a - b &= 6 \end{aligned}$$

$$(I) (1) - 2(-1) + c = 5$$

$$1 + 2 + c = 5$$

$$3 + c = 5$$

$$\frac{-3 \quad -3}{-3 \quad -3}$$

$$c = 2$$

$$3) \begin{aligned} (I) 3x + y - z &= 11 \\ (II) x + 3y &= z + 13 \\ (III) x + y - 3z &= 11 \end{aligned}$$

$$(II)x + 3y = z + 13$$

$$\frac{-z - z}{-z - z}$$

$$(II) x + 3y - z = 13$$

$$A: -2x + 2y = 2$$

$$B: \underline{-8x - 2y = -22}$$

$$\frac{-10x}{-10} = \frac{20}{-10}$$

$$x = 2$$

$$(2, 3, -2)$$

$$(I) -1(3x + y - z) = 11(-1)$$

$$(II) \quad x + 3y - z = 13$$

$$\underline{-3x - y + z = -11}$$

$$A: -2x + 2y = 2$$

$$A: -2(2) + 2y = 2$$

$$-4 + 2y = 2$$

$$\frac{+4 \quad +4}{+4 \quad +4}$$

$$\frac{2y}{2} = \frac{6}{2}$$

$$y = 3$$

$$(I) -3(3x + y - z) = 11(-3)$$

$$(III) \quad x + y - 3z = 11$$

$$\underline{-9x - 3y + 3z = -33}$$

$$B: -8x - 2y = -22$$

$$(I) 3(2) + 3 - z = 11$$

$$6 + 3 - z = 11$$

$$9 - z = 11$$

$$\frac{-9 \quad -9}{-9 \quad -9}$$

$$\frac{-z}{-1} = \frac{2}{-1}$$

$$z = -2$$

5) (I) $x + 6y + 3z = 4$ (II) $2x + y + 2z = 3$ (III) $3x - 2y + z = 0$	(I) $x + 6y + 3z = 4$ (III) $3(3x - 2y + z) = (0)3$ $x + 6y + 3z = 4$ $\underline{9x - 6y + 3z = 0}$ A: $10x + 6z = 4$	(II) $2(2x + y + 2z) = (3)2$ (III) $3x - 2y + z = 0$ $\underline{4x + 2y + 4z = 6}$ B: $7x + 5z = 6$
A: $-5(10x + 6z) = 4(-5)$ B: $6(7x + 5z) = 6(6)$ $-50x - 30z = -20$ $\underline{42x + 30z = 36}$ $\frac{-8x}{-8} = \frac{16}{-8}$ $x = -2$	A: $10(-2) + 6z = 4$ $-20 + 6z = 4$ $\underline{+20 \quad +20}$ $\frac{6z}{6} = \frac{24}{6}$ $z = 4$ (-2, -1, 4)	(I) $(-2) + 6y + 3(4) = 4$ $-2 + 6y + 12 = 4$ $10 + 6y = 4$ $\underline{-10 \quad -10}$ $\frac{6y}{6} = \frac{-6}{6}$ $y = -1$
7) (I) $x + y + z = 6$ (II) $2x - y - z = -3$ (III) $x - 2y + 3z = 6$	(I) $x + y + z = 6$ (II) $\underline{2x - y - z = -3}$ $\frac{3x}{3} = \frac{3}{3}$ $x = 1$	(II) $2(1) - y - z = -3$ $2 - y - z = -3$ $\underline{-2 \quad -2}$ A: $-y - z = -5$
A: $3(-y - z) = (-5)3$ B: $-2y + 3z = 5$ $\underline{-3y - 3z = -15}$ $-\frac{5y}{-5} = -\frac{10}{-5}$ $y = 2$	A: $-(2) - z = -5$ $\underline{+2 \quad +2}$ $-\frac{z}{-1} = -\frac{3}{-1}$ $z = 3$	(III) $1 - 2y + 3z = 6$ $\underline{-1 \quad -1}$ B: $-2y + 3z = 5$
(1, 2, 3)		
9) (I) $x + y - z = 0$ (II) $x - y - z = 0$ (III) $x + y + 2z = 0$	(I) $x + y - z = 0$ (II) $\underline{x - y - z = 0}$ A: $2x - 2z = 0$	(II) $x - y - z = 0$ (III) $\underline{x + y + 2z = 0}$ B: $2x + z = 0$
A: $(-1)(2x - 2z) = 0(-1)$ B: $2x + z = 0$ $\underline{-2x + 2z = 0}$ $\frac{3z}{0} = 0$ $x = 0$	A: $2x - 2(0) = 0$ $\frac{2x}{2} = \frac{0}{2}$ $x = 0$	(I) $0 + y - 0 = 0$ $y = 0$
(0, 0, 0)		

$$\begin{array}{lll}
11) \text{ (I)} -2x + y - 3z = 1 & \text{(I)} -2x + y - 3z = 1 & \text{(I)} 2(-2x + y - 3z) = (1)2 \\
\text{(II)} x - 4y + z = 6 & \text{(II)} 2(x - 4y + z) = 6(2) & \text{(III)} 4x + 16y + 4z = 24 \\
\text{(III)} 4x + 16y + 4z = 24 & \begin{array}{r} -2x + y - 3z = 1 \\ \underline{2x - 8y + 2z = 12} \end{array} & \begin{array}{r} \underline{-4x + 2y - 6z = 2} \\ B: 18y - 2z = 26 \end{array} \\
A: & -7y - z = 13 &
\end{array}$$

$$\begin{array}{lll}
A: -2(-7y - z) = 13(-2) & A: -7(0) - z = 13 & \text{(I)} -2x + 0 - 3(-13) = 1 \\
B: 18y - 2z = 26 & \begin{array}{r} -\frac{z}{-1} = \frac{13}{-1} \\ z = -13 \end{array} & \begin{array}{r} -2x + 39 = 1 \\ \underline{-39 \quad -39} \\ -\frac{2x}{-2} = -\frac{38}{-2} \\ x = 19 \end{array} \\
\begin{array}{r} \underline{14y + 2z = -26} \\ \frac{32y}{32} = \frac{0}{32} \\ y = 0 \end{array} & (19, 0, -13) &
\end{array}$$

$$\begin{array}{lll}
13) \text{ (I)} 2x + y - 3z = 0 & \text{(I)} 4(2x + y - 3z) = (0)4 & \text{(II)} 4(x - 4y + z) = (0)4 \\
\text{(II)} x - 4y + z = 0 & \text{(II)} x - 4y + z = 0 & \text{(III)} 4x + 16y + 4z = 0 \\
\text{(III)} 4x + 16y + 4z = 0 & \begin{array}{r} -8x + 4y - 12z = 0 \\ \underline{-8x + 4y - 12z = 0} \end{array} & \begin{array}{r} \underline{4x - 16y + 4z = 0} \\ B: 8x + 8z = 2 \end{array} \\
A: 8(9x - 11z) = 0(8) & A: 9x - 11z = 0 & \\
B: -9(8x + 8z) = 2(-9) & A: 9x - 11(0) = 0 & \text{(I)} 2(0) + y - 3(0) = 0 \\
\begin{array}{r} 72x - 88z = 0 \\ \underline{-72x - 72z = 0} \\ \frac{-160z}{-160} = 0 \\ z = 0 \end{array} & \begin{array}{r} \frac{9x}{9} = \frac{0}{9} \\ x = 0 \end{array} & y = 0 \\
& (0, 0, 0) &
\end{array}$$

$$\begin{array}{lll}
15) \text{ (I)} 3x + 2y + 2z = 3 & \text{(II)} x + 2y - z = 5 & \text{(I)} 3x + 2y + 2z = 3 \\
\text{(II)} x + 2y - z = 5 & \text{(III)} 2x - 4y + z = 0 & \text{(II)} 2(x + 2y - z) = (5)2 \\
\text{(III)} 2x - 4y + z = 0 & A: 3x - 2y = 5 & \begin{array}{r} 3x + 2y + 2z = 3 \\ \underline{2x + 4y - 2z = 10} \\ B: 5x + 6x = 13 \end{array} \\
A: 3(3x - 2y) = (5)5 & & \\
B: 5x + 6y = 13 & & \\
\begin{array}{r} \underline{9x - 6y = 15} \\ \frac{14x}{14} = \frac{28}{14} \\ x = 2 \end{array} & A: 3(2) - 2y = 5 & \text{(I)} 3(2) + 2\left(\frac{1}{2}\right) + 2z = 3 \\
& \begin{array}{r} 6 - 2y = 5 \\ \underline{-6 \quad -6} \\ -\frac{2y}{-2} = -\frac{1}{-2} \\ y = \frac{1}{2} \end{array} & \begin{array}{r} 6 + 1 + 2z = 3 \\ 7 + 2z = 3 \\ \underline{-7 \quad -7} \\ \frac{2z}{2} = -\frac{4}{2} \\ z = -2 \end{array} \\
(2, \frac{1}{2}, -2) & &
\end{array}$$

$$\begin{aligned}
 17) \quad (I) \quad & x - 2y + 3z = 4 \\
 & (II) \quad 2x - y + z = -1 \\
 & (III) \quad 4x + y + z = 1
 \end{aligned}$$

$$\begin{aligned}
 A: \quad & 3(6x + 2y) = (0)3 \\
 B: \quad & -2(9x + 5z) = 6(-2)
 \end{aligned}$$

$$\begin{aligned}
 & 18x + 6z = 0 \\
 & \underline{-18x - 10z = -12} \\
 & \quad -\frac{4z}{-4} = -\frac{12}{-4} \\
 & \quad z = 3
 \end{aligned}$$

$$(-1, 2, 3)$$

$$\begin{aligned}
 (II) \quad & 2x - y + z = -1 \\
 (III) \quad & \underline{4x + y + z = 1} \\
 A: \quad & 6x + 2z = 0
 \end{aligned}$$

$$\begin{aligned}
 A: \quad & 6x + 2(3) = 0 \\
 & 6x + 6 = 0 \\
 & \underline{-6 \quad -6} \\
 & \quad \frac{6x}{6} = \frac{-6}{6} \\
 & \quad x = -1
 \end{aligned}$$

$$\begin{aligned}
 (I) \quad & x - 2y + 3z = 4 \\
 (III) \quad & 2(4x + y + z) = (1)2 \\
 & \quad x - 2y + 3z = 4 \\
 & \underline{8x + 2y + 2z = 2} \\
 B: \quad & 9x + 5z = 6
 \end{aligned}$$

$$\begin{aligned}
 (I) \quad & (-1) - 2y + 3(3) = 4 \\
 & \quad 8 - 2y = 4 \\
 & \quad \underline{-8 \quad -8} \\
 & \quad \quad \frac{-2y}{-2} = \frac{-4}{-2} \\
 & \quad \quad y = 2
 \end{aligned}$$

$$\begin{aligned}
 19) \quad (I) \quad & x - y + 2z = 0 \\
 & (II) \quad x - 2y + 3z = -1 \\
 & (III) \quad 2x - 2y + z = -3
 \end{aligned}$$

$$\begin{aligned}
 (I) \quad & x - (2) + 2(1) = 0 \\
 & \quad x - 2 + 2 = 0 \\
 & \quad x = 0
 \end{aligned}$$

$$\begin{aligned}
 (I) \quad & (-1)(x - y + 2z) = 0(-1) \\
 (II) \quad & x - 2y + 3z = -1 \\
 & \underline{-x + y - 2z = 0}
 \end{aligned}$$

$$\begin{aligned}
 A: \quad & -y + z = -1 \\
 & \quad -y + (1) = -1 \\
 & \quad \underline{-1 \quad -1} \\
 & \quad \quad -\frac{y}{-1} = \frac{-2}{-1} \\
 & \quad \quad y = 2
 \end{aligned}$$

$$\begin{aligned}
 (I) \quad & (-2)(x - y + 2z) = 0(-2) \\
 (III) \quad & 2x - 2y + z = -3 \\
 & \underline{-2x + 2y - 4z = 0}
 \end{aligned}$$

$$\begin{aligned}
 & \quad \quad \underline{-\frac{3z}{-3} = -\frac{3}{-3}} \\
 & \quad \quad z = 1
 \end{aligned}$$

$$(0, 2, 1)$$

$$\begin{aligned}
 21) \quad (I) \quad & 4x - 3y + 2z = 40 \\
 & (II) \quad 5x + 9y - 7z = 47 \\
 & (III) \quad 9x + 8y - 3z = 97
 \end{aligned}$$

$$\begin{aligned}
 A: \quad & 7(17x - z) = (167)7 \\
 B: \quad & 59x + 7z = 611 \\
 & \underline{119x - 7z = 1169} \\
 & \quad \frac{178x}{178} = \frac{1780}{178} \\
 & \quad x = 10
 \end{aligned}$$

$$(10, 2, 3)$$

$$\begin{aligned}
 (I) \quad & 3(4x - 3y + 2z) = (40)3 \\
 (II) \quad & 5x + 9y - 7z = 47 \\
 & \underline{12x - 9y + 6z = 120} \\
 A: \quad & 17x - z = 167
 \end{aligned}$$

$$\begin{aligned}
 A: \quad & 17(10) - z = 167 \\
 & 170 - z = 167 \\
 & \underline{-170 \quad -170} \\
 & \quad \quad -\frac{z}{-1} = \frac{-3}{-1} \\
 & \quad \quad z = 3
 \end{aligned}$$

$$\begin{aligned}
 (I) \quad & 8(4x - 3y + 2z) = (40)8 \\
 (III) \quad & 3(9x + 8y - 3z) = (97)3 \\
 & \quad 32x - 24y + 16z = 320 \\
 & \underline{27x + 24y - 9z = 291}
 \end{aligned}$$

$$\begin{aligned}
 B: \quad & 59x + 7z = 611 \\
 (I) \quad & 4(10) - 3y + 2(3) = 40 \\
 & \quad 46 - 3y = 40 \\
 & \quad \underline{-46 \quad -46} \\
 & \quad \quad \frac{-3y}{-3} = \frac{-6}{-3} \\
 & \quad \quad y = 2
 \end{aligned}$$

$$\begin{array}{l}
23) \text{ (I)} 3x + 3y - 2z = 13 \\
\text{ (II)} 6x + 2y - 5z = 13 \\
\text{ (III)} 5x - 2y - 5z = -1 \\
\hline
A: 19(11x - 10z) = (12)19 \\
B: (-10)(21x - 19z) = 23(-10) \\
\begin{array}{r}
209x - 190z = 228 \\
-210 + 190z = -230 \\
\hline
-\frac{x}{-1} = -\frac{2}{-1} \\
x = 2
\end{array} \\
(2, 3, 1)
\end{array}$$

$$\begin{array}{l}
\text{ (II)} 6x + 2y - 5z = 13 \\
\text{ (III)} 5x - 2y - 5z = -1 \\
\hline
A: 11x - 10z = 12 \\
A: 11(2) - 10z = 12 \\
22 - 10z = 12 \\
-22 \quad -22 \\
\hline
-\frac{10z}{-10} = -\frac{10}{-10} \\
z = 1
\end{array}$$

$$\begin{array}{l}
\text{ (I)} 2(3x + 3y - 2z) = (13)2 \\
\text{ (III)} 3(5x - 2y - 5z) = (-1)3 \\
\begin{array}{r}
6x + 6y - 4z = 26 \\
15x - 6y - 15z = -3 \\
\hline
B: 21x - 19z = 23 \\
\text{ (I)} 3(2) + 3y - 2(1) = 13 \\
4 + 3y = 13 \\
-4 \quad -4 \\
\hline
\frac{3y}{3} = \frac{9}{3} \\
y = 3
\end{array}
\end{array}$$

$$\begin{array}{l}
25) \text{ (I)} 3x - 4y + 2z = 1 \\
\text{ (II)} 2x + 3y - 3z = -1 \\
\text{ (III)} x + 10y - 8z = 7 \\
\hline
A: -34y + 26z = -20 \\
B: -2(-17y + 13z) = -15(-2) \\
\begin{array}{r}
-34y + 26z = -20 \\
34y - 26z = 30 \\
\hline
0 = 10
\end{array}
\end{array}$$

$$\begin{array}{l}
\text{ (I)} 3x - 4y + 2z = 1 \\
\text{ (III)} (-3)(x + 10y - 8z) = 7(-3) \\
\begin{array}{r}
3x - 4y + 2z = 1 \\
-3x - 30y + 24z = -21 \\
\hline
A: -34y + 26z = -20
\end{array}
\end{array}$$

$$\begin{array}{l}
\text{ (II)} 2x + 3y - 3z = -1 \\
\text{ (III)} (-2)(x + 10y - 8z) = 7(-2) \\
\begin{array}{r}
2x + 3y - 3z = -1 \\
-2x - 20y + 16z = -14 \\
\hline
B: -17y + 13z = -15
\end{array}
\end{array}$$

$$\begin{array}{l}
27) \text{ (I)} m + 6n + 3p = 8 \\
\text{ (II)} 3m + 4n = -3 \\
\text{ (III)} 5m + 7n = 1 \\
\hline
\text{ (I)} (-25) + 6(18) + 3p = 8 \\
-25 + 108 + 3p = 8 \\
83 + 3p = 8 \\
-83 \quad -83 \\
\hline
\frac{3p}{3} = -\frac{75}{3} \\
p = -25
\end{array}$$

$$\begin{array}{l}
\text{ (II)} (-5)(3m + 4n) = (-3)(-5) \\
\text{ (III)} 3(5m + 7n) = (1)3 \\
\begin{array}{r}
-15m - 20n = 15 \\
15m + 21n = 3 \\
\hline
n = 18
\end{array} \\
(-25, 18, -25)
\end{array}$$

$$\begin{array}{l}
\text{ (II)} 3m + 4(18) = -3 \\
3m + 72 = -3 \\
-72 \quad -72 \\
\hline
\frac{3m}{3} = -\frac{75}{3} \\
m = -25
\end{array}$$

$$\begin{aligned}
 29) \text{ (I)} \quad & -2w + 2x + 2y - 2z = -10 \\
 \text{ (II)} \quad & w + x + y + z = -5 \\
 \text{ (III)} \quad & 3w + 2x + 2y + 4z = -11 \\
 \text{ (IV)} \quad & w + 3x - 2y + 2z = -6
 \end{aligned}$$

$$\begin{aligned}
 \text{ (I)} \quad & -2w + 2x + 2y - 2z = -10 \\
 \text{ (II)} \quad & (-2)(w + x + y + z) = (-5)(-2) \\
 & -2w + 2x + 2y - 2z = -10 \\
 & \underline{-2w - 2x - 2y - 2z = 10} \\
 \text{ A:} \quad & -4w - 4z = 0
 \end{aligned}$$

$$\begin{aligned}
 \text{ A:} \quad & 3(-4w - 4z) = 0(3) \\
 \text{ B:} \quad & 2(5w + 6z) = (-1)2 \\
 & -12w - 12z = 0 \\
 & \underline{10w + 12z = -2} \\
 & -\frac{2w}{-2} = -\frac{2}{-2} \\
 & w = 1
 \end{aligned}$$

$$\begin{aligned}
 \text{ (III)} \quad & 3(1) + 2x + 2y + 4(-1) = -11 \\
 & 3 + 2x + 2y - 4 = -11 \\
 & 2x + 2y - 1 = -11 \\
 & \quad \quad \quad \underline{\quad \quad +1 \quad +1} \\
 \text{ C:} \quad & 2x + 2y = -10
 \end{aligned}$$

$$\begin{aligned}
 \text{ C:} \quad & 2x + 2y = -10 \\
 \text{ D:} \quad & 3x - 2y = -5 \\
 & \frac{5x}{5} = -\frac{15}{5} \\
 & x = -3
 \end{aligned}$$

$$(1, -3, -2, -1)$$

$$\begin{aligned}
 \text{ (I)} \quad & (-1)(-2w + 2x + 2y - 2z) = (-10)(-1) \\
 \text{ (III)} \quad & 3w + 2x + 2y + 4z = -11 \\
 & \underline{2w - 2x - 2y + 2z = 10} \\
 \text{ B:} \quad & 5w + 6z = -1
 \end{aligned}$$

$$\begin{aligned}
 \text{ A:} \quad & -4(1) - 4z = 0 \\
 & -4 - 4z = 0 \\
 & \underline{\quad +4 \quad \quad +4} \\
 & -\frac{4z}{-4} = \frac{4}{-4} \\
 & z = -1
 \end{aligned}$$

$$\begin{aligned}
 \text{ (IV)} \quad & (1) + 3x - 2y + 2(-1) = -6 \\
 & 1 + 3x - 2y - 2 = -6 \\
 & 3x - 2y - 1 = -6 \\
 & \quad \quad \quad \underline{\quad \quad +1 \quad +1} \\
 \text{ D:} \quad & 3x - 2y = -5
 \end{aligned}$$

$$\begin{aligned}
 \text{ C:} \quad & 2(-3) + 2y = -10 \\
 & -6 + 2y = -10 \\
 & \underline{\quad +6 \quad \quad +6} \\
 & \frac{2y}{2} = -\frac{4}{2} \\
 & y = -2
 \end{aligned}$$

31) (I) $w + x + y + z = 2$

(II) $w + 2x + 2y + 4z = 1$

(III) $-w + x - y - z = -2$

(IV) $-w + 3x + y - z = -2$

(I) $w + x + y + z = 2$

(III) $-w + x - y - z = -6$

$$\begin{array}{r} \frac{2x}{2} = -\frac{4}{2} \\ x = -2 \end{array}$$

(II) $w + 2x + 2y + 4z = 1$

(IV) $-2 + 3x + y - z = -2$

$5x + 3y + 3z = -1$

$5(-2) + 3(4) + 3z = -1$

$-10 + 12 + 3z = -1$

$2 + 3z = -1$

$\frac{-2 \quad -2}{3z} = \frac{-3}{3}$

$z = -1$

(III)(-1)(-w + x - y - z) = (-6)(-1)

(IV) $-w + 3x + y - z = -2$

$w - x + y + z = 6$

$2x + 2y = 4$

$2(-2) + 2y = 4$

$-4 + 2y = 4$

$+4 \quad +4$

$\frac{2y}{2} = \frac{8}{2}$

$y = 4$

(I) $w + (-2) + (4) + (-1) = 2$

$w + 1 = 2$

$\frac{-1 \quad -1}{w} = 1$

$w = 1$

$(1, -2, 4, -1)$

4.5

- 1) A collection of dimes and quarters is worth \$15.25. There are 103 coins in all. How many of each is there?

N	V	T
D	10	10D
Q	25	25Q
103		1525

$(-10)(D + Q) = (103)(-10)$

$10D + 25Q = 1525$

$\frac{-10D - 10Q}{15Q} = \frac{-1030}{15}$

$\frac{15Q}{15} = \frac{495}{15}$

$Q = 33$

$D + 33 = 103$

$\frac{-33 \quad -33}{D} = 70$

$D = 70$

70 dimes

33 Quarters

- 3) The attendance at a school concert was 578. Admission was \$2.00 for adults and \$1.50 for children. The total receipts were \$985.00. How many adults and how many children attended?

N	V	T
A	2	2A
C	1.5	1.5C
578		985

$-2(A + C) = (578)(-2)$

$2A + 1.5C = 985$

$\frac{-2A - 2C}{-0.5C} = \frac{-1156}{-0.5}$

$\frac{-0.5C}{-0.5} = \frac{-1156}{-0.5}$

$C = 342$

236 Adults

342 Children

$A + 342 = 578$

$\frac{-342 \quad -342}{A} = 236$

$A = 236$

- 5) A boy has \$2.25 in nickels and dimes. If there are twice as many dimes as nickels, how many of each kind has he?

N	V	T
N	5	5N
D=2N	10	20N
		225

$$5N + 20N = 225$$

$$\frac{25N}{25} = \frac{225}{25}$$

$$N = 9$$

$$D = 2(9) = 18$$

9 Nickels

18 Dimes

- 7) A collection of 27 coins consisting of nickels and dimes amounts to \$2.25. How many coins of each kind are there?

N	V	T
N	5	5N
D	10	10D
27		225

$$(-10)(N + D) = (27)(-10)$$

$$5N + 10D = 225$$

$$\frac{-10N - 10D}{-5} = \frac{-270}{-5}$$

$$\frac{-5N}{-5} = \frac{-45}{-5}$$

$$N = 9$$

$$9 + D = 27$$

$$\frac{-9 \quad -9}{\quad \quad \quad}$$

$$D = 18$$

18 Dimes

9 Nickels

- 9) There were 429 people at a play. Admission was \$1 each for adults and 75 cents each for children. The receipts were \$372.50. How many children and how many adults attended?

N	V	T
A	1	A
C	.75	.75C
429		372.50

$$(-1)(A + C) = (429)(-1)$$

$$A + .75C = 372.5$$

$$\frac{-A - C}{-.25} = \frac{-429}{-.25}$$

$$\frac{-25C}{-.25} = \frac{-56.5}{-.25}$$

$$C = 226$$

$$A + 226 = 429$$

$$\frac{-226 \quad -226}{\quad \quad \quad}$$

$$A = 203$$

203 Adults

226 Children

- 11) There were 203 tickets sold for a volleyball game. For activity-card holders, the price was \$1.25 each and for non-card holders the price was \$2 each. The total amount of money collected was \$310. How many of each type of ticket was sold?

N	V	T
A	1.25	1.25A
N	2	2N
203		310

$$-2(A + N) = (203)(-2)$$

$$1.25A + 2N = 310$$

$$\frac{-2A - 2N}{-.75} = \frac{-406}{-.75}$$

$$\frac{-.75A}{-.75} = \frac{-96}{-.75}$$

$$A = 128$$

$$128 + N = 203$$

$$\frac{-128 \quad -128}{\quad \quad \quad}$$

$$N = 75$$

75 Non Card

128 Activity Card

- 13) At a recent Vikings game \$445 in admission tickets was taken in. The cost of a student ticket was \$1.50 and the cost of a non-student ticket was \$2.50. A total of 232 tickets were sold. How many students and how many nonstudents attended the game?

N	V	T	$-1.5(5 + N) = (232)(-1.5)$
5	1.5	1.55	$1.55 + 2.5N = 445$
N	2.5	2.5N	$-1.55 - 1.5N = 348$
232		445	$N = 97$

$$S + 97 = 232$$

$$\begin{array}{r} -97 \quad -97 \\ \hline S = 135 \end{array}$$

97 Non – Students
135 Students

- 15) A coin purse contains 18 coins in nickels and dimes. The coins have a total value of \$1.15. Find the number of nickels and dimes in the coin purse.

N	V	T	$-5(N + D) = (18)(-5)$
N	5	5N	$5N + 10D = 115$
D	10	100	$-5N - 5D = -90$
18		115	$\frac{5D}{5} = \frac{25}{5}$

$$N + 5 = 18$$

$$\begin{array}{r} -5 \quad -5 \\ \hline N = 13 \end{array}$$

$$D = 5$$

13 Nickels

5 Dimes

- 17)) A postal clerk sold some 15¢ stamps and some 25¢ stamps. Altogether, 15 stamps were sold for a total cost of \$3.15. How many of each type of stamps were sold?

N	V	T	$-15(F + T) = (15)(-15)$	$F + 9 = 15$
F	15	15F	$15F + 25T = 315$	$\frac{-9 \quad -9}{}$
T	25	25T	$-15F - 15T = -225$	$F = 6$
15		315	$\frac{10T}{10} = \frac{90}{10}$	

$$T = 9$$

6 Fifteen cents, 9 twenty – five cents

- 19) The total value of dimes and quarters in a bank is \$6.05. There are six more quarters than dimes. Find the number of each type of coin in the bank.

N	V	T	$10D + 25D + 150D = 605$	$Q = 13 + 6$
D	10	10D	$35D + 150 = 605$	$Q = 19$
Q=D+6	25	25D+150	$-150 \quad -150$	
		605	$\frac{35D}{35} = \frac{455}{35}$	13 Dimes

$$D = 13$$

19 Quarters

- 21) A coin bank contains nickels and dimes. The number of dimes is 10 less than twice the number of nickels. The total value of all the coins is \$2.75. Find the number of each type of coin in the bank.

N	V	T
N	5	5N
D=2N-10	10	20N-100
		275

$$5N + 20N - 100 = 275$$

$$25N - 100 = 275$$

$$\begin{array}{r} +100 + 100 \\ \hline \end{array}$$

$$\frac{25N}{25} = \frac{375}{25}$$

$$N = 15$$

$$D = 2(15) - 10$$

$$D = 30 - 20$$

$$D = 10$$

20 Dimes

15 Nickels

- 23) A bank teller cashed a check for \$200 using twenty dollar bills and ten dollar bills. In all, twelve bills were handed to the customer. Find the number of twenty dollar bills and the number of ten dollar bills.

N	V	T
W	20	20W
T	10	10T
12		200

$$-10(W + T) = (12)(-10)$$

$$20W + 10T = 200$$

$$\begin{array}{r} -10W - 10T = -120 \\ \hline \end{array}$$

$$\frac{10W}{10} = \frac{80}{10}$$

$$W = 8$$

$$8 + T = 12$$

$$\begin{array}{r} -8 \quad -8 \\ \hline \end{array}$$

$$T = 4$$

4 Tens

8 Twenties

- 25) A total of \$27000 is invested, part of it at 12% and the rest at 13%. The total interest after one year is \$3385. How much was invested at each rate?

N	V	T
x	.12	.12x
y	.13	.13y
27000		3385

$$-.12(x + y) = (27000)(-.12)$$

$$.12x + .13y = 3385$$

$$\begin{array}{r} -.12x - .12y = -3240 \\ \hline \end{array}$$

$$\frac{-.01y}{-.01} = \frac{-145}{-.01}$$

$$y = 14500$$

$$x + 14500 = 27000$$

$$\begin{array}{r} -14500 - 14500 \\ \hline \end{array}$$

$$x = 12500$$

\$12,500 @12%

\$14,500 @ 13%

- 27) A total of \$9000 is invested, part of it at 10% and the rest at 12%. The total interest after one year is \$1030. How much was invested at each rate?

N	V	T
x	.10	.1x
y	.12	.12y
9000		1030

$$-.1(x + y) = (9000)(-.1)$$

$$.1 + .12y = 1030$$

$$\begin{array}{r} -.1x - .1y = -900 \\ \hline \end{array}$$

$$\frac{.02y}{.02} = \frac{130}{.02}$$

$$y = 6500$$

$$x + 6500 = 9000$$

$$\begin{array}{r} =6500 - 6500 \\ \hline \end{array}$$

$$x = 2500$$

\$2500 @ 10%

\$6500 @ 12%

29) An inheritance of \$10000 is invested in 2 ways, part at 9.5% and the remainder at 11%. The combined annual interest was \$1038.50. How much was invested at each rate?

N	V	T
x	.095	.095x
y	.11	.11y
10000		1038.50

$$\begin{aligned}
 -.095(x + y) &= (10000)(-.095) & x + 5900 &= 10000 \\
 .095x + .11y &= 1038.50 & \underline{-5900} & \underline{-5900} \\
 \hline
 -.095x - .095y &= -950 & & x = 4100 \\
 \frac{0.015y}{.015} &= \frac{88.5}{.015} & & \\
 y &= 5900 & \$4100 @ 9.5\% & \\
 & & \$5900 @ 11\% &
 \end{aligned}$$

31) Jason earned \$256 interest last year on his investments. If \$1600 was invested at a certain rate of return and \$2400 was invested in a fund with a rate that was double the rate of the first fund, find the two rates of interest.

N	V	T
1600	x	1600x
2400	2x	4800x
		256

$$\begin{aligned}
 1600x + 4800x &= 256 \\
 \frac{6400x}{6400} &= \frac{256}{6400} \\
 x &= 0.04 & \$1600 @ 4\% \\
 2x &= 0.08 & \$2400 @ 8\%
 \end{aligned}$$

33) A total of \$8500 is invested, part of it at 6% and the rest at 3.5%. The total interest after one year is \$385. How much was invested at each rate?

N	V	T
x	.06	.06x
y	.035	.035y
8500		385

$$\begin{aligned}
 -.035(x + y) &= (8500)(-.035) & 3500 + y &= 8500 \\
 .06x + .035y &= 385 & \underline{-3500} & \underline{-3500} \\
 \hline
 -.035x - .035y &= -297.5 & & y = 5000 \\
 \frac{.025x}{.025} &= \frac{87.5}{.025} & & \\
 x &= 3500 & \$3500 @ 6\% & \\
 & & \$5000 @ 3.5\% &
 \end{aligned}$$

35) A total of \$15000 is invested, part of it at 8% and the rest at 11%. The total interest after one year is \$1455. How much was invested at each rate?

N	V	T
x	.08	.08x
y	.11	.11y
15000		1455

$$\begin{aligned}
 -.08(x + y) &= (15000)(-.08) & x + 8500 &= 15000 \\
 .08x + .11y &= 1455 & \underline{-8500} & \underline{-8500} \\
 \hline
 -.08x - .08y &= -1200 & & x = 6500 \\
 \frac{.03y}{.03} &= \frac{255}{.03} & & \\
 y &= 8500 & \$6500 @ 8\% & \\
 & & \$8500 @ 11\% &
 \end{aligned}$$

37) A total of \$6000 is invested, part of it at 4.25% and the rest at 5.75%. The total interest after one year is \$300. How much was invested at each rate?

N	V	T
x	.0425	.0425x
y	.0575	.0575y
6000		300

$$\begin{aligned}
 -.0425(x + y) &= (6000)(-.0425) & x + 3000 &= 6000 \\
 .0425x + .0575y &= 300 & \underline{-3000} & \underline{-3000} \\
 \hline
 -.0425x - .0425y &= -255 & & x = 3000 \\
 \frac{.015y}{.015} &= \frac{45}{.015} & & \\
 y &= 3000 & & \$3000 @ 4.25\% \\
 & & & \$3000 @ 5.75\%
 \end{aligned}$$

39) A total of \$11000 is invested, part of it at 6.8% and the rest at 8.2%. The total interest after one year is \$797. How much was invested at each rate?

N	V	T
x	.068	.068x
y	.082	.082y
11000		797

$$\begin{aligned}
 -.068(x + y) &= (11000)(-.068) & x + 3500 &= 11000 \\
 .068x + .082y &= 797 & \underline{-3500} & \underline{-3500} \\
 \hline
 -.068x - .068y &= -748 & & x = 7500 \\
 \frac{.014y}{.014} &= \frac{49}{.014} & & \\
 y &= 3500 & & \$7500 @ 6.8\% \\
 & & & \$3500 @ 8.2\%
 \end{aligned}$$

42) Samantha earned \$1480 in interest last year on her investments. If \$5000 was invested at a certain rate of return and \$11000 was invested in a fund with a rate that was two-thirds the rate of the first fund, find the two rates of interest.

N	V	T
5000	x	5000x
11000	$\frac{2}{3}x$	$\frac{22000}{3}x$
		1480

$$\begin{aligned}
 3\left(5000x + \frac{22000}{3}x\right) &= (1480)3 \\
 15000x + 22000x &= 4440 & & \$5000 @ 12\% \\
 \frac{37000x}{37000} &= \frac{4440}{37000} & & \$11000 @ 8\% \\
 x &= .12 \\
 \frac{2}{3}(.12) &= .08
 \end{aligned}$$

44) 30 coins having a value of \$3.30 consists of nickels, dimes and quarters. If there are twice as many quarters as dimes, how many coins of each kind were there?

N	V	T
N	5	5N
D	10	10D
Q=2D	25	25D
30		330

$$\begin{aligned}
 N + D + 2D &= 30 & (-5)(N + 3D) &= (30)(-5) \\
 5N + 10D + 50D &= 330 & 5N + 60D &= 330 \\
 & & \underline{-5N - 15D} & \underline{-150} \\
 & & \frac{45D}{45} &= \frac{180}{45} \\
 & & D &= 4 \\
 N + 3(4) &= 30 \\
 N + 12 &= 30 \\
 -12 & -12 \\
 N &= 18 \\
 Q &= 2(4) = 8
 \end{aligned}$$

18 Nickels
4 Dimes
8 Quarters

4.6

- 1) A tank contains 8000 liters of a solution that is 40% acid. How much water should be added to make a solution that is 30% acid?

A	P	T
8000	.4	3200
w	0	0
8000+w	.3	2400+.3w

$$3200 = 2400 + .3w$$

$$\underline{-2400 \quad -2400}$$

$$\frac{800}{.3} = \frac{.3w}{.3}$$

$$w = 2,666.67 L.$$

- 3) Of 12 pounds of salt water 10% is salt; of another mixture 3% is salt. How many pounds of the second should be added to the first in order to get a mixture of 5% salt?

A	P	T
12	.1	1.2
x	.03	.03x
12+x	.05	.6+.05x

$$1.2 + .03x = .6 + .05x$$

$$\underline{-0.03x \quad -0.03x}$$

$$1.2 = .6 + .02x$$

$$\underline{-0.6 \quad -0.6}$$

$$\frac{.6}{.02} = \frac{.02x}{.02}$$

$$x = 30 lbs$$

- 5) How many pounds of a 4% solution of borax must be added to 24 pounds of a 12% solution of borax to obtain a 10% solution of borax?

A	P	T
x	.04	.04x
24	.12	2.88
x+24	.10	.1x+.24

$$.04x + 2.88 = .1x + 2.4$$

$$\underline{-0.04x \quad -0.04x}$$

$$2.88 = .06x + 2.4$$

$$\underline{-2.4 \quad -2.4}$$

$$\frac{.48}{.06} = \frac{.06x}{.06}$$

$$x = 8 lbs$$

- 7) A 100 LB bag of animal feed is 40% oats. How many pounds of oats must be added to this feed to produce a mixture which is 50% oats?

A	P	T
100	.4	40
x	1	X
100+x	.5	50+.5x

$$40 + x = 50 + .5x$$

$$\underline{-0.5x \quad -0.5x}$$

$$40 + .5x = 50$$

$$\underline{-40 \quad -40}$$

$$\frac{.5x}{.5} = \frac{10}{.5}$$

$$x = 20 lbs$$

- 9) How many pounds of tea that cost \$4.20 per pound must be mixed with 12 lb of tea that cost \$2.25 per pound to make a mixture that costs \$3.40 per pound?

A	P	T
x	4.2	4.2x
12	2.25	27
x+12	3.40	3.4x+40.8

$$\begin{aligned}
 4.2x + 27 &= 3.4x + 40.8 \\
 -3.4x &\quad - 3.4x \\
 \hline
 0.8x + 27 &= 40.8 \\
 -27 &\quad - 27 \\
 \hline
 0.8x &= 13.8 \\
 \frac{0.8x}{0.8} &= \frac{13.8}{.8} \\
 x &= 12.25 \text{ lbs}
 \end{aligned}$$

- 11) How many kilograms of hard candy that cost \$7.50 per kilogram must be mixed with 24 kg of jelly beans that cost \$3.25 per kilogram to make a mixture that sells for \$4.50 per kilogram?

A	P	T
x	7.5	7.5x
24	3.25	78
x+24	4.5	4.5x+108

$$\begin{aligned}
 7.5x + 78 &= 4.5x + 108 \\
 -4.5x &\quad - 4.5x \\
 \hline
 3x + 78 &= 108 \\
 -78 &\quad - 78 \\
 \hline
 3x &= 30 \\
 \frac{3x}{3} &= \frac{30}{3} \\
 x &= 10 \text{ kg}
 \end{aligned}$$

- 13) How many pounds of lima beans that cost 90¢ per pound must be mixed with 16 lb of corn that cost 50¢ per pound to make a mixture of vegetables that costs 65¢ per pound?

A	P	T
x	.9	.9x
16	.5	8
x+16	.65	.65x+10.4

$$\begin{aligned}
 .9x + 8 &= .65x + 10.4 \\
 -.65x &\quad -.65x \\
 \hline
 .25x + 8 &= 10.4 \\
 -8 &\quad - 8 \\
 \hline
 .25x &= 2.4 \\
 \frac{.25x}{.25} &= \frac{2.4}{.25} \\
 x &= 9.6 \text{ lbs}
 \end{aligned}$$

- 15) Solution A is 50% acid and solution B is 80% acid. How much of each should be used to make 100cc. of a solution that is 68% acid?

A	P	T
A	.5	.5A
B	.8	.8B
100	.68	68

$$\begin{aligned}
 -.5(A + B) &= (100)(-.5) \\
 .5A + .8B &= 68 \\
 -.5A - .5B &= -50 \\
 \hline
 .3B &= 18 \\
 \frac{.3B}{.3} &= \frac{18}{.3} \\
 B &= 60
 \end{aligned}$$

$$\begin{aligned}
 A + 60 &= 100 \\
 -60 &\quad - 60 \\
 \hline
 A &= 40
 \end{aligned}$$

60 cc of 80%
40 cc of 50%

- 17) A farmer has some cream which is 21% butterfat and some which is 15% butter fat. How many gallons of each must be mixed to produce 60 gallons of cream which is 19% butterfat?

A	P	T
A	.21	.21A
B	.15	.15B
60	.19	11.4

$$-.15(A + B) = (60)(-.15)$$

$$.21A + .15B = 11.4$$

$$-.15A - .15B = -9$$

$$\frac{.06A}{.06} = \frac{2.4}{.06}$$

$$A = 40$$

$$40 + B = 60$$

$$\frac{-40}{-40} \quad \frac{-40}{-40}$$

$$B = 20$$

40 gal 21%

20 gal 15%

- 19) A chemist wants to make 50ml of a 16% acid solution by mixing a 13% acid solution and an 18% acid solution. How many milliliters of each solution should the chemist use?

A	P	T
x	.13	.13x
y	.18	.18y
50	.16	8

$$-.13(x + y) = (50)(-.13)$$

$$.13x + .18y = 8$$

$$-.13x - .13y = -6.5$$

$$\frac{.05y}{.05} = \frac{1.5}{.05}$$

$$y = 30$$

$$x + 30 = 50$$

$$\frac{-30}{-30} \quad \frac{-30}{-30}$$

$$x = 20$$

20 mL 13%

30 mL 18%

- 21) A paint that contains 21% green dye is mixed with a paint that contains 15% green dye. How many gallons of each must be used to make 60 gal of paint that is 19% green dye?

A	P	T
x	.21	.21x
y	.15	.15y
60	.19	11.4

$$-.15(x + y) = (60)(-.15)$$

$$.21x + .15y = 11.4$$

$$-.15x - .15y = -9$$

$$\frac{.06x}{.06} = \frac{2.5}{.06}$$

$$x = 40$$

$$40 + y = 60$$

$$\frac{-40}{-40} \quad \frac{-40}{-40}$$

$$y = 20$$

40 gal 21%

20 gal 15%

- 23) To make a weed and feed mixture, the Green Thumb Garden Shop mixes fertilizer worth \$4.00/lb. with a weed killer worth \$8.00/lb. The mixture will cost \$6.00/lb. How much of each should be used to prepare 500 lb. of the mixture?

A	P	T
x	4	4x
y	8	8y
500	6	3000

$$-4(x + y) = (500)(-4)$$

$$4x + 8y = 3000$$

$$-4x - 4y = -2000$$

$$\frac{4y}{4} = \frac{1000}{4}$$

$$y = 250$$

$$x + 250 = 500$$

$$\frac{-250}{-250} \quad \frac{-250}{-250}$$

$$x = 250$$

250 lbs @ \$4

250 lbs @ \$8

25) A grocer wishes to mix sugar at 9 cents per pound with sugar at 6 cents per pound to make 60 pounds at 7 cents per pound. What quantity of each must he take?

A	P	T
x	9	9x
y	6	6y
60	7	420

$$\begin{array}{r}
 -6(x + y) = (60)(-6) \\
 9x + 6y = 420 \\
 \hline
 -6x - 6y = -360 \\
 \hline
 \frac{3x}{3} = \frac{60}{3} \\
 x = 20
 \end{array}$$

$$\begin{array}{r}
 20 + y = 60 \\
 \hline
 -20 \quad -20 \\
 \hline
 y = 40
 \end{array}$$

20 lbs @ 9¢
40 lbs @ 6¢

27) A goldsmith combined an alloy that costs \$4.30 per ounce with an alloy that costs \$1.80 per ounce. How many ounces of each were used to make a mixture of 200 oz costing \$2.50 per ounce?

A	P	T
x	4.30	4.3x
y	1.80	1.80y
200	2.50	500

$$\begin{array}{r}
 -1.8(x + y) = (200)(-1.8) \\
 4.3x + 1.8y = 500 \\
 \hline
 -1.8x - 1.8y = -360 \\
 \hline
 \frac{2.5x}{2.5} = \frac{140}{2.5} \\
 x = 56
 \end{array}$$

$$\begin{array}{r}
 56 + y = 200 \\
 \hline
 -56 \quad -56 \\
 \hline
 y = 144
 \end{array}$$

56 oz. @ \$4.30
144 oz. @ \$1.80

29) The manager of a garden shop mixes grass seed that is 60% rye grass with 70 lb of grass seed that is 80% rye grass to make a mixture that is 74% rye grass. How much of the 60% mixture is used?

A	P	T
x	.6	.6x
70	.8	56
x+70	.74	.74x+51.8

$$\begin{array}{r}
 .6x + 56 = .74x + 51.8 \\
 \hline
 -.6x \quad - .6x \\
 \hline
 56 = .45x + 51.8 \\
 \hline
 -51.8 \quad - 51.8 \\
 \hline
 \frac{4.2}{.14} = \frac{14x}{.14} \\
 30 \text{ lbs} = x
 \end{array}$$

31) A caterer made an ice cream punch by combining fruit juice that cost \$2.25 per gallon with ice cream that costs \$3.25 per gallon. How many gallons of each were used to make 100 gal of punch costing \$2.50 per pound?

A	P	T
x	2.25	2.25x
y	3.25	3.25y
100	2.5	250

$$\begin{array}{r}
 -2.25(x + y) = (100)(-2.25) \\
 2.25x + 3.25y = 250 \\
 \hline
 -2.25x - 2.25y = -225 \\
 \hline
 y = 25
 \end{array}$$

$$\begin{array}{r}
 x + 25 = 100 \\
 \hline
 -25 \quad -25 \\
 \hline
 x = 75
 \end{array}$$

75 gal @ \$2.25
25 gal @ \$3.25

33) A carpet manufacturer blends two fibers, one 20% wool and the second 50% wool. How many pounds of each fiber should be woven together to produce 600 lb of a fabric that is 28% wool?

A	P	T
x	.2	.2x
y	.5	.5y
600	.28	168

$$\begin{aligned}
 -2(x + y) &= (600)(-.2) & x + 160 &= 600 \\
 .2x + .5y &= 168 & \underline{-160 - 160} & \\
 -2x - .2y &= -120 & x &= 440 \\
 \frac{.3y}{.3} &= \frac{48}{.3} & & \\
 y &= 160 & &
 \end{aligned}$$

440 lbs @ 20%
160 lbs @ 50%

35) The manager of a specialty food store combined almonds that cost \$4.50 per pound with walnuts that cost \$2.50 per pound. How many pounds of each were used to make a 100 lb mixture that cost \$3.24 per pound?

A	P	T
x	4.50	4.5x
y	2.50	2.5y
100	3.24	324

$$\begin{aligned}
 -2.5(x + y) &= (100)(-2.5) & 37 + y &= 100 \\
 4.5x + 2.5y &= 324 & \underline{-37 - 37} & \\
 -2.5x - 2.5y &= -250 & y &= 63 \\
 \frac{2x}{2} &= \frac{74}{2} & & \\
 x &= 37 & &
 \end{aligned}$$

37 lbs @ \$4.50
63 lbs @ \$2.50

37) How many ounces of dried apricots must be added to 18 oz of a snack mix that contains 20% dried apricots to make a mixture that is 25% dried apricots?

A	P	T
x	1	x
18	.2	3.6
x+18	.25	.25x+4.5

$$\begin{aligned}
 x + 3.6 &= .25x + 4.5 \\
 \underline{-.25x - .25x} & \\
 .75x + 3.6 &= 4.5 \\
 \underline{-3.6 - 3.6} & \\
 \frac{.75x}{.75} &= \frac{0.9}{.75} \\
 x &= 1.2 \text{ oz}
 \end{aligned}$$

39) How many ounces of pure bran flakes must be added to 50 oz. of cereal that is 40% bran flakes to produce a mixture that is 50% bran flakes?

A	P	T
x	1	x
50	.4	20
x+50	.5	.5x+25

$$\begin{aligned}
 x + 20 &= .5x + 25 \\
 \underline{-.5x - .5x} & \\
 .5x + 20 &= 25 \\
 \underline{-20 - 20} & \\
 \frac{.5x}{.5} &= \frac{5}{.5} \\
 x &= 10 \text{ oz}
 \end{aligned}$$

41) How many grams of pure water must be added to 50 g of pure acid to make a solution that is 40% acid?

A	P	T
w	0	0
50	1	50
w+50	.4	.4w+20

$$50 = .4w + 20$$

$$\begin{array}{r} -20 \quad \quad -20 \\ \hline \end{array}$$

$$\frac{30}{.4} = \frac{.4w}{.4}$$

$$75g = w$$

43) How many ounces of pure water must be added to 50 oz of a 15% saline solution to make a saline solution that is 10% salt?

A	P	T
x	0	0
50	.15	7.5
x+50	.10	.1x+5

$$7.5 = .1x + 5$$

$$\begin{array}{r} -5 \quad \quad -5 \\ \hline \end{array}$$

$$\frac{2.5}{.1} = \frac{.1x}{.1}$$

$$25oz = x$$

Chapter 5: Polynomials

5.1

$$1) 4 \cdot 4^4 \cdot 4^4 = 4^9$$

$$3) 4 \cdot 2^2 = 2^2 \cdot 2^2 = 2^4$$

$$5) 3m \cdot 4mn = 12m^2n$$

$$7) 2m^4n^2 \cdot 4nm^2 = 8m^6n^3$$

$$9) (3^3)^4 = 3^{12}$$

$$11) (4^4)^2 = 4^8$$

$$13) (2u^3v^2)^2 = 4u^6v^4$$

$$15) (2a^4)^4 = 2^4a^{16} = 16a^{16}$$

$$17) \frac{4^5}{4^3} = 4^2$$

$$19) \frac{3^2}{3} = 3$$

$$21) \frac{3nm^2}{3n} = m^2$$

$$23) \frac{4x^3y^4}{3xy^3} = \frac{4x^2y}{3}$$

$$25) (x^3y^4 \cdot 2x^2y^3)^2 \\ (2x^5y^7)^2 \\ 2^2x^{10}y^{14} \\ 4x^{10}y^{14}$$

$$27) 2x(x^4y^4)^4 \\ 2x(x^{16}y^{16}) \\ 2x^{17}y^{16}$$

$$29) \frac{2x^7y^5}{3x^3y \cdot 4x^2y^3} = \frac{2x^7y^5}{12x^5y^4} = \frac{x^2y}{6}$$

$$31) \left(\frac{(2x)^3}{x^3}\right)^2 = \left(\frac{2^3x^3}{x^3}\right)^2 = \left(\frac{8x^3}{x^3}\right)^2 = 8^2 = 64$$

$$33) \left(\frac{2y^{17}}{(2x^2y^4)^4}\right)^3 = \left(\frac{2y^{17}}{2^4x^8y^{16}}\right)^3 = \left(\frac{2y^{17}}{16x^8y^{16}}\right)^3 = \\ \left(\frac{y}{8x^8}\right)^3 = \frac{y^3}{8^3x^{24}} = \frac{y^3}{512x^{24}}$$

$$35) \left(\frac{2mn^4 \cdot 2m^4n^4}{mn^4}\right)^3 = \left(\frac{4m^5n^8}{mn^4}\right)^3 = (4m^4n^4)^3 = \\ 4^3m^{12}n^{12} = 64m^{12}n^{12}$$

$$37) \frac{2xy^5 \cdot 2x^2y^3}{2xy^4 \cdot y^3} = \frac{4x^3y^8}{2xy^7} = 2x^2y$$

$$39) rr \frac{q^3r^2 \cdot (2p^2q^2r^3)^2}{2p^3} = \frac{q^3r^2(2^2p^4q^4r^2)}{2p^3} = \\ \frac{q^3r^2 \cdot 4p^4q^4r^6}{2p^3} = \frac{4p^4q^7r^8}{2p^3} = 2pq^7r^8$$

$$41) \left(\frac{zy^3 \cdot z^3x^4y^4}{x^3y^3z^3}\right)^4 = \left(\frac{z^4y^7x^4}{x^3y^3z^3}\right)^4 = (xy^4z)^4 = \\ x^4y^{16}z^4$$

$$43) \frac{2x^2y^2z^6 \cdot 2zx^2y^2}{(x^2z^3)^2} = \frac{4x^4y^4z^7}{x^4z^6} = 4y^4z$$

5.2

$$1) 2x^4y^{-2}(2xy^3)^4 = 2x^4y^{-2}(2^4x^4y^{12}) = \\ 2^5x^8y^{10} = 32x^8y^{10}$$

$$3) (a^4b^{-3})^3 2a^3b^{-2} = a^{12}b^9 \cdot 2a^3b^{-2} = \\ 2a^{15}b^{-11} = \frac{2a^{15}}{b^{11}}$$

$$5) (2x^2y^2)^4x^{-4} = 2^4x^8y^8x^{-4} = 16x^4y^8$$

$$7) (x^3y^4) \cdot 3x^{-4}y^4 = x^9y^{12} \cdot 3x^0y^4 = x^5y^{16}$$

$$9) \frac{2x^{-3}y^2}{3x^{-3}y^3 \cdot 3x^0} = \frac{2y^2x^3}{x^3 \cdot 3y^3 \cdot 3x^0} = \frac{2y^3x^3}{9x^3y^3} = \frac{2}{9y}$$

$$11) \frac{4xy^{-3} \cdot x^{-4}y^0}{4y^{-1}} = \frac{4xy^0y}{4y^3x^4} = \frac{4xy}{4y^3x^4} = \frac{1}{x^3y^2}$$

$$13) \frac{u^2v^{-1}}{2u^0v^4 \cdot 2uv} = \frac{u^2}{v \cdot 2u^0v^4 \cdot 2uv} = \frac{u^2}{4uv^6} = \frac{u}{4v^6}$$

$$15) \frac{u^2}{4u^0v^3 \cdot 3v^2} = \frac{u^2}{12v^5}$$

$$17) \frac{2y}{(x^0y^2)^4} = \frac{2y}{x^0y^8} = \frac{2}{y^7}$$

$$19) \left(\frac{2a^2b^3}{a^{-1}}\right)^4 = (2a^2a b^3)^4 = (2a^3b^3)^4 = 2^4 a^{12} b^{12} = 16a^{12}b^{12}$$

$$21) \frac{2nm^4}{(2m^2n^2)^4} = \frac{2nm^4}{2^4m^8n^8} = \frac{1}{2^3m^4n^7} = \frac{1}{8m^4n^7}$$

$$23) \frac{(2mn)^4}{m^0n^{-2}} = \frac{2^4m^4n^4}{m^0n^{-2}} = 2^4m^4n^4n^2 = 16m^4n^6$$

$$25) \frac{y^3 \cdot x^{-3}y^2}{(x^4y^2)^3} = \frac{y^3x^{-3}y^2}{x^{12}y^6} = \frac{y^3y^2}{x^3x^{12}y^6} = \frac{y^5}{x^{15}y^6} = \frac{1}{x^{15}y}$$

$$27) \frac{2u^{-2}v^3(2uv^4)^{-1}}{2u^{-4}v^0} = \frac{2u^{-2}v^3 \cdot 2^{-1}u^{-1}v^{-4}}{2u^{-4}v^0} = \frac{2v^3 \cdot u^4}{u^2 2uv^4 \cdot 2v^0} = \frac{2v^3u^4}{4u^3v^4} = \frac{u}{2v}$$

$$29) \left(\frac{2x^0y^4}{y^4}\right)^3 = (2)^3 = 8$$

$$31) \frac{y(2x^4y^2)^2}{2x^4y^0} = \frac{y(2^2x^8y^4)}{2x^4y^0} = \frac{4x^8y^5}{2x^4y^0} = 2x^4y^5$$

$$33) \frac{2yzx^2}{2x^4y^4z^{-2}(zy^2)^4} = \frac{2yzx^2}{2x^4y^4z^{-2}z^4y^8} = \frac{2yz^3x^2}{2x^4y^{12}z^4} = \frac{1}{x^2y^{11}z}$$

$$35) \frac{2kh^0 \cdot 2h^{-3}k^0}{(2kj^3)^2} = \frac{2kh^0 \cdot 2h^{-3}k^0}{2^2k^2j^6} = \frac{2k \cdot 2}{h^3 \cdot 4k^2j^6} = \frac{4k}{4k^2h^3j^6} = \frac{1}{kh^3j^6}$$

$$37) \frac{(cb^3)^2 \cdot 2a^{-3}b^2}{(a^3b^{-2}c^3)^3} = \frac{c^2b^6 \cdot 2a^{-3}b^2}{a^9b^{-6}c^9} = \frac{c^2b^6 2b^2b^6}{a^3a^9c^9} = \frac{2b^{14}c^2}{a^{12}c^9} = \frac{2b^{14}}{a^{12}c^7}$$

$$39) \frac{(yx^{-4}z^2)^{-1}}{z^3x^2y^3z^{-1}} = \frac{y^{-1}x^4z^{-2}}{x^3x^2y^3z^{-1}} = \frac{x^4z}{yz^2x^3x^2y^3} = \frac{x^4z}{x^2y^4z^5} = \frac{x^2}{y^4z^4}$$

5.3

$$1) 885$$

$$8.85 \times 10^2$$

$$3) 0.081$$

$$8.1 \times 10^{-2}$$

$$5) 0.039$$

$$3.9 \times 10^{-2}$$

$$7) 8.7 \times 10^5$$

$$870,000$$

$$9) 9 \times 10^{-4}$$

$$0.0009$$

$$11) 2 \times 10^0$$

$$2$$

$$13) (7 \times 10^{-1})(2 \times 10^{-3})$$

$$14 \times 10^{-4}$$

$$1.4 \times 10^1 \times 10^{-4}$$

$$1.4 \times 10^{-3}$$

$$15) (5.26 \times 10^{-5})(3.16 \times 10^{-2})$$

$$16.6216 \times 10^{-7}$$

$$1.66216 \times 10^1 \times 10^{-7}$$

$$1.66216 \times 10^{-6}$$

$$17) (2.6 \times 10^{-2})(6 \times 10^{-2})$$

$$15.6 \times 10^{-4}$$

$$1.56 \times 10^1 \times 10^{-4}$$

$$1.56 \times 10^{-3}$$

$$19) \frac{4.9 \times 10^1}{2.7 \times 10^{-3}} = 1.81 \times 10^4$$

$$21) \frac{5.33 \times 10^{-6}}{9.62 \times 10^{-2}} = 0.554 \times 10^{-4} =$$

$$5.54 \times 10^{-1} \times 10^{-4} = 5.54 \times 10^{-5}$$

$$23) (5.5 \times 10^{-5})^2$$

$$30.25 \times 10^{-10}$$

$$3.025 \times 10^1 \times 10^{-10}$$

$$3.025 \times 10^{-9}$$

$$25) (7.8 \times 10^{-2})^5$$

$$28.872 \times 10^{-10}$$

$$2.8872 \times 10^1 \times 10^{-10}$$

$$2.8872 \times 10^{-9}$$

$$27) (8.03 \times 10^4)^{-4}$$

$$0.000241 \times 10^{-16}$$

$$2.41 \times 10^{-4} \times 10^{-16}$$

$$2.41 \times 10^{-20}$$

$$29) \frac{6.1 \times 10^{-6}}{5.1 \times 10^{-4}} = 1.196 \times 10^{-2}$$

$$31) (3.6 \times 10^0)(6.1 \times 10^{-3})$$

$$21.96 \times 10^{-3}$$

$$2.196 \times 10^1 \times 10^{-3}$$

$$2.196 \times 10^{-2}$$

$$33) (1.8 \times 10^{-5})^{-3}$$

$$0.1715 \times 10^{15}$$

$$1.715 \times 10^{-1} \times 10^{15}$$

$$1.715 \times 10^{14}$$

$$35) \frac{9 \times 10^4}{7.83 \times 10^{-2}} = 1.149 \times 10^6$$

$$37) \frac{3.22 \times 10^{-3}}{7 \times 10^{-6}} = 0.46 \times 10^3 =$$

$$4.6 \times 10^{-1} \times 10^3 = 4.6 \times 10^2$$

$$39) \frac{2.4 \times 10^{6-6}}{6.5 \times 10^0} = 0.3692 \times 10^{-6} =$$

$$3.692 \times 10^{-1} \times 10^{-6} = 3.692 \times 10^{-7}$$

$$41) \frac{6 \times 10^3}{5.8 \times 10^{-3}} = 1.034 \times 10^6$$

5.4

$$1) -a^3a^2 + 6a - 21 \text{ at } a = -4$$

$$-(-4)^3 - (-4)^2 + 6(-4) - 21$$

$$-(-64) - (16) + 6(-4) - 21$$

$$64 - 16 - 24 - 21$$

$$3$$

$$3) n^3 - 7n^2 + 15n - 20 \text{ when } n = 2$$

$$(2)^3 - 7(2)^2 + 15(2) - 20$$

$$8 - 7(4) + 15(2) - 20$$

$$8 - 28 + 30 - 20$$

$$-10$$

$$5) -5n^4 - 11n^3 - 9n^2 - n - 5 \text{ when } n = -1$$

$$-5(-1)^4 - 11(-1)^3 - 9(-1)^2 - (-1) - 5$$

$$-5(1) - 11(-1) - 9(1) - 1(-1) - 5$$

$$-5 + 11 - 9 + 1 - 5$$

$$-7$$

$$7) x^2 + 9x + 23 \text{ when } x = -3$$

$$(-3)^2 + 9(-3) + 23$$

$$9 + 9(-3) + 23$$

$$9 - 27 + 23$$

$$5$$

- 9) $x^4 - 6x^3 + x^2 - 24$ when $x = 6$
 $(6)^4 - 6(6)^3 + (6)^2 - 24$
 $1296 - 6(216 + 36 - 24)$
 $1296 - 1296 + 36 - 24$
 12
- 11) $(5p - 5p^4) - (8p - 8p^4)$
 $5p - 5p^5 - 8p + 8p^4$
 $3p^4 - 3p$
- 13) $(3n^2 + n^3) - (2n^3 - 7n^2)$
 $3n^2 + n^3 - 2n^3 - 7n^2$
 $-n^3 + 10n^2$
- 15) $(8n + n^4) - (3n - 4n^4)$
 $8n + n^4 - 3n + 4n^4$
 $5n^4 + 5n$
- 17) $(1 + 5p^3) - (1 - 8p^3)$
 $1 + 5p^3 - 1 + 8p^3$
 $13p^3$
- 19) $(5n^4 + 6n^3) + (8 - 3n^3 - 5n^4)$
 $3n^3 + 8$
- 21) $(3 + b^4) + (7 + 2b + b^4)$
 $2b^4 + 2b + 10$
- 39) $(8 - b + 7b^3) - (3b^4 + 7b - 8 - 7b^2) + (3 - 3b + 6b^3)$
 $8 - b + 7b^3 - 3b^4 - 7b + 8 + 7b^2 + 3 - 3b + 6b^3$
 $-3b^4 + 13b^3 - 7b^2 - 11b + 19$
- 41) $(8x^4 + 2x^3 + 2x) + (2x + 2 - 2x^3 - x^4) - (x^3 + 5x^4 + 8x)$
 $8x^4 + 2x^3 + 2x + 2x + 2 - 2x^3 - x^4 - x^3 - 5x^4 - 8x$
 $2x^4 - x^3 - 4x + 2$
- 23) $(8x^3 + 1) - (5x^4 - 6x^3 + 2)$
 $8x^3 + 1 - 5x^4 + 6x^3 - 2$
 $-5x^4 + 14x^3 - 1$
- 25) $(2a + 2a^4) - (3a^2 - 5a^4 + 4a)$
 $2a + 2a^4 - 3a^2 + 5a^4 - 4a$
 $7a^4 - 3a^2 - 2a$
- 27) $(4p^2 - 3 - 2p) - (3p^2 - 6p + 3)$
 $4p^2 - 3 - 2p - 3p^2 + 6p - 3$
 $p^2 + 4p - 6$
- 29) $(4b^3 + 7b^2 - 3) + (8 + 5b^2 + b^3)$
 $5b^3 + 12b^2 + 5$
- 31) $(3 + 2n^2 + 4n^4) + (n^3 - 7n^2 - 4n^4)$
 $n^3 - 5n^2 + 3$
- 33) $(n - 5n^4 + 7) + (n^2 - 7n^4 - n)$
 $-12n^4 + n^2 + 7$
- 35) $(8r^4 - 5r^3 + 5r^2) + (2r^2 + 2r^3 - 7r^4 + 1)$
 $r^4 - 3r^3 + 7r^2 + 1$
- 37) $(2n^2 + 7n^4 - 2) + (2 + 2n^3 + 4n^2 + 2n^4)$
 $9n^4 + 2n^3 + 6n^2$

5.5

- 1) $6(p - 7)$
 $6p - 42$
- 3) $2(6x + 3)$
 $12x + 6$
- 5) $5m^4(4m + 4)$
 $20m^5 + 20m^4$
- 7) $(4n + 6)(8n + 8)$
 $32n^2 + 32n + 48n + 48$
 $32n^2 + 80n + 48$
- 9) $(8b + 3)(7b - 5)$
 $56b^2 - 40b + 21b - 15$
 $56b^2 - 19b - 15$
- 11) $(4x + 5)(2x + 3)$
 $8x^2 + 12x + 10x + 15$
 $8x^2 + 22x + 15$
- 13) $(3v - 4)(5v - 2)$
 $15v^2 - 6v - 20v + 8$
 $15v^2 - 26v + 8$
- 27) $(6x + 3y)(6x^2 - 7xy + 4y^2)$
 $36x^3 - 42x^2y + 24xy^2 + 18x^2y - 21xy^2 + 12y^3$
 $36x^3 - 24x^2y + 3xy^2 + 12y^3$
- 29) $(8n^2 + 4n + 6)(6n^2 - 6n + 6)$
 $48n^4 - 40n^3 + 48n^2 + 24n^3 - 20n^2 + 24n + 36n^2 - 30n + 36$
 $48n^4 - 16n^3 + 64n^2 - 6n + 36$
- 31) $(5k^2 + 3k + 3)(3k^2 + 3k + 6)$
 $15k^4 + 15k^3 + 30k^2 + 9k^3 + 9k^2 + 18k + 9k^2 + 9k + 18$
 $15k^4 + 24k^3 + 48k^2 + 27k + 18$
- 33) $3(3x - 4)(2x + 1)$
 $3(6x^2 + 3x - 8x - 4)$
 $3(6x^2 - 5x - 5)$
 $18x^2 - 15x - 12$
- 15) $(6x - 7)(4x + 1)$
 $24x^2 + 6x - 28x - 7$
 $24x^2 - 22x - 7$
- 17) $(5x + y)(6x - 4y)$
 $30x^2 - 20xy + 6xy - 4y^2$
 $30x^2 - 14xy - 4y^2$
- 19) $(x + 3y)(3x + 4y)$
 $3x^2 - 4xy + 9xy + 12y^2$
 $3x^2 + 13xy + 12y^2$
- 21) $(7x + 5y)(8x + 3y)$
 $56x^2 + 21xy + 40xy + 15y^2$
 $56x^2 + 61xy + 15y^2$
- 23) $(r - 7)(6r^2 - 4 + 5)$
 $6r^3 - r^2 + 5r - 42r^2 + 7r - 35$
 $6r^3 - 43r^2 + 12r - 35$
- 25) $(6n - 4)(2n^2 - 2n + 5)$
 $12n^3 - 12n^2 + 30n - 8n^2 + 8n - 20$
 $12n^3 - 20n^2 + 38n - 20$
- 35) $3(2x + 1)(4x - 5)$
 $3(8x^2 - 10x + 4x - 5)$
 $3(8x^2 - 6x - 5)$
 $24x^2 - 18x - 15$

$$37) 7(x-5)(x-2)$$

$$7(x^2 - 2x - 5x + 10)$$

$$7(x^2 - 7x + 10)$$

$$7x^2 - 49x + 70$$

$$39) 6(4x-1)(4x+1)$$

$$6(16x^2 + 4x - 4x - 1)$$

$$6(16x^2 - 1)$$

$$96x^2 - 6$$

5.6

$$1) (x+8)(x-8)$$

$$x^2 - 64$$

$$17) (a+5)^2$$

$$2(5a) = 10a$$

$$a^2 + 10a + 25$$

$$29) (2x+2y)^2$$

$$2(4xy) = 8xy$$

$$4x^2 + 8xy + 4y^2 = y^2$$

$$3) (1+3p)(1-3p)$$

$$1 - 9p^2$$

$$19) (x-8)^2$$

$$2(-8x) = -16x$$

$$x^2 - 16x + 64$$

$$31) (5+2r)^2$$

$$2(10r) = 20r$$

$$25 + 20r + 4r^2$$

$$5) (1-7n)(1+7n)$$

$$1 - 49n^2$$

$$21) (p+7)^2$$

$$2(7p) = 14p$$

$$p^2 + 14p + 49$$

$$33) (2+5x)^2$$

$$2(10x) = 20x$$

$$4 + 20x + 25x^2$$

$$7) (5n-8)(5n+8)$$

$$25n^2 - 64$$

$$23) (7-5n)^2$$

$$2(-35n) = -70n$$

$$49 - 70n + 25n^2$$

$$35) (4v-7)(4v+7)$$

$$16v^2 - 49$$

$$11) (4y-x)(4y+x)$$

$$16y^2 - x^2$$

$$25) (5m-8)^2$$

$$2(-40m) = -80m$$

$$25m^2 - 80m + 64$$

$$37) (n-5)(n+5)$$

$$n^2 - 25$$

$$13) (4m-8n)(4m+8n)$$

$$16m^2 - 64n^2$$

$$27) (5x+7y)^2$$

$$2(35xy) = 70xy$$

$$25x^2 + 70xy + 49y^2$$

$$39) (4k+2)^2$$

$$2(8k) = 16k$$

$$16k^2 + 16k + 4$$

$$15) (6x-2y)(6x+2y)$$

$$36x^2 - 4y^2$$

5.7

$$1) \frac{20x^4+x^3+2x^2}{4x^3} = \frac{20x^4}{4x^3} + \frac{x^3}{4x^3} + \frac{2x^2}{4x^3} = 5x + \frac{1}{4} + \frac{1}{2x}$$

$$3) \frac{20n^4+n^3+40n^2}{10n} = \frac{20n^4}{10n} + \frac{n^3}{10n} + \frac{40n^2}{10n} = 2n^3 + \frac{n^2}{10} + 4n$$

$$5) \frac{12x^4+24x^3+3x^2}{6x} = \frac{12x^4}{6x} + \frac{24x^3}{6x} + \frac{3x^2}{6x} = 2x^3 + 4x^2 + \frac{x}{2}$$

$$7) \frac{10n^4+50n^3+2n^2}{10n^2} = \frac{10n^4}{10n^2} + \frac{50n^3}{10n^2} + \frac{2n^2}{10n^2} = n^2 + 5n + \frac{1}{5}$$

$$9) \frac{x^2-2x-71}{x+8} \quad \begin{array}{r} x - 10 + \frac{9}{x+8} \\ x + 8 \overline{)x^2 - 2x - 71} \\ \underline{-x^2 + (-8x)} \\ -10x - 71 \\ \underline{+10x + 80} \\ 9 \end{array}$$

$$11) \frac{n^2+13n+32}{n+5} \quad \begin{array}{r} n + 8 - \frac{8}{n+5} \\ n + 5 \overline{)n^2 + 13n + 32} \\ \underline{-n^2 - 5n} \\ 8n + 32 \\ \underline{-8n - 40} \\ -8 \end{array}$$

$$13) \frac{v^2-2v-89}{v-10} \quad \begin{array}{r} v + 8 - \frac{9}{v-10} \\ v - 10 \overline{)v^2 - 2v - 89} \\ \underline{-v^2 + 10v} \\ 8v - 89 \\ \underline{-8v + 80} \\ 9 \end{array}$$

$$15) \frac{a^2-4a-38}{a-8} \quad \begin{array}{r} a + 4 - \frac{6}{a-8} \\ a - 8 \overline{)a^2 - 4a - 38} \\ \underline{-a^2 + 8a} \\ 4a - 38 \\ \underline{-4a + 32} \\ -6 \end{array}$$

$$17) \frac{45p^2+56p+19}{9p+4} \quad \begin{array}{r} 5p + 4 + \frac{3}{9p+4} \\ 9p + 4 \overline{)45p^2 + 56p + 19} \\ \underline{-45p^2 - 20p} \\ 36p + 19 \\ \underline{-36p - 16} \\ 3 \end{array}$$

$$19) \frac{10x^2 - 32x + 9}{10x - 2} \quad x - 3 + \frac{3}{10x - 2}$$

$$10x - 2 \overline{) 10x^2 - 32x + 9}$$

$$\underline{-10x^2 + 2x}$$

$$-30x + 9$$

$$\underline{+30x - 6}$$

$$3$$

$$21) \frac{4r^2 - r - 1}{4r + 3} \quad r - 1 + \frac{2}{4r + 3}$$

$$4r + 3 \overline{) 4r^2 - r - 1}$$

$$\underline{-4r^2 - 3r}$$

$$-4r - 1$$

$$\underline{+4r + 3}$$

$$2$$

$$23) \frac{n^2 - 4}{n - 2} \quad n + 2$$

$$n - 2 \overline{) n^2 - 0n - 4}$$

$$\underline{-n^2 - 2n}$$

$$-2n - 4$$

$$\underline{+2n + 4}$$

$$0$$

$$25) \frac{27b^2 + 87b + 35}{3b + 8} \quad 9b + 5 - \frac{5}{3b + 8}$$

$$3b + 8 \overline{) 27b^2 + 87b + 35}$$

$$\underline{-27b^2 - 72b}$$

$$15b + 35$$

$$\underline{-15b - 40}$$

$$-5$$

$$27) \frac{4x^2 - 33x + 28}{4x - 5} \quad x - 7 - \frac{7}{4x - 5}$$

$$4x - 5 \overline{) 4x^2 - 33x + 28}$$

$$\underline{-4x^2 + 5x}$$

$$28x + 28$$

$$\underline{-28x - 35}$$

$$-7$$

$$29) \frac{a^3+15a^2+49a-55}{a+7} \quad a^2 + 8a - 7 - \frac{6}{a+7}$$

$$a + 7 \overline{\begin{array}{r} a^3 + 15a^2 + 49a - 55 \\ -a^3 - 7a^2 \\ \hline 8a^2 + 49a \\ -8a^2 - 56a \\ \hline -7a - 55 \\ +7a + 55 \\ \hline 0 \end{array}}$$

$$31) \frac{x^3-26x-41}{x+4} \quad x^2 - 4x - 10 - \frac{1}{x+4}$$

$$x + 4 \overline{\begin{array}{r} x^3 - 0x^2 - 26x - 41 \\ -x^3 - 4x^2 \\ \hline -4x^2 - 26x \\ +4x^2 + 16x \\ \hline -10x - 41 \\ +10x + 40 \\ \hline 1 \end{array}}$$

$$33) \frac{3n^3+9n^2-64n-68}{n+6} \quad 3n^2 - 9n - 10 - \frac{8}{n+6}$$

$$n + 6 \overline{\begin{array}{r} 3n^3 + 9n^2 - 64n - 68 \\ -3n^3 - 18n^2 \\ \hline -9n^2 - 64n \\ +9n^2 + 54n \\ \hline -10n - 68 \\ +10n + 60 \\ \hline -8 \end{array}}$$

$$35) \frac{x^3-46x+22}{x+7} \quad x^2 - 7x + 3 + \frac{1}{x+7}$$

$$x + 7 \overline{\begin{array}{r} x^3 + 0x^2 - 46x + 22 \\ -x^3 - 7x \\ \hline -7x - 46x \\ +7x + 49x \\ \hline 3x + 22 \\ -3x - 21 \\ \hline 1 \end{array}}$$

$$37) \frac{9p^3+45p^2+27p-5}{9p+9} \quad p^2 + 4p - 1 + \frac{4}{9p+9}$$

$$9p + 9 \overline{) 9p^3 + 45p^2 + 27p - 5}$$

$$\underline{-9p^3 - 9p}$$

$$36p^2 + 27p$$

$$\underline{-36p^2 - 36p}$$

$$-9p - 5$$

$$\underline{+9p + 9}$$

$$4$$

$$39) \frac{r^3-r^2-16r+8}{r-4} \quad r^2 + 3r - 4 - \frac{8}{r-4}$$

$$r - 4 \overline{) r^3 - r^2 - 16r + 8}$$

$$\underline{-r^3 + 4r^2}$$

$$3r^2 - 16r$$

$$\underline{-3r^2 + 12r}$$

$$-4r + 8$$

$$\underline{+4r - 16}$$

$$-8$$

$$41) \frac{12n^3+12n^2-15n-4}{2n+3} \quad 6n^2 - 3n - 3 + \frac{5}{2n+3}$$

$$2n + 3 \overline{) 12n^3 + 12n^2 - 15n - 4}$$

$$\underline{-12n^3 - 18n^2}$$

$$-6n^2 - 15n$$

$$\underline{+6n^2 + 9n}$$

$$-6n - 4$$

$$\underline{+6n + 9}$$

$$5$$

$$43) \frac{4v^3-21v^2+6v+19}{4v+3} \quad v^2 - 6v + 6 + \frac{1}{4v+3}$$

$$4v + 3 \overline{) 4v^3 - 21v^2 + 6v + 19}$$

$$\underline{-4v^3 - 3v^2}$$

$$-24v^2 + 6v$$

$$\underline{+24v^2 + 18v}$$

$$24v + 19$$

$$\underline{-24v - 18}$$

$$1$$

Chapter 6: Factoring

6.1

1) $9 + 8x$
 $1(9 + 8x)$

3) $45x^2 - 25$
 $5(9x^2 - 5)$

5) $56 - 35p$
 $7(8 - 5p)$

7) $7ab - 36a^2b$
 $7ab(1 - 5a)$

9) $-3a^2b + 6a^3b^2$
 $-3a^2b(1 - 2ab)$

11) $-5x^2 - 5x^3 - 15x^4$
 $-5x^2(1 + x + 3x^2)$

13) $20x^4 - 30x + 30$
 $10(2x^4 - 3x + 3)$

15) $28m^4 + 40m^3 + 8$
 $4(7m^4 + 10m^3 + 2)$

17) $30b^9 + 5ab - 15a^2$
 $5(6b^9 + ab - 3a^2)$

19) $-48a^2b^2 - 56a^3b - 56a^5b$
 $-8a^2b(6b + 7a + 7a^3)$

21) $20x^8y^2z^2 + 15x^5y^2z + 35x^3y^3z$
 $5x^3y^2z(4x^5z + 3x^2 + 7y)$

23) $50x^2y + 10y^2 + 70xz^2$
 $10(5x^2y + y^2 + 7xz^2)$

25) $30qpr - 5qp + 5q$
 $5q(6pr - p + 1)$

27) $-18n^5 + 3n^3 - 21n + 3$
 $-3(6n^5 - n^3 + 7n - 1)$

29) $-40x^{11} - 20x^{12} + 50x^{13} - 50x^{14}$
 $-10x^{11}(4 + 22x - 5x^2 + 5x^3)$

31) $-32mn^8 + 4m^6n + 12mn^4 + 16mn$
 $-4mn(8n^7 - m^5 - 3n^3 - 4)$

6.2

1) $40r^3 - 8r^2 - 25r + 5$
 $8r^2(5r - 1) - 5(5r - 1)$
 $(5r - 1)(8r^2 - 5)$

3) $3n^2 - 2n^2 - 9n + 6$
 $n^2(3n - 2) - 3(3n - 2)$
 $(3n - 2)(n^2 - 3)$

5) $15b^3 + 21b^2 - 35b - 49$
 $3b^2(5b + 7) - 7(5b + 7)$
 $(5b + 7)(3b^2 - 7)$

7) $3x^3 + 15x^2 + 2x + 10$
 $3x^2(x + 5) + 2(x + 5)$
 $(x + 5)(3x^2 + 2)$

9) $35x^3 - 28x^2 - 20x + 16$
 $7x^2(5x - 4) - 4(5x - 4)$
 $(5x - 4)(7x^2 - 4)$

11) $7xy - 49x + 5y - 35$
 $7x(y - 7) + 5(y - 7)$
 $(y - 7)(7x + 5)$

$$13) 32xy + 40x^2 + 12y + 15x$$

$$8x(4y + 5x) + 3(4y + 5x)$$

$$(4y + 5x)(8x + 3)$$

$$15) 16xy - 56x + 2y - 7$$

$$8x(2y - 7) + 1(2y - 7)$$

$$(2y - 7)(8x + 1)$$

$$17) 2xy - 8x^2 + 7y^3 - 28y^2x$$

$$2x(y - 4x) + 7y^2(y - 4x)$$

$$(y - 4x)(2x + 7y^2)$$

$$19) 40xy + 35x - 8y^2 - 7y$$

$$5x(8y + 7) - y(8y + 7)$$

$$(8y + 7)(5x - y)$$

$$21) 32uv - 20u + 24v - 15$$

$$4u(8v - 5) + 3(8v - 5)$$

$$(8v - 5)(4u + 3)$$

$$23) 10xy + 30 + 25x + 12y$$

$$10(xy + 3) + 1(25x + 12y)$$

No!

$$10xy + 25x + 12y + 30$$

$$5x(2y + 5) + 6(2y + 5)$$

$$(2y + 5)(5x + 6)$$

$$25) 3uv + 14u - 6u^2 - 7v$$

$$u(3v + 14) - 1(6u^2 + 7v)$$

No!

$$3uv - 6u^2 - 7v + 14u$$

$$3u(v - 2u) - 7(v - 2u)$$

$$(v - 2u)(3u - 7)$$

$$27) 16xy - 3x - 6x^2 + 8y$$

$$x(16y - 3) - 1(6x^2 - 8y)$$

No!

$$16xy - 6x^2 + 8y - 3x$$

$$2x(8y - 3x) + 1(8y - 3x)$$

$$(8y - 3x)(2x + 1)$$

6.3

$$1) p^2 + 17p + 72$$

$$(p + 8)(p + 9)$$

$$3) n^2 - 9n + 8$$

$$(n - 8)(n - 1)$$

$$5) x^2 - 9x - 10$$

$$(x - 10)(x + 1)$$

$$7) b^2 + 12b + 32$$

$$(b + 4)(b + 8)$$

$$9) x^2 + 3x - 70$$

$$(x + 10)(x - 7)$$

$$11) n^2 - 8n + 15$$

$$(n - 3)(n - 5)$$

$$13) p^2 + 15p + 54$$

$$(p + 9)(p + 6)$$

$$15) n^2 - 15n + 56$$

$$(n - 7)(n - 8)$$

$$17) u^2 - 8uv + 15v^2$$

$$(u - 5v)(u - 3v)$$

$$19) m^2 - 2mn - 8n^2$$

$$(m + 4n)(m - 2n)$$

$$21) x^2 - 11xy - 12y^2$$

$$(x - 9y)(x - 2y)$$

$$24) x^2 + xy - 12y^2$$

$$(x + 4y)(x - 3y)$$

$$26) \begin{array}{l} x^2 + 4xy - 12y^2 \\ (x + 6y)(x - 2y) \end{array} \quad \begin{array}{c} -12 \\ 6 \times -2 \\ 4 \end{array}$$

$$27) \begin{array}{l} 5a^2 + 60a + 100 \\ 5(a^2 + 12a + 20) \\ 5(a + 10)(a + 2) \end{array} \quad \begin{array}{c} 20 \\ 10 \times 2 \\ 12 \end{array}$$

$$29) \begin{array}{l} 6a^2 + 24a - 192 \\ 6(a^2 + 4a - 32) \\ 6(a + 8)(a - 4) \end{array} \quad \begin{array}{c} -32 \\ 8 \times -4 \\ 3 \end{array}$$

$$31) \begin{array}{l} 6x^2 + 18xy + 12y^2 \\ 6(x^2 + 3xy + 2y^2) \\ 6(x + y)(x + 2y) \end{array} \quad \begin{array}{c} 2 \\ 1 \times 2 \\ 3 \end{array}$$

$$35) \begin{array}{l} 6x^2 + 96xy + 378y^2 \\ 6(x^2 + 16xy + 63y^2) \\ 6(x + 9y)(x + 7y) \end{array} \quad \begin{array}{c} 63 \\ 9 \times 7 \\ 16 \end{array}$$

6.4

$$1) \begin{array}{l} 7x^2 - 48x + 36 \\ 7x^2 - 6x - 42x + 36 \\ x(7x - 6) - 6(7x - 6) \\ (7x - 6)(x - 6) \end{array} \quad \begin{array}{c} 252 \\ -6 \times -42 \\ -48 \end{array}$$

$$11) \begin{array}{l} 2b^2 - b - 3 \\ 2b^2 + 2b - 3b - 3 \\ 2b(b + 1) - 3(b + 1) \\ (b + 1)(2b - 3) \end{array} \quad \begin{array}{c} -3 \\ 2 \times -3 \\ -1 \end{array}$$

$$3) \begin{array}{l} 7b^2 + 15b + 2 \\ 7b^2 + b + 14b + 2 \\ b(7b + 1) + 2(7b + 1) \\ (7b + 1)(b + 2) \end{array} \quad \begin{array}{c} 14 \\ 1 \times 14 \\ 15 \end{array}$$

$$13) \begin{array}{l} 5k^2 + 13k + 6 \\ 5k^2 + 10k + 3k + 6 \\ 5k(k + 2) + 3(k + 2) \\ (k + 2)(5k + 3) \end{array} \quad \begin{array}{c} 30 \\ 10 \times 3 \\ 13 \end{array}$$

$$5) \begin{array}{l} 5a^2 - 13a - 28 \\ 5a^2 + 7a - 20a - 28 \\ a(5a + 7) - 4(5a + 7) \\ (5a + 7)(a - 4) \end{array} \quad \begin{array}{c} -140 \\ 7 \times -20 \\ -13 \end{array}$$

$$15) \begin{array}{l} 3x^2 - 17x + 20 \\ 3x^2 - 12x - 5x + 20 \\ 3x(x - 4) - 5(x - 4) \\ (x - 4)(3x - 5) \end{array} \quad \begin{array}{c} 60 \\ -12 \times -5 \\ -17 \end{array}$$

$$7) \begin{array}{l} 2x^2 - 5x + 2 \\ 2x^2 - 4x - x + 2 \\ 2x(x - 2) - 1(x - 2) \\ (x - 2)(2x - 1) \end{array} \quad \begin{array}{c} 4 \\ -4 \times -1 \\ -5 \end{array}$$

$$17) \begin{array}{l} 3x^2 + 17xy + 10y^2 \\ 3x^2 + 15xy + 2xy + 10y^2 \\ 3x(x + 5y) + 2y(x + 5y) \\ (x + 5y)(3x + 2y) \end{array} \quad \begin{array}{c} 30 \\ 15 \times 2 \end{array}$$

$$9) \begin{array}{l} 2x^2 + 19x + 35 \\ 2x^2 + 14x + 5x + 35 \\ 2x(x + 7) + 5(x + 7) \\ (x + 7)(2x + 5) \end{array} \quad \begin{array}{c} 70 \\ 14 \times 5 \\ 19 \end{array}$$

$$19) \begin{array}{l} 5x^2 + 28xy - 49y^2 \\ 5x^2 + 35xy - 7xy - 49y^2 \\ 5x(x + 7y) - 7y(x + 7y) \\ (x + 7y)(5x - 7y) \end{array} \quad \begin{array}{c} -245 \\ 35 \times -7 \\ 28 \end{array}$$

$$\begin{aligned}
 21) \quad & 6x^2 - 39x - 21 \\
 & 3(2x^2 - 13x - 7) \\
 & 3(2x^2 - 14x + x - 7) \\
 & 3(2x(x - 7) + 1(x - 7)) \\
 & 3(x - 7)(2x + 1)
 \end{aligned}$$

$$\begin{aligned}
 23) \quad & 21k^2 - 87k - 90 \\
 & 3(7k^2 - 29k - 30) \\
 & 3(7k^2 + 6k - 35k - 30) \\
 & 3(k(7k + 6) - 5(7k + 6)) \\
 & 3(7k + 6)(k - 5)
 \end{aligned}$$

$$\begin{aligned}
 25) \quad & 14x^2 - 60x + 16 \\
 & 2(7x^2 - 30x + 8) \\
 & 2(7x^2 - 2x - 28x + 8) \\
 & 2(x(7x - 2) - 4(7x - 2)) \\
 & 2(7x - 2)(x - 4)
 \end{aligned}$$

$$\begin{aligned}
 28) \quad & 6x^2 + 29x + 20 \\
 & 6x^2 + 5x + 24x + 20 \\
 & x(6x + 5) + 4(6x + 5) \\
 & (6x + 5)(x + 4)
 \end{aligned}$$

$$\begin{aligned}
 30) \quad & 4k^2 - 17k + 4 \\
 & 4k^2 - 16k - k + 4 \\
 & 4k(k - 4) - 1(k - 4) \\
 & (k - 4)(4k - 1)
 \end{aligned}$$

$$\begin{aligned}
 33) \quad & 4x^2 + 9xy + 2y^2 \\
 & 4x^2 + 8xy + xy + 2y^2 \\
 & 4x(x + 2y) + y(x + 2y) \\
 & (x + 2y)(4 + y)
 \end{aligned}$$

$$\begin{aligned}
 33) \quad & 4m^2 - 9mn - 9n^2 \\
 & 4m^2 - 12mn + 3mn - 9n^2 \\
 & 4m(m - 3n) + 3n(m - 3n) \\
 & (m - 3n)(4m + 3n)
 \end{aligned}$$

$$\begin{aligned}
 37) \quad & 4x^2 + 13xy + 3y^2 \\
 & 4x^2 + 12xy + xy + 3y^2 \\
 & 4x(x + 3y) + y(x + 3y) \\
 & (x + 3y)(4x + y)
 \end{aligned}$$

$$\begin{aligned}
 39) \quad & 12x^2 + 62xy + 70y^2 \\
 & 2(6x^2 + 31xy + 35y^2) \\
 & 2(6x^2 + 21xy + 10xy + 35y^2) \\
 & 2(3x(2x + 7y) + 5y(2x + 7y)) \\
 & 2(2x + 7y)(3x + 5y)
 \end{aligned}$$

$$\begin{aligned}
 40) \quad & 24x^2 - 52xy + 8y^2 \\
 & 4(6x^2 - 13xy + 2y^2) \\
 & 4(6x^2 - 12xy - xy + 2y^2) \\
 & 4(6x(x - 2y) - y(x - 2y)) \\
 & 4(x - 2y)(6x - y)
 \end{aligned}$$

6.5

$$\begin{aligned}
 1) \quad & r^2 - 16 \\
 & (r) (4) \\
 & (r + 4)(r - 4)
 \end{aligned}$$

$$\begin{aligned}
 3) \quad & v^2 - 25 \\
 & (v) (5) \\
 & (v + 5)(v - 5)
 \end{aligned}$$

$$\begin{aligned}
 5) \quad & p^2 - 4 \\
 & (p) (2) \\
 & (p + 2)(p - 2)
 \end{aligned}$$

$$\begin{aligned}
 7) \quad & 9k^2 - 4 \\
 & (3k) (2) \\
 & (3k + 2)(3k - 2)
 \end{aligned}$$

$$\begin{aligned}
 9) \quad & 3x^2 - 27 \\
 & 3(x^2 - 9) \\
 & (x) (3) \\
 & 3(x + 3)(x - 3)
 \end{aligned}$$

$$\begin{aligned}
 11) \quad & 16x^2 - 36 \\
 & 4(4x^2 - 9) \\
 & (2x) (3) \\
 & 4(2x + 3)(2x - 3)
 \end{aligned}$$

- 13) $18a^2 - 50b^2$
 $2(9a^2 - 25b^2)$
 $(3a) (5b)$
 $2(3a + 5b)(3a - 5b)$
- 15) $a^2 - 2a + 1$ ~~$\begin{array}{c} 1 \\ 1 \times 1 \\ 2 \end{array}$~~
 $(a - 1)^2$
- 17) $x^2 + 6x + 9$ ~~$\begin{array}{c} 9 \\ 3 \times 3 \\ 6 \end{array}$~~
 $(x + 3)^2$
- 19) $x^2 - 6x + 9$ ~~$\begin{array}{c} 9 \\ -3 \times -3 \\ -3 \end{array}$~~
 $(x + 3)^2$
- 21) $25p^2 - 10p + 1$ ~~$\begin{array}{c} 25 \\ -5 \times -5 \\ -10 \end{array}$~~
 $(5p - 1)^2$
- 23) $25a^2 + 30ab + 9b^2$ ~~$\begin{array}{c} 225 \\ 15 \times 15 \\ 30 \end{array}$~~
 $(5a + 3b)^2$
- 25) $4a^2 - 20ab + 25b^2$ ~~$\begin{array}{c} 100 \\ -10 \times -10 \\ -20 \end{array}$~~
 $(2a - 5b)^2$
- 27) $8x^2 - 24xy + 18y^2$ ~~$\begin{array}{c} 36 \\ -6 \times -6 \\ -12 \end{array}$~~
 $2(4x^2 - 12xy + 9y^2)$
 $2(2x - 3y)^2$
- 29) $8 - m^3$
 $(2) (m)$
 $(2 - m)(4 + 2m + m^2)$
- 31) $x^3 - 64$
 $(x) (4)$
 $(x - 4)(x^2 + 4x + 16)$
- 33) $216 - u^3$
 $(6) (u)$
 $(6 - u)(36 + 6u + u^2)$
- 35) $125a^3 - 64$
 $(5a) (4)$
 $(5a - 4)(25a^2 + 20a + 16)$
- 37) $64x^3 + 27y^3$
 $(4x) (3y)$
 $(4x + 3y)(16x^2 - 12xy + 9y^2)$
- 39) $54x^3 + 250y^3$
 $2(27x^3 + 125y^3)$
 $(3x) (5y)$
 $2(3x + 5y)(9x^2 - 15xy + 25y^2)$
- 41) $a^4 - 81$
 $(a^2) (9)$
 $(a^2 + 9)(a^2 - 9)$
 $(a) (3)$
 $(a^2 + 9)(a + 3)(a - 3)$
- 43) $16 - z^4$
 $(4) (z^2)$
 $(4 + z^2)(4 - z^2)$
 $(2) (z)$
 $(4 + z^2)(2 + z)(2 - z)$
- 45) $x^4 - y^4$
 $(x^2) (y^2)$
 $(x^2 + y^2)(x^2 - y^2)$
 $(x) (y)$
 $(x^2 + y^2)(x + y)(x - y)$
- 47) $m^4 - 81b^4$
 $(m^2) (9b^2)$
 $(m^2 + 9b^2)(m^2 - 9b^2)$
 $(m) (3b)$
 $(m^2 + 9b^2)(m + 3b)(m - 3b)$

6.6

- 1) $24az - 18ah + 60yz - 45yh$
 $3(8az - 6ah + 20yz - 15yh)$
 $3(2a(4z - 3h) + 5y(4z - 3h))$
 $3(4z - 3h)(2a + 5y)$
- 3) $5u^2 - 9uv + 4v^2$
 $5u^2 - 4uv - 5uv + 4v^2$
 $u(5u - 4v) - v(5u - 4v)$
 $(5u - 4v)(u - v)$
- 5) $-2x^3 + 128y^3$
 $-2(x^3 - 64y^3)$
 $(x) (4y)$
 $-2(x - 4y)(x^2 + 4xy + 16y^2)$
- 7) $5n^3 + 7n^2 - 6n$
 $n(5n^2 + 7n - 6)$
 $n(5n^2 + 10n - 3n - 6)$
 $n(5n(n + 2) - 3(n + 2))$
 $n(n + 2)(5n - 3)$
- 9) $54u^3 - 16$
 $2(27u^3 - 8)$
 $(3u) (2)$
 $2(3u - 2)(9u^2 + 6u + 4)$
- 11) $n^2 - n$
 $n(n - 1)$
- 13) $x^2 - 4xy + 3y^2$
 $x^2 - xy - 3xy + 3y^2$
 $x(x - y) - 3y(x - y)$
 $(x - y)(x - 3y)$
- 15) $9x^2 - 25y^2$
 $(3x) (5y)$
 $(3x + 5y)(3x - 5y)$
- 17) $m^2 - 4n^2$
 $(m) (2n)$
 $(m + 2n)(m - 2n)$
- 19) $36b^2c - 16xd - 24b^2d + 24xc$
 $4(9b^2c - 4xd - 6b^2d + 6xc)$
 ~~$4(1(9b^2c - 4xd) - 6(b^2d + xc))$~~
 $4(9b^2c - 6b^2d + 6xc - 4xd)$
 $4(3b^2(3c - 2d) + 2x(3c - 2d))$
 $4(3c - 2d)(3b^2 + 2x)$
- 21) $128 + 54x^3$
 $2(64 + 27x^3)$
 $(4) (3x)$
 $2(4 + 3x)(16 - 12x + 9x^2)$
- 23) $2x^3 + 6x^2y - 20y^2x$
 $2x(x^2 + 3xy - 10y^2)$
 $2x(x + 5y)(x - 2y)$
- 25) $n^3 + 7n^2 + 10n$
 $n(n^2 + 7n + 10)$
 $n(n + 5)(n + 2)$
- 27) $27x^3 - 64$
 $(3x) (4)$
 $(3x - 4)(9x^2 + 12x + 16)$
- 29) $5x^2 + 2x$
 $x(5x + 2)$
- 31) $3k^3 - 27k^2 + 60k$
 $3k(k^2 - 9k + 20)$
 $3k(k - 4)(k - 5)$
- 33) $mn - 12x + 3m - 4xn$
 ~~$1(mn - 12x) + 1(3m - 4xn)$~~
 $mn + 3m - 4xn - 12x$
 $m(n + 3) - 4x(n + 3)$
 $(n + 3)(m - 4x)$
- 35) $16x^2 - 8xy + y^2$
 $(4x - y)^2$

$$\begin{aligned}
 37) \quad & 27m^2 - 48n^2 \\
 & 3(9m^2 - 16n^2) \\
 & (3m) (4n) \\
 & 3(3m + 4n)(3m - 4n)
 \end{aligned}$$

$$\begin{aligned}
 41) \quad & 2m^2 + 6mn - 20n^2 \\
 & 2(m^2 + 3mn - 10n^2) \\
 & 2(m + 5n)(m - 2n)
 \end{aligned}$$

$$\begin{aligned}
 39) \quad & 9x^3 + 21x^2y - 60y^2x \\
 & 3x(3x^2 + 7xy - 20y^2) \\
 & 3x(3x^2 + 12xy - 5xy - 20y^2) \\
 & 3x(3x(x + 4y) - 5y(x + 4y)) \\
 & 3x(x + 4y)(3x - 5y)
 \end{aligned}$$

6.7

$$\begin{aligned}
 1) \quad & (k - 7)(k + 2) = 0 \\
 & k - 7 = 0 \quad k + 2 = 0 \\
 & \frac{+7}{k = 7} \quad \frac{-2}{k = -2}
 \end{aligned}$$

$$\begin{aligned}
 9) \quad & 7x^2 + 26x + 15 = 0 \\
 & 7x^2 + 5x + 21x + 15 = 0 \\
 & x(7x + 5) + 3(7x + 5) = 0 \\
 & (7x + 5)(x + 3) = 0 \\
 & 7x + 5 = 0 \quad x + 3 = 0 \\
 & \frac{-5}{7x = -\frac{5}{7}} \quad \frac{-3}{x = -3} \\
 & x = -\frac{5}{7}
 \end{aligned}$$

$$\begin{aligned}
 3) \quad & (x - 1)(x + 4) = 0 \\
 & x - 1 = 0 \quad x + 4 = 0 \\
 & \frac{+1}{x = 1} \quad \frac{-4}{x = -4}
 \end{aligned}$$

$$\begin{aligned}
 5) \quad & 6x^2 - 150 = 0 \\
 & 6(x^2 - 25) = 0 \\
 & 6(x + 5)(x - 5) = 0 \\
 & x + 5 = 0 \quad x - 5 = 0 \\
 & \frac{-5}{x = -5} \quad \frac{+5}{x = 5}
 \end{aligned}$$

$$\begin{aligned}
 11) \quad & 5n^2 - 9n - 2 = 0 \\
 & 5n^2 - 10n + n - 2 = 0 \\
 & 5n(n - 2) + 1(n - 2) = 0 \\
 & (n - 2)(5n + 1) = 0 \\
 & n - 2 = 0 \quad 5n + 1 = 0 \\
 & \frac{+2}{n = 2} \quad \frac{-1}{\frac{5n}{5} = \frac{-1}{5}} \\
 & n = -\frac{1}{5}
 \end{aligned}$$

$$\begin{aligned}
 7) \quad & 2n^2 + 10n - 28 = 0 \\
 & 2(n^2 + 5n - 14) = 0 \\
 & 2(n + 7)(n - 2) = 0 \\
 & n + 7 = 0 \quad n - 2 = 0 \\
 & \frac{-7}{n = -7} \quad \frac{+2}{n = 2}
 \end{aligned}$$

$$\begin{aligned}
 13) \quad & x^2 - 4x - 8 = -8 \\
 & x^2 - 4x = 0 \\
 & x(x - 4) = 0 \\
 & x = 0 \quad x - 4 = 0 \\
 & \frac{+4}{x = 4}
 \end{aligned}$$

$$15) x^2 - 4x - 1 = -5$$

$$\begin{array}{r} +5 \quad +5 \\ x^2 - 4x + 4 = 0 \end{array}$$

$$(x-4)(x-1) = 0$$

$$x-4 = 0 \quad x-1 = 0$$

$$\begin{array}{r} +4 \quad +4 \quad +1 \quad +1 \\ x = 4 \quad x = 1 \end{array}$$

$$23) x^2 - 6x = 16$$

$$\begin{array}{r} -16 \quad -16 \\ x^2 - 6x - 16 = 0 \end{array}$$

$$(x-8)(x+2) = 0$$

$$x-8 = 0 \quad x+2 = 0$$

$$\begin{array}{r} +8 \quad +8 \quad -2 \quad -2 \\ x = 8 \quad x = -2 \end{array}$$

$$17) 49p^2 + 371p - 241 = 0$$

$$7(7p^2 + 53p - 24) = 0$$

$$7(7p^2 - 3p + 56p - 24) = 0$$

$$7(p(7p-3) + 8(7p-3)) = 0$$

$$7(7p-3)(p+8) = 0$$

$$7p-3 = 0 \quad p+8 = 0$$

$$\begin{array}{r} +3 \quad +3 \quad -8 \quad -8 \\ \frac{7p}{7} = \frac{3}{7} \quad p = -8 \\ p = \frac{3}{7} \end{array}$$

$$25) 3v^2 + 7v = 40$$

$$\begin{array}{r} -40 \quad -40 \\ 3v^2 + 7v - 40 = 0 \end{array}$$

$$3v^2 + 15v - 8v - 40 = 0$$

$$3v(v+5) - 8(v+5) = 0$$

$$(v+5)(3v-8) = 0$$

$$v+5 = 0 \quad 3v-8 = 0$$

$$\begin{array}{r} -5 \quad -5 \quad +8 \quad +8 \\ v = -5 \quad \frac{3v}{3} = \frac{8}{3} \\ v = \frac{8}{3} \end{array}$$

$$19) 7x^2 + 17x - 20 = -8$$

$$\begin{array}{r} +8 \quad +8 \\ 7x^2 + 17x - 12 = 0 \end{array}$$

$$7x^2 - 4x + 21x - 12 = 0$$

$$x(7x-4) + 3(7x-4) = 0$$

$$(7x-4)(x+3) = 0$$

$$7x-4 = 0 \quad x+3 = 0$$

$$\begin{array}{r} +4 \quad +4 \quad -3 \quad -3 \\ \frac{7x}{7} = \frac{4}{7} \quad x = -3 \\ x = \frac{4}{7} \end{array}$$

$$27) 35x^2 + 120x = -45$$

$$\begin{array}{r} +45 \quad +45 \\ 35x^2 + 120x + 45 = 0 \end{array}$$

$$5(7x^2 + 24x + 9) = 0$$

$$5(7x^2 + 3x + 21x + 9) = 0$$

$$5(x(7x+3) + 3(7x+3)) = 0$$

$$5(7x+3)(x+3) = 0$$

$$7x+3 = 0 \quad x+3 = 0$$

$$\begin{array}{r} -3 \quad -3 \quad -3 \quad -3 \\ \frac{7x}{7} = \frac{-3}{7} \quad x = -3 \\ x = -\frac{3}{7} \end{array}$$

$$21) 7r^2 + 84 = -49r$$

$$\begin{array}{r} +49r \quad +49r \\ 7r^2 + 49r + 84 = 0 \end{array}$$

$$7(r^2 + 7r + 12) = 0$$

$$7(r^2 + 7r + 12) = 0$$

$$7(r+4)(r+3) = 0$$

$$r+4 = 0 \quad r+3 = 0$$

$$\begin{array}{r} -4 \quad -4 \quad -3 \quad -3 \\ r = -4 \quad r = -3 \end{array}$$

$$29) 4k^2 + 18k - 23 = 6k - 7$$

$$\begin{array}{r} -6k + 7 \quad -6k + 7 \\ 4k^2 + 12k - 16 = 0 \end{array}$$

$$4(k^2 + 3k - 4) = 0$$

$$4(k+4)(k-1) = 0$$

$$k+4 = 0 \quad k-1 = 0$$

$$\begin{array}{r} -4 \quad -4 \quad +1 \quad +1 \\ k = -4 \quad k = 1 \end{array}$$

$$\begin{aligned}
 31) \quad & 9x^2 - 46 + 7x = 7x + 8x^2 + 3 \\
 & \underline{-8x^2 - 3 \quad -7x \quad -7x \quad -8x^2 - 3} \\
 & x^2 - 49 = 0 \\
 & (x + 7)(x - 7) = 0 \\
 & x + 7 = 0 \quad x - 7 = 0 \\
 & \underline{-7 \quad -7} \quad \underline{+7 \quad +7} \\
 & x = -7 \quad x = 7
 \end{aligned}$$

$$33) 2m^2 + 19m + 40 = -2m$$

$$\begin{aligned}
 & \underline{\quad +2m \quad \quad \quad +2m} \\
 & 2m^2 + 21m + 40 = 0 \\
 & 2m^2 + 4m + 16m + 40 = 0 \\
 & m(2m + 5 + 8(2m + 5)) = 0 \\
 & (2m + 5)(m + 8) = 0 \\
 & 2m + 5 = 0 \quad m + 8 = 0 \quad \begin{array}{l} \diagup 80 \\ 5 \quad \diagdown 16 \\ 21 \end{array} \\
 & \underline{-5 \quad -5} \quad \underline{-8 \quad -8} \\
 & \frac{2m}{2} = \frac{-5}{2} \quad m = -8 \\
 & m = -\frac{5}{2}
 \end{aligned}$$

$$\begin{aligned}
 35) \quad & 40p^2 + 183p - 168 = p + 5p^2 \\
 & \underline{-5p^2 \quad \quad -p \quad \quad -p \quad -5p^2} \\
 & 35p^2 + 182p - 168 = 0 \\
 & 7(5p^2 + 26p - 24) = 0 \quad \begin{array}{l} \diagup -120 \\ -4 \quad \diagdown 30 \\ 26 \end{array} \\
 & 7(5p^2 - 4p + 30p - 24) = 0 \\
 & 7(p(5p - 4) + 6(5p - 4)) = 0 \\
 & 7(5p - 4)(p + 6) = 0 \\
 & 5p - 4 = 0 \quad p + 6 = 0 \\
 & \underline{+4 \quad +4} \quad \underline{-6 \quad -6} \\
 & \frac{5p}{5} = \frac{4}{5} \quad p = -6 \\
 & p = \frac{4}{5}
 \end{aligned}$$

Chapter 7: Rational Expressions

7.1

1) $\frac{3k^2+30k}{k+10}$

$k + 10 \neq 0$

$\frac{-10}{-10} = \frac{-10}{-10}$

$k \neq -10$

3) $\frac{15n^2}{10n+25}$

$10n + 25 \neq 0$

$\frac{-25}{-25} = \frac{-25}{-25}$

$\frac{10n}{10} \neq \frac{-25}{10}$

$n \neq -\frac{5}{2}$

5) $\frac{10m^2+8m}{10m}$

$\frac{10m}{10} \neq \frac{0}{10}$

$m \neq 0$

7) $\frac{r^2+3r+12}{5r+10}$

$5r + 10 \neq 0$

$\frac{-10}{-10} = \frac{-10}{-10}$

$\frac{5r}{5} \neq \frac{-10}{5}$

$r \neq -2$

9) $\frac{b^2+12b+32}{b^2+4b-32}$

$b^2 + 4b - 32 \neq 0$

$(b + 8)(b - 4) \neq 0$

$b + 8 \neq 0 \quad b - 4 \neq 0$

$\frac{-8}{-8} = \frac{-8}{-8} \quad \frac{+4}{+4} = \frac{+4}{+4}$

$b \neq -8 \quad b \neq 4$

11) $\frac{21x^2}{18x} = \frac{7x}{6}$

13) $\frac{24a}{40a^2} = \frac{3}{5a}$

15) $\frac{32x^3}{8x^4} = \frac{4}{x}$

17) $\frac{18m-24}{60} = \frac{6(3m-4)}{60} = \frac{3m-4}{10}$

19) $\frac{20}{4+2p} = \frac{20}{2(2+p)} = \frac{10}{2+p}$

21) $\frac{x+1}{x^2+8x+7} = \frac{x+1}{(x+7)(x+1)} = \frac{1}{x+7}$

23) $\frac{32x^2}{28x^2+28x} = \frac{32x^2}{28x(x+1)} = \frac{8x}{7(x+1)}$

25) $\frac{n^2+4n-12}{n^2-7n+10} = \frac{(n+6)(n-2)}{(n-5)(n-2)} = \frac{n+6}{n-5}$

27) $\frac{9v+54}{v^2-4v-60} = \frac{9(v+6)}{(v-10)(v+6)} = \frac{9}{v-10}$

29) $\frac{12x^2-42x}{30x^2-42x} = \frac{6x(2x-7)}{6x(5x-7)} = \frac{2x-7}{5x-7}$

31) $\frac{6a-10}{10a+4} = \frac{2(3a-5)}{2(5a+2)} = \frac{3a-5}{5a+2}$

33) $\frac{2n^2+19n-10}{9n+90} = \frac{(2n-1)(n+10)}{9(n+10)} = \frac{2n-1}{9}$

35) $\frac{8m+16}{20m-12} = \frac{8(m+2)}{4(5m-3)} = \frac{2(m+2)}{5m-3}$

37) $\frac{2x^2-10x+8}{3x^2-7x+4} = \frac{2(x-4)(x-1)}{(3x-4)(x-1)} = \frac{2(x-4)}{3x-4}$

39) $\frac{7n^2-32n+16}{4n-16} = \frac{(7n-4)(n-4)}{4(n-4)} = \frac{7n-4}{4}$

41) $\frac{n^2-2n+1}{6n+6} = \frac{(n-1)^2}{6(n+1)}$

43) $\frac{7a^2-26a-45}{6a^2-34a+20} = \frac{(7a+9)(a-5)}{2(3a-2)(a-5)} = \frac{7a+9}{2(3a-2)}$

7.2

$$1) \frac{4 \cancel{8x^2} \cdot \cancel{9}}{\cancel{9} \cdot \cancel{2}} = 4x^2$$

$$3) \frac{\cancel{9n} \cdot 7}{\cancel{2n} \cdot 5n} = \frac{63}{10n}$$

$$5) \frac{\cancel{5x^2} \cdot \cancel{6^3}}{\cancel{2^4} \cdot \cancel{5}} = \frac{3x^2}{2}$$

$$7) \frac{\cancel{7(m-6)} \cdot 5m \cdot \cancel{7(m-5)}}{\cancel{m-6} \cdot \cancel{7(m-5)}} = 5m$$

$$9) \frac{7r}{7r(r+10)} \div \frac{r-6}{(r-6)^2} = \frac{\cancel{7r}}{\cancel{7r}(r+10)} \cdot \frac{(r-6)^{\cancel{2}}}{\cancel{r-6}} = \frac{r-6}{r+10}$$

$$11) \frac{25n+25}{5} \cdot \frac{4}{30n+30} = \frac{\cancel{25(n+1)}}{\cancel{5}} \cdot \frac{\cancel{4^2}}{\cancel{3} \cdot \cancel{10(n+1)}} = \frac{2}{3}$$

$$13) \frac{x-10}{35x+21} \div \frac{7}{3x+21} = \frac{x-10}{35x+21} \cdot \frac{35x+21}{7} = \frac{x-10}{\cancel{7(5x+3)}} \cdot \frac{\cancel{7(5x+3)}}{7} = \frac{x-10}{7}$$

$$15) \frac{x^2-6x-7}{x+5} \cdot \frac{x+5}{x-7} = \frac{\cancel{(x-7)}(x+1)}{\cancel{x+5}} \cdot \frac{\cancel{x+5}}{\cancel{x-7}} = x+1$$

$$17) \frac{8k}{24k^2-40k} \div \frac{1}{15k-25} = \frac{8k}{24k^2-40k} \cdot \frac{15k-25}{1} = \frac{\cancel{8k}}{\cancel{8k}(3k-5)} \cdot \frac{5(3k-5)}{1} = 5$$

$$19) (n-8) \cdot \frac{6}{10n-80} = \frac{\cancel{n-8}}{1} \cdot \frac{\cancel{6^3}}{\cancel{10}(n-8)} = \frac{3}{5}$$

$$21) \frac{4m+36}{m+9} \cdot \frac{m-5}{5m^2} = \frac{4(\cancel{m+9})}{\cancel{m+9}} \cdot \frac{m-5}{5m^2} = \frac{4(m-5)}{5m^2}$$

$$23) \frac{3x-6}{12x-24} \cdot (x+3) = \frac{\cancel{3(x-2)}}{\cancel{4} \cdot \cancel{12}(x-2)} \cdot \frac{x+3}{1} = \frac{x+3}{4}$$

$$25) \frac{b+2}{40b^2-24b} \cdot (5b-3) = \frac{b+2}{8b(\cancel{5b-3})} \cdot \frac{\cancel{5b-3}}{1} = \frac{b+2}{8b}$$

$$27) \frac{n-7}{6n-12} \cdot \frac{12-6n}{n^2+13n+42} = \frac{\cancel{n-7}}{\cancel{6}(n-2)} \cdot \frac{\cancel{-6}(n-2)}{\cancel{(n-7)}(n-6)} = \frac{-1}{n-6}$$

$$29) \frac{27a+36}{9a+63} \div \frac{6a+8}{2} = \frac{27a+36}{9a+63} \cdot \frac{2}{6a+8} = \frac{\cancel{9(3a+4)}}{\cancel{9}(a+7)} \cdot \frac{\cancel{2}}{\cancel{2}(3a+4)} = \frac{1}{a+7}$$

$$31) \frac{x^2-12x+32}{x^2-6x-16} \cdot \frac{7x^2+14x}{7x^2+21x} = \frac{\cancel{(x-8)}(x-4)}{\cancel{(x-8)}(x+2)} \cdot \frac{\cancel{7x}(x+2)}{\cancel{7x}(x+3)} = \frac{x-4}{x+3}$$

$$33) (10m^2 + 100m) \cdot \frac{18m^3 - 36m^2}{20m^2 - 40m} = \frac{\cancel{10}m(m+10)}{1} \cdot \frac{\cancel{18}m^2(\cancel{m-2})}{\cancel{20}m(\cancel{m-2})} = 9m^2(m+10)$$

$$35) \frac{7p^2 + 25p + 12}{6p + 48} \cdot \frac{3p - 8}{21p^2 - 44p - 32} = \frac{(7p+4)(p+3)}{6(p+8)} \cdot \frac{3p-8}{(7p+4)(3p-8)} = \frac{p+3}{6(p+8)}$$

$$37) \frac{10b^2}{30b+20} \cdot \frac{30b+20}{2b^2+10b} = \frac{\cancel{10}b^2}{10(3b+2)} \cdot \frac{\cancel{10}(3b+2)}{2b(b+5)} = \frac{5b}{b+5}$$

$$39) \frac{7r^2 - 53r - 24}{7r+2} \div \frac{49r+21}{49r+14} = \frac{7r^2 - 53r - 24}{7r+2} \cdot \frac{49r+14}{49r+21} = \frac{(7r+3)(r-8)}{7r+2} \cdot \frac{7(7r+2)}{7(7r+3)} = r - 8$$

$$41) \frac{x^2-1}{2x-4} \cdot \frac{x^2-4}{x^2-x-2} \div \frac{x^2+x-2}{3x-6} = \frac{x^2-1}{2x-4} \cdot \frac{x^2-4}{x^2-x-2} \cdot \frac{3x-6}{x^2+x-2} = \frac{(x+1)(x-1)}{2(x-2)} \cdot \frac{(x+2)(x-2)}{(x-2)(x+1)} \cdot \frac{3(x-2)}{(x+2)(x-1)} = \frac{3}{2}$$

$$43) \frac{x^2+3x+9}{x^2+x-12} \cdot \frac{x^2+2x-8}{x^3-27} \div \frac{x^2-4}{x^2-6x+9} = \frac{x^2+3x+9}{x^2+x-12} \cdot \frac{x^2+2x-8}{x^3-27} \cdot \frac{x^2-6x+9}{x^2-4} = \frac{\cancel{x^2+3x+9}}{(x+4)(x-3)} \cdot \frac{(x+4)(x-2)}{(x-3)(x^2+3x+9)} \cdot \frac{(x-3)^2}{(x-2)(x+2)} = \frac{1}{x+2}$$

7.3

$$1) \frac{(6)3}{(6)8} = \frac{?}{48}$$

$$\frac{18}{48}$$

$$11) 2a^3, 6a^4b^2, 4a^3b^5$$

$$12a^4b^5$$

$$3) \frac{(y)a}{(y)x} = \frac{?}{xy}$$

$$\frac{ay}{xy}$$

$$13) x^3 - 3x, \quad x - 3, x$$

$$x(x^2 - 3)$$

$$x(x - 3)$$

$$5) \frac{(3a^2c^3)}{(3a^2c^3)} \frac{2}{3a^3b^2c} = \frac{?}{9a^3b^2c^4}$$

$$\frac{6a^2c^3}{9a^3b^2c^4}$$

$$15) x + 2, x - 4$$

$$(x + 2)(x - 4)$$

$$7) \frac{(x-4)}{(x-4)} \frac{2}{x+4} = \frac{?}{x^2-16}$$

$$\frac{(x+4)(x-4)}{x^2-16}$$

$$\frac{2x-8}{(x+4)(x-4)}$$

$$17) x^2 - 25, \quad x + 5$$

$$(x + 5)(x - 5)$$

$$(x + 5)(x - 5)$$

$$9) \frac{(x+3)}{(x+3)} \frac{(x-4)}{(x+2)} = \frac{?}{x^2+5x+6}$$

$$\frac{(x+2)(x+3)}{x^2+5x+6}$$

$$19) x^2 + 3x + 2, \quad x^2 + 5x + 6$$

$$(x + 1)(x + 2) (x + 2)(x + 3)$$

$$(x + 1)(x + 2)(x + 3)$$

$$\frac{x^2-4x+3x-12}{(x+2)(x+3)} = \frac{x^2-x-12}{(x+2)(x+3)}$$

$$21) \frac{(2a^3)}{(2a^3)} \frac{3a}{5b^2}, \frac{2}{10a^3b} \frac{(b)}{(b)}$$

$$LCD = 10a^3b^2$$

$$\frac{6a^4}{10a^3b^2}, \frac{2b}{10a^3b^2}$$

$$23) \frac{(x+2)}{(x+2)} \frac{x+2}{x-3} , \frac{(x-3)}{(x+2)} \frac{(x-3)}{(x-3)}$$

$$LCD = (x-3)(x+2)$$

$$\frac{x^2+4x+4}{(x-3)(x+2)} , \frac{x^2-6x+9}{(x-3)(x+2)}$$

$$25) \frac{(x-4)}{(x-4)} \frac{x}{x^2-16} , \frac{3x}{x^2-8x+16} \frac{(x+4)}{(x+4)}$$

$$(x-4)(x+4) (x-4)(x-4)$$

$$LCD = (x-4)^2(x+4)$$

$$27) \frac{4x}{x^2-x-6} , \frac{x+2}{x-3} \frac{(x+2)}{(x+2)}$$

$$(x-3)(x+2)$$

$$LCD: (x-3)(x+2)$$

$$\frac{4x}{(x-3)(x+2)} , \frac{x^2+4x+4}{(x-3)(x+2)}$$

7.4

$$1) \frac{2}{a+3} + \frac{4}{a+3} = \frac{6}{a+3}$$

$$3) \frac{t^2+4t}{t-1} + \frac{2t-7}{t-1} = \frac{t^2+6t-7}{t-1} = \frac{(t+7)(t-1)}{t-1} = t+7$$

$$5) \frac{2x^2+3}{x^2-6x+5} + \frac{-x^2+5x+(-9)}{x^2-6x+5} = \frac{x^2+5x-6}{x^2-6x+5} =$$

$$\frac{(x+6)(x-1)}{(x+5)(x-1)} = \frac{x+6}{x+5}$$

$$7) \frac{(4)}{(4)} \frac{5}{6r} + \frac{-5(3)}{8r(3)}$$

$$LCD : 24r$$

$$\frac{20}{24r} + \frac{-15}{24r} = \frac{5}{24r}$$

$$9) \frac{(2)}{(2)} \frac{8}{9t^3} + \frac{5(3t)}{6t^2(3t)}$$

$$LCD : 18t^3$$

$$\frac{16}{18t^3} + \frac{15t}{18t^3} = \frac{16+15t}{18t^3}$$

$$11) \frac{(2)}{(2)} \frac{a+2}{2} + \frac{-a+4}{4}$$

$$LCD : 4$$

$$\frac{2a+4}{4} + \frac{-a+4}{4} = \frac{a+8}{4}$$

$$13) \frac{x-1}{4x} + \frac{-2x-3}{x} \frac{(4)}{(4)}$$

$$LCD : 4x$$

$$\frac{x-1}{4x} + \frac{-8x-12}{4x} = \frac{-7x-13}{4x}$$

$$15) \frac{(y)}{(y)} \frac{5x+3y}{2x^2y} + \frac{-3x-4y}{xy^2} \frac{(2x)}{(2x)}$$

$$LCD : 2x^2y^2$$

$$\frac{5xy+3y^2}{2x^2y^2} + \frac{-6x^2-8xy}{2x^2y^2} = \frac{-6x^2-3xy+3y^2}{2x^2y^2}$$

$$17) \frac{(z+1)}{(z+1)} \frac{2z}{z-1} + \frac{-3z}{z+1} \frac{(z-1)}{(z-1)}$$

$$LCD : (z-1)(z+1)$$

$$\frac{2z^2+2z}{(z-1)(z+1)} + \frac{-3z^2+3z}{(z-1)(z+1)} = \frac{-a^2+5z}{(z-1)(z+1)}$$

$$19) \frac{8}{x^2-4} + \frac{-3}{x+2} \frac{(x-2)}{(x-2)}$$

$$LCD : (x+2)(x-2)$$

$$\frac{8}{(x+2)(x-2)} + \frac{-3x+6}{(x+2)(x-2)} = \frac{-3x+14}{(x+2)(x-2)}$$

$$21) \frac{(4)}{(4)} \frac{t}{t-3} + \frac{-5}{4t-12}$$

$$LCD : 4(t-3)$$

$$\frac{4t}{4(t-3)} + \frac{-5}{4(t-3)} = \frac{4t-5}{4(t-3)}$$

$$23) \frac{(3)}{(3)} \frac{2}{5x^2+5x} + \frac{-4}{3x+3} \frac{(5x)}{(5x)}$$

$$5x(x+1) 3(x+1)$$

$$LCD: 15x(x+1)$$

$$\frac{6}{15x(x+1)} + \frac{-20x}{15x(x+1)} = \frac{6-20x}{15x(x+1)}$$

$$25) \frac{(y+t)}{(y+t)} \frac{t}{y-t} + \frac{-y}{y+t} \frac{(y-t)}{(y-t)}$$

$$LCD: (y+t)(y-t)$$

$$\frac{yt+t^2}{(y+t)(y-t)} + \frac{-y^2+yt}{(y+t)(y-t)} = \frac{t^2+2yt-y^2}{(y+t)(y-t)}$$

$$27) \frac{(x+1)}{(x+1)} \frac{x}{x^2+5x+6} + \frac{-2}{x^2+3x+2} \frac{(x+3)}{(x+1)(x+2)}$$

$$LCD: (x+1)(x+2)(x+3)$$

$$\frac{x^2+x}{(x+1)(x+2)(x+3)} + \frac{-2x-6}{(x+1)(x+2)(x+3)} = \frac{x^2-x-6}{(x+1)(x+2)(x+3)} = \frac{(x-3)(x+2)}{(x+1)(x+2)(x+3)} = \frac{x-3}{(x+1)(x+3)}$$

$$29) \frac{(x+6)}{(x+6)} \frac{x}{x^2+15x+56} + \frac{-7}{x^2+13x+42} \frac{(x+8)}{(x+7)(x+6)}$$

$$LCD: (x+6)(x+7)(x+8)$$

$$\frac{x^2+6x}{(x+6)(x+7)(x+8)} + \frac{-7x-56}{(x+6)(x+7)(x+8)} = \frac{x^2-x-56}{(x+6)(x+7)(x+8)} = \frac{(x-8)(x+7)}{(x+6)(x+7)(x+8)} = \frac{x-8}{(x+6)(x+8)}$$

$$31) \frac{(x+3)}{(x+3)} \frac{5x}{x^2-x-6} + \frac{-18}{x^2-9} \frac{(x+2)}{(x+3)(x-3)}$$

$$LCD: (x+2)(x-3)(x+3)$$

$$\frac{5x^2+15x}{(x+2)(x-3)(x+3)} + \frac{-18x-36}{(x+2)(x-3)(x+3)} = \frac{5x^2-3x-36}{(x+2)(x-3)(x+3)} = \frac{(5x+12)(x-3)}{(x+2)(x-3)(x+3)} = \frac{5x+12}{(x+2)(x+3)}$$

$$33) \frac{(x+3)}{(x+3)} \frac{2x}{x^2-1} + \frac{-4}{x^2+2x-3} \frac{(x+1)}{(x+1)(x-1)}$$

$$LCD: (x+3)(x+1)(x-1)$$

$$\frac{2x^2+6x}{(x+3)(x+1)(x-1)} + \frac{-4x-4}{(x+3)(x+1)(x-1)} = \frac{2x^2+2x-4}{(x+3)(x+1)(x-1)} = \frac{2(x+2)(x-1)}{(x+3)(x+1)(x-1)} = \frac{2(x+2)}{(x+3)(x+1)}$$

$$35) \frac{(x+2)}{(x+2)} \frac{x+1}{x^2-2x-35} + \frac{x+6}{x^2+7x+10} \frac{(x-7)}{(x+5)(x+2)}$$

$$LCD: (x+2)(x+5)(x-7)$$

$$\frac{x^2+x+2x+2}{(x+2)(x+5)(x-7)} + \frac{x^2-7x+6x-42}{(x+2)(x+5)(x-7)} = \frac{2x^2+2x-40}{(x+2)(x+5)(x-7)} = \frac{2(x+5)(x-4)}{(x+2)(x+5)(x-7)} = \frac{2(x-4)}{(x-7)(x+2)}$$

$$37) \frac{(-1)}{(-1)} \frac{4-a^2}{a^2-9} + \frac{-a+2}{3-a} \frac{(a+3)}{(a+3)}$$

$$LCD: (-1)(a+3)(a-3)$$

$$\frac{a^2-4}{(-1)(a+3)(a-3)} + \frac{-a^2-3a+2a+6}{(-1)(a+3)(a-3)} = \frac{-a+2}{(-1)(a+3)(a-3)}$$

$$39) \frac{(2z+1)}{(2z+1)} \frac{2z}{1-2z} + \frac{(-1)(2z-1)}{(-1)(2z-1)} \frac{3z}{2z+1} + \frac{-3}{4z^2-1} \frac{(-1)}{(-1)}$$

$$LCD: (-1)(2z-1)(2z+1)$$

$$\frac{4z^2+2z}{(-1)(2z-1)(2z+1)} + \frac{-6z^2+3z}{(-1)(2z-1)(2z+1)} + \frac{3}{(-1)(2z-1)(2z+1)} = \frac{-2z^2+5z+3}{(-1)(2z-1)(2z+1)} = \frac{(-1)(2z+1)(z-3)}{(-1)(2z-1)(2z+1)} = \frac{z-3}{2z-1}$$

$$41) \frac{(x+3)}{x+3} \frac{2x-3}{x^2+3x+2} + \frac{3x-1}{x^2+5x+6} \frac{(x+1)}{(x+1)}$$

$$(x+1)(x+2)(x+3)(x+2)$$

$$LCD: (x+1)(x+2)(x+3)$$

$$\frac{2x^2-3x+6x-9}{(x+1)(x+2)(x+3)} + \frac{3x^2+3x-x-1}{(x+1)(x+2)(x+3)} = \frac{(5x^2+5x-10)}{(x+1)(x+2)(x+3)} = \frac{5(x+2)(x-1)}{(x+1)(x+2)(x+3)} = \frac{5(x-1)}{(x+1)(x+3)}$$

$$43) \frac{(x+5)}{(x+5)} \frac{(2x+7)}{(x^2-2x-3)} + \frac{-3x+2}{x^2+6x+5} \frac{(x-3)}{(x-3)}$$

$$(x-3)(x+1) \quad (x+5)(x+1)$$

$$LCD: (x+1)(x-3)(x+5)$$

$$\frac{2x^2+7x+10x+35}{(x+1)(x-3)(x+5)} + \frac{(-3x^2+9x+2x-6)}{(x+1)(x-3)(x+5)} = \frac{-x^2+28x+29}{(x+1)(x-3)(x+5)} = \frac{-1(x-29)(x+1)}{(x+1)(x-3)(x+5)} = \frac{(-1)(x-29)}{(x-3)(x+5)}$$

7.5

$$1) \frac{(x^2)1 + \frac{1}{x}(x^2)}{(x^2)1 - \frac{1}{x^2}(x^2)} = \frac{x^2+x}{x^2-1} = \frac{x(x+1)}{(x+1)(x-1)} = \frac{x}{x-1}$$

$$3) \frac{(a)a - 2(a)}{(a)\frac{4}{a} - a(a)} = \frac{a^2 - 2a}{4 - a^2} = \frac{a(a-2)(-1)}{(2+a)(2-a)} = \frac{-a}{a+2}$$

$$5) \frac{(a^2)\frac{1}{a^2} - \frac{1}{a}(a^2)}{(a^2)\frac{1}{a^2} + \frac{1}{a}(a^2)} = \frac{1-a}{1+a}$$

$$7) \frac{(x+2)2 - \frac{4}{x+2}(x+2)}{(x+2)5 - \frac{10}{x+2}(x+2)} = \frac{2x+4-4}{5x+10-10} = \frac{2x}{5x} = \frac{2}{5}$$

$$9) \frac{(2a-3)\frac{3}{(2a-3)} + 2(2a-3)}{(2a-3)\frac{-6}{(2a-3)} - 4(2a-3)} = \frac{3+4a-6}{-6-8a+12} = \frac{4a-3}{-8a+6} = \frac{4a-3}{-2(4a-3)} = -\frac{1}{2}$$

$$11) \frac{x(x+1)\frac{x}{x+1} - \frac{1}{x}x(x+1)}{x(x+1)\frac{x}{x+1} + \frac{1}{x}x(x+1)} = \frac{x^2-x-1}{x^2+x+1}$$

$$13) \frac{(x^2)\frac{3}{x}}{(x^2)\frac{9}{x^2}} = \frac{3x}{9} = \frac{x}{3}$$

$$15) \frac{(16a^2b^2)\frac{a^2-b^2}{4a^2b}}{(16a^2b^2)\frac{a+b}{16ab^2}} = \frac{4b(a^2-b^2)}{a(a+b)} = \frac{4b(a+b)(a-b)}{a(a+b)} = \frac{4b(a-b)}{a}$$

$$17) \frac{(x^2)1 - \frac{3}{x}(x^2) - \frac{10}{x^2}(x^2)}{(x^2)1 + \frac{11}{x}(x^2) + \frac{18}{x^2}(x^2)} = \frac{x^2-3x-10}{x^2+11x+18} = \frac{(x-5)(x+2)}{(x+9)(x+2)} = \frac{x-5}{x+9}$$

$$19) \frac{(3x-4)1 - \frac{2x}{(3x-4)}(3x-4)}{(3x-4)x - \frac{32}{(3x-4)}(3x-4)} = \frac{3x-4-2x}{3x^2-4x-32} = \frac{x-4}{(3x+8)(x-4)} = \frac{1}{3x+8}$$

$$21) \frac{(x-4)x - (x-4)1 + \frac{2}{(x-4)}(x-4)}{(x-4)x + (x-4)3 + \frac{6}{(x-4)}(x-4)} = \frac{x^2-4x-x+4+2}{x^2-4x+3x-12+6} = \frac{x^2-5x+6}{x^2-x-6} = \frac{(x-2)(x+3)}{(x-3)(x+2)} = \frac{(x-2)}{x+2}$$

$$23) \frac{(2x+3)x - (2x+3)4 + \frac{9}{(2x+3)}(2x+3)}{(2x+3)x + (2x+3)3 - \frac{5}{(2x+3)}(2x+3)} = \frac{2x^2+3x-8x-12+9}{2x^2+3x+6x+9-5} = \frac{(2x+1)(x-3)}{(2x+1)(x+4)} = \frac{x-3}{x+4}$$

$$25) \frac{b(b+3)\frac{2}{b} - \frac{5}{b+3}b(b+3)}{b(b+3)\frac{3}{b} + \frac{3}{b+3}b(b+3)} = \frac{2b+6-5b}{3b+9+3b} = \frac{-3b+6}{6b+9} = \frac{(-3)(b-2)}{3(2b+3)} = \frac{(-1)(b-2)}{2b+3}$$

$$27) \frac{(a^2b^2)\frac{2}{b^2} - (a^2b^2)\frac{5}{ab} - \frac{3}{a^2}(a^2b^2)}{(a^2b^2)\frac{2}{b^2} - (a^2b^2)\frac{7}{ab} + \frac{3}{a^2}(a^2b^2)} = \frac{2a^2-5ab-3b^2}{2a^2+7ab+3b^2} = \frac{(2a+b)(a-3b)}{(2a+b)(a+3b)} = \frac{a-3b}{a+3b}$$

$$29) \frac{(y+2)(y-2)\frac{y}{y+2} - \frac{y}{y-2}(y+2)(y-2)}{(y+2)(y-2)\frac{y}{y+2} + \frac{y}{y-2}(y+2)(y-2)} = \frac{y^2-2y-y^2-2y}{y^2-2y+y^2+2y} = \frac{-4y}{2y^2} = -\frac{2}{y}$$

$$31) \frac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}} = \frac{(x^2y^2)\frac{1}{x^2} - \frac{1}{y^2}(x^2y^2)}{(x^2y^2)\frac{1}{x} + \frac{1}{y}(x^2y^2)} = \frac{y^2-x^2}{xy^2+x^2y} = \frac{(y+x)(y-x)}{xy(y+x)} = \frac{y-x}{xy}$$

$$33) \frac{x^{-3}y - xy^{-3}}{x^{-2} - y^{-2}} = \frac{(x^3y^3)\frac{y}{x^3} - \frac{x}{y^3}(x^3y^3)}{(x^3y^3)\frac{1}{x^2} - \frac{1}{y^2}(x^3y^3)} = \frac{y^4-x^4}{xy^3-x^3y} = \frac{(y^2+x^2)(y^2-x^2)}{xy(y^2-x^2)} = \frac{y^2+x^2}{xy}$$

$$35) \frac{x^{-2}-6x^{-1}+9}{x^2-9} = \frac{(x^2)\frac{1}{x^2} - (x^2)\frac{6}{x} + 9(x^2)}{(x^2)x^2-9} = \frac{1-6x+9x^2}{x^2(x^2-9)} = \frac{(1-3x)^2}{(x^2)(x+3)(x-3)}$$

7.6

$$1) \frac{10}{a} = \frac{6}{8}$$

$$\frac{80}{6} = \frac{6a}{6}$$

$$13.3 = a$$

$$3) \frac{7}{6} = \frac{2}{k}$$

$$\frac{7k}{7} = \frac{12}{7}$$

$$k = 1.71$$

$$5) \frac{6}{x} = \frac{8}{2}$$

$$\frac{12}{8} = \frac{8x}{8}$$

$$1.5 = x$$

$$7) \frac{m-1}{5} = \frac{8}{2}$$

$$2(m-1) = 40$$

$$2m - 2 = 40$$

$$\begin{array}{r} + 2 \quad + 2 \\ \hline 2m = 42 \\ \frac{2m}{2} = \frac{42}{2} \\ m = 21 \end{array}$$

$$9) \frac{2}{9} = \frac{10}{p-4}$$

$$2(p-4) = 90$$

$$2p - 8 = 90$$

$$\begin{array}{r} + 8 \quad + 8 \\ \hline 2p = 98 \\ \frac{2p}{2} = \frac{98}{2} \\ p = 49 \end{array}$$

$$11) \frac{b-10}{7} = \frac{b}{4}$$

$$4(b-10) = 7b$$

$$4b - 40 = 7b$$

$$\begin{array}{r} -4b \quad - 4b \\ \hline -40 = 3b \\ \frac{-40}{3} = \frac{3b}{3} \\ -13.3 = b \end{array}$$

$$13) \frac{x}{5} = \frac{x+2}{9}$$

$$9x = 5(x+2)$$

$$9x = 5x + 10$$

$$\begin{array}{r} -5x \quad - 5x \\ \hline 4x = 10 \\ \frac{4x}{4} = \frac{10}{4} \\ x = 2.5 \end{array}$$

$$15) \frac{3}{10} = \frac{a}{a+2}$$

$$3(a+2) = 10a$$

$$3a + 6 = 10a$$

$$\begin{array}{r} -3a \quad - 3a \\ \hline 6 = 7a \\ \frac{6}{7} = \frac{7a}{7} \\ 0.86 = a \end{array}$$

$$17) \frac{v-5}{v+6} = \frac{4}{9}$$

$$9(v-5) = 4(v+6)$$

$$9v - 45 = 4v + 24$$

$$\begin{array}{r} -4v \quad - 4v \\ \hline 5v - 45 = 24 \\ + 45 \quad + 45 \\ \hline 5v = 69 \\ \frac{5v}{5} = \frac{69}{5} \\ v = 13.8 \end{array}$$

$$19) \frac{7}{x-1} = \frac{4}{x-6}$$

$$7(x-6) = 4(x-1)$$

$$7x - 42 = 4x - 4$$

$$\begin{array}{r} -4x \quad - 4x \\ \hline 3x - 42 = -4 \\ + 42 \quad + 42 \\ \hline 3x = 38 \\ \frac{3x}{3} = \frac{38}{3} \\ x = 12.67 \end{array}$$

$$\begin{aligned}
21) \quad \frac{x+5}{5} &= \frac{6}{x-2} \\
(x+5)(x-2) &= 30 \\
x^2 + 5x - 2x - 10 &= 30 \\
x^2 + 3x - 10 &= 30 \\
&\quad \underline{-30 \quad -30} \\
x^2 + 3x - 40 &= 0 \\
(x+8)(x-5) &= 0 \\
x+8=0 \quad x-5=0 \\
\underline{-8 \quad -8} \quad \underline{+5 \quad +5} \\
x = -8 \quad x = 5
\end{aligned}$$

$$\begin{aligned}
27) \quad \frac{n+4}{3} &= \frac{-3}{n-2} \\
(n+4)(n-2) &= -9 \\
n^2 - 2n + 4n - 8 &= -9 \\
n^2 + 2n - 8 &= -9 \\
&\quad \underline{+9 \quad +9} \\
n^2 + 2n + 1 &= 0 \\
(n+1)^2 &= 0 \\
n+1 &= 0 \\
\underline{-1 \quad -1} \\
n &= -1
\end{aligned}$$

$$\begin{aligned}
23) \quad \frac{m+3}{4} &= \frac{11}{m-4} \\
(m+3)(m-4) &= 44 \\
m^2 - 4m + 3m - 12 &= 44 \\
m^2 - m - 12 &= 44 \\
&\quad \underline{-44 \quad -44} \\
m^2 - m - 56 &= 0 \\
(m-8)(m+7) &= 0 \\
m-8=0 \quad m+7=0 \\
\underline{+8 \quad +8} \quad \underline{-7 \quad -7} \\
m = 8 \quad m = -7
\end{aligned}$$

$$\begin{aligned}
29) \quad \frac{3}{x+4} &= \frac{x+2}{5} \\
15 &= (x+4)(x+2) \\
15 &= x^2 + 2x + 4x + 8 \\
15 &= x^2 + 6x + 8 \\
\underline{-15 \quad \quad \quad -15} \\
0 &= x^2 + 6x - 7 \\
0 &= (x+7)(x-1) \\
x+7=0 \quad x-1=0 \\
\underline{-7 \quad -7} \quad \underline{+1 \quad +1} \\
x = -7 \quad x = 1
\end{aligned}$$

$$\begin{aligned}
25) \quad \frac{2}{p+4} &= \frac{p+5}{3} \\
6 &= (p+4)(p+5) \\
6 &= p^2 + 5p + 4p + 20 \\
6 &= p^2 + 9p + 20 \\
\underline{-6 \quad \quad \quad -6} \\
0 &= p^2 + 9p + 14 \\
0 &= (p+7)(p+2) \\
p+7=0 \quad p+2=0 \\
\underline{-7 \quad -7 \quad -2 \quad -2} \\
p &= -7, -2
\end{aligned}$$

31) The currency in Western Samoa is the Tala. The exchange rate is approximately \$0.70 to 1 Tala. At this rate, how many dollars would you get if you exchanged 13.3 Tala?

$$\begin{aligned}
\frac{T}{\$} &= \frac{1}{0.70} = \frac{13.3}{x} \\
x &= \$9.31
\end{aligned}$$

- 39) Kali reduced the size of a painting to a height of 1.3 in. What is the new width if it was originally 5.2 in. tall and 10 in. wide?

$$\frac{h}{w} = \frac{5.2}{10} = \frac{1.3}{x}$$

$$x = 2.5 \text{ in}$$

- 41) A bird bath that is 5.3 ft tall casts a shadow that is 25.4 ft long. Find the length of the shadow that a 8.2 ft adult elephant casts.

$$\frac{h}{s} = \frac{5.3}{25.4} = \frac{8.2}{x}$$

$$x = 39.3 \text{ ft}$$

- 43) The Vikings led the Timberwolves by 19 points at the half. If the Vikings scored 3 points for every 2 points the Timberwolves scored, what was the half time score?

$$\frac{V}{T} = \frac{(x+19)}{x} = \frac{3}{2}$$

$$2(x + 19) = 3x$$

$$2x + 38 = 3x$$

$$\begin{array}{r} -2x \qquad \qquad -2x \\ \hline 38 = x \end{array}$$

Timberwolves: 38

Vikings: 57

- 45) Computer Services Inc. charges \$8 more for a repair than Low Cost Computer Repair. If the ratio of the costs is 3 : 6, what will it cost for the repair at Low Cost Computer Repair?

$$\frac{CSI}{LCCR} = \frac{x+8}{x} = \frac{6}{3}$$

$$3(x + 8) = 6x$$

$$3x + 24 = 6x$$

$$\begin{array}{r} -3x \qquad \qquad -3x \\ \hline 24 = 3x \\ \frac{24}{3} = \frac{3x}{3} \\ \$8 = x \end{array}$$

7.7

$$1) (2x) 3x - (2x) \frac{1}{2} - (2x) \frac{1}{x} = 0(2x)$$

LCD: $2x$

$$\frac{2x}{2} \neq \frac{0}{2}$$

* $x \neq 0$ *

$$6x^2 - x - 2 = 0$$

$$(2x + 1)(3x - 2) = 0$$

$$2x + 1 = 0 \quad 3x - 2 = 0$$

$$\frac{-1 \quad -1}{2} \quad \frac{+2 \quad +2}{3}$$

$$\frac{2x}{2} = \frac{-1}{2} \quad \frac{3x}{3} = \frac{2}{3}$$

$$x = -\frac{1}{2} \quad x = \frac{2}{3}$$

$$3) x(x - 4) + \frac{20}{(x-4)} (x - 4) = \frac{5x}{(x-4)} (x - 4) - 2(x - 4)$$

LCD: $(x - 4)$

$$x - 4 \neq 0$$

$$\frac{+4 \quad +4}{x - 4}$$

* $x \neq 4$ *

$$x^2 - 4x + 20 = 5x - 2x + 8$$

$$x^2 - 4x + 20 = 3x + 8$$

$$\frac{-3x - 8 \quad -3x - 8}{x^2 - 7x + 12 = 0}$$

$$(x - 4)(x - 3) = 0$$

$$x - 4 = 0 \quad x - 3 = 0$$

$$\frac{+4 \quad +4}{x - 4} \quad \frac{+3 \quad +3}{x - 3}$$

$$\frac{\cancel{x - 4}}{x - 4} \quad x = 3$$

$$5) x(x - 3) + \frac{6}{(x-3)} (x - 3) = \frac{2x}{(x-3)} (x - 3)$$

LCD = $x - 3$

$$x - 3 \neq 0$$

$$\frac{+3 \quad +3}{x - 3}$$

* $x \neq 3$ *

$$x^2 - 3x + 6 = 2x$$

$$\frac{-2x \quad -2x}{x^2 - 5x + 6 = 0}$$

$$(x - 2)(x - 3) = 0$$

$$x - 2 = 0 \quad x - 3 = 0$$

$$\frac{+2 \quad +2}{x - 2} \quad \frac{+3 \quad +3}{x - 3}$$

$$x = 2 \quad \frac{\cancel{x - 3}}{x - 3}$$

$$7) \frac{2x}{3x-4} (6x-1)(3x-4) = \frac{4x+5}{6x-1} (6x-1)(3x-4) - \frac{3}{3x-4} (6x-1)(3x-4)$$

$$LCD : (6x-1)(3x-4)$$

$$6x-1 \neq 0 \quad 3x-4 \neq 0$$

$$\frac{+1}{6x} \frac{+1}{6} \quad \frac{+4}{3x} \frac{+4}{3}$$

$$\frac{6x}{6} \neq \frac{1}{6} \quad \frac{3x}{3} \neq \frac{4}{3}$$

$$* x \neq \frac{1}{6} * * x \neq \frac{4}{3} *$$

$$12x^2 - 2x = 12x^2 - 16x + 15x - 20 - 18x + 3$$

$$12x^2 - 2x = 12x^2 - 19x - 17$$

$$\underline{-12x^2} \quad \underline{-12x^2}$$

$$-2x = -19x - 17$$

$$\underline{+19x} \quad \underline{+19x}$$

$$\frac{17x}{17} = -\frac{17}{17}$$

$$x = -1$$

$$9) \frac{3m}{2m-5} (2)(2m-5)(3m+1) - \frac{7}{3m+1} (2)(2m-5)(3m+1) = \frac{3}{2} (2)(2m-5)(3m+1)$$

$$LCD: (2)(2m-5)(3m+1)$$

$$2m-5 \neq 0 \quad 3m+1 \neq 0$$

$$\frac{+5}{2m} \frac{+5}{2} \quad \frac{-1}{3} \frac{-1}{3}$$

$$\frac{2m}{2} \neq \frac{5}{2} \quad \frac{3m}{3} \neq -\frac{1}{3}$$

$$* x \neq \frac{5}{2} * * x \neq -\frac{1}{3} *$$

$$18m^2 + 6m - 28m + 70 = 18m^2 - 39m - 15$$

$$18m^2 - 22m + 70 = 18m^2 - 39m - 15$$

$$\underline{-18m^2} \quad \underline{-18m^2}$$

$$-22m + 70 = -39m - 15$$

$$\underline{+39m} \quad \underline{+39m}$$

$$17m + 70 = -15$$

$$\underline{-70} \quad \underline{-70}$$

$$\frac{17m}{17} = \frac{-85}{17}$$

$$m = -5$$

$$11) \frac{4-x}{1-x} (1-x)(3-x) = \frac{12}{3-x} (1-x)(3-x)$$

$$LCD : (1-x)(3-x)$$

$$1-x \neq 0 \quad 3-x \neq 0$$

$$\frac{+x}{1-x} \frac{+x}{3-x} \frac{+x}{3-x} \frac{+x}{3-x}$$

$$* 1 \neq x * * 3 \neq x *$$

$$12 - 4x - 3x + x^2 = 12 - 12x$$

$$x^2 - 7x + 12 = 12 - 12x$$

$$\frac{+12x - 12}{x^2 - 7x + 12} \quad \frac{-12 + 12x}{x^2 - 7x + 12}$$

$$x^2 + 5x = 0$$

$$x(x+5) = 0$$

$$x = 0 \quad x + 5 = 0$$

$$\frac{-5}{x} = \frac{-5}{x}$$

$$x = -5$$

$$13) \frac{7}{y-3} (2)(y-3)(y-4) - \frac{1}{2} (2)(y-3)(y-4) = \frac{y-2}{y-4} (2)(y-3)(y-4)$$

$$LCD : (2)(y-3)(y-4)$$

$$y-3 \neq 0 \quad y-4 \neq 0$$

$$\frac{+3}{y-3} \frac{+3}{y-3} \frac{+4}{y-4} \frac{+4}{y-4}$$

$$*y \neq 3 * *y \neq 4 *$$

$$14y - 56 - y^2 + 3y + 4y - 12 = 2y^2 - 6y - 4y + 12$$

$$-y^2 + 21y - 68 = 2y^2 - 10y + 12$$

$$\frac{+y^2 - 21y + 68}{-y^2 + 21y - 68} \quad \frac{+y^2 - 21y + 68}{-y^2 + 21y - 68}$$

$$0 = 3y^2 - 31y + 80$$

$$0 = (3y - 16)(y - 5)$$

$$3y - 16 = 0 \quad y - 5 = 0$$

$$\frac{+16}{3y-16} \frac{+16}{3y-16} \quad \frac{+5}{y-5} \frac{+5}{y-5}$$

$$\frac{3y}{3} = \frac{16}{3} \quad y = 5$$

$$15) \frac{1}{x+2} (x+2)(x-2) + \frac{1}{2-x} (x+2)(x-2) = \frac{3x+8}{x^2-4} (x+2)(x-2)$$

$$LCD : (x+2)(x-2)$$

$$x+2 \neq 0 \quad x-2 \neq 0$$

$$\frac{-2}{x+2} \frac{-2}{x+2} \quad \frac{+2}{x-2} \frac{+2}{x-2}$$

$$x \neq -2 \quad x \neq 2$$

$$x - 2 + x + 2 = 3x + 8$$

$$2x = 3x + 8$$

$$\frac{-3x}{2x} \quad \frac{-3x}{2x}$$

$$-\frac{x}{-1} = \frac{8}{-1}$$

$$x = -8$$

$$17) \frac{(x+1)}{x-1} (6)(x-1)(x+1) + \frac{-x+1}{x+1} (6)(x-1)(x+1) = \frac{5}{6} (6)(x-1)(x+1)$$

$$LCD: (6)(x-1)(x+1)$$

$$x-1 \neq 0 \quad x+1 \neq 0$$

$$\frac{+1 \quad +1 \quad -1 \quad -1}{* x \neq 1 \quad * x \neq -1 *$$

$$6x^2 + 6x + 6x + 6 - 6x^2 + 6x + 6x - 6 = 5x^2 - 5$$

$$24x = 5x^2 - 5$$

$$\frac{-24x \quad -24x}{0 = 5x^2 - 24x - 5}$$

$$0 = 5x^2 - 24x - 5$$

$$0 = (5x+1)(x-5)$$

$$5x+1=0 \quad x-5=0$$

$$\frac{-1 \quad -1}{\frac{5x}{5} = \frac{-1}{5}} \quad \frac{+5 \quad +5}{x=5}$$

$$\frac{5x}{5} = \frac{-1}{5}$$

$$x = -\frac{1}{5}$$

$$19) \frac{3}{2x+1} \frac{(2x+1)(2x-1)}{1} + \frac{-2x-1}{2x-1} \frac{(2x+1)(2x-1)}{1} = 1 \frac{(2x+1)(2x-1)}{1} - \frac{8x^2}{4x^2-1} \frac{(2x+1)(2x-1)}{1}$$

$$LCD: (2x+1)(2x-1)$$

$$2x+1 \neq 0 \quad 2x-1 \neq 0$$

$$\frac{-1 \quad -1}{\frac{2x}{2} \neq \frac{-1}{2}} \quad \frac{+1 \quad +1}{\frac{2x}{2} \neq \frac{1}{2}}$$

$$\frac{2x}{2} \neq \frac{-1}{2} \quad \frac{2x}{2} \neq \frac{1}{2}$$

$$* x \neq -\frac{1}{2} * \quad * x \neq \frac{1}{2} *$$

$$6x - 3 - 4x^2 - 2x - 2x - 1 = 4x^2 - 1 - 8x^2$$

$$-4x^2 + 2x - 4 = -4x^2 - 1$$

$$\frac{+4x^2}{2x-4=-1} \quad \frac{+4x^2}{+4 \quad +4}$$

$$2x-4=-1$$

$$\frac{+4 \quad +4}{\frac{2x}{2} = \frac{3}{2}}$$

$$\frac{2x}{2} = \frac{3}{2}$$

$$x = \frac{3}{2}$$

$$21) \frac{x-2}{x+3} (x+3)(x-2) - \frac{1}{x-2} (x+3)(x-2) = \frac{1}{x^2+x-6} (x+3)(x-2)$$

$$(x-2)(x+3)$$

$$LCD : (x+3)(x-2)$$

$$x+3 \neq 0 \quad x-2 \neq 0$$

$$\frac{-3}{-3} \quad \frac{-3}{-3} \quad \frac{+2}{+2} \quad \frac{+2}{+2}$$

$$* x \neq -3 \quad * x \neq 2 *$$

$$x^2 - 4x + 4 - x - 3 = 1$$

$$x^2 - 5x + 1 = 1$$

$$\frac{-1}{-1} \quad \frac{-1}{-1}$$

$$x^2 - 5x = 0$$

$$x(x-5) = 0$$

$$x = 0 \quad x - 5 = 0$$

$$\frac{+5}{+5} \quad \frac{+5}{+5}$$

$$x = 5$$

$$23) \frac{3}{x+2} + \frac{x-1}{x+5} = \frac{5(x+4)}{6(x+4)}$$

$$\frac{5x+20}{6x+24}$$

$$\frac{3}{x+2} (6)(x+2)(x+5) + \frac{x-1}{x+5} (6)(x+2)(x+5) = \frac{5}{6} (6)(x+2)(x+5)$$

$$LCD : (6)(x+2)(x+5)$$

$$x+2 \neq 0 \quad x+5 \neq 0$$

$$\frac{-2}{-2} \quad \frac{-2}{-2} \quad \frac{-5}{-5} \quad \frac{-5}{-5}$$

$$* x \neq -2 \quad * x \neq -5 *$$

$$18x + 90 + 6x^2 + 12x - 6x - 12 = 5x^2 + 25x + 10x + 50$$

$$6x^2 + 24x + 78 = 5x^2 + 35x + 50$$

$$\frac{-5x^2 - 35x - 50}{-5x^2 - 35x - 50}$$

$$x^2 - 11x + 28 = 0$$

$$(x-7)(x-4) = 0$$

$$x-7 = 0 \quad x-4 = 0$$

$$\frac{+7}{+7} \quad \frac{+7}{+7} \quad \frac{+4}{+4} \quad \frac{+4}{+4}$$

$$x = 7 \quad x = 4$$

$$25) \frac{x}{x-1} (x+1)(x-1) - \frac{2}{x+1} (x+1)(x-1) = \frac{4x^2}{x^2-1} (x+1)(x-1)$$

$$(x+1)(x-1)$$

$$LCD : (x+1)(x-1)$$

$$x+1 \neq 0 \quad x-1 \neq 0$$

$$\frac{-1 \quad -1 \quad +1 \quad +1}{* x \neq -1 * \quad * x \neq 1 *}$$

$$x^2 + x - 2x + 2 = 4x^2$$

$$x^2 - x + 2 = 4x^2$$

$$\frac{-x^2 + x - 2 \quad -x^2 + x - 2}{0 = 3x^2 + x - x}$$

$$0 = 3x^2 + x - x$$

$$0 = (3x-2)(x+1)$$

$$3x-2=0 \quad x+1=0$$

$$\frac{+2 \quad +2}{\frac{3x}{3} = \frac{2}{3}} \quad \frac{-1 \quad -1}{x = -1}$$

$$x = \frac{2}{3}$$

$$x = \frac{2}{3}$$

$$27) \frac{2x}{x+1} (x+1)(x+5) - \frac{3}{x+5} (x+1)(x+5) = \frac{-8x^2}{x^2+6x+5} (x+1)(x+5)$$

$$(x+1)(x+5)$$

$$LCD : (x+1)(x+5)$$

$$x+1 \neq 0 \quad x+5 \neq 0$$

$$\frac{-1 \quad -1 \quad -5 \quad -5}{* x \neq -1 * \quad * x \neq -5 *}$$

$$2x^2 + 10x - 3x - 3 = -8x^2$$

$$2x^2 + 7x - 3 = -8x^2$$

$$\frac{+8x^2 \quad +8x^2}{10x^2 + 7x - 3 = 0}$$

$$10x^2 + 7x - 3 = 0$$

$$(10x-3)(x+1) = 0$$

$$10x-3=0 \quad x+1=0$$

$$\frac{+3 \quad +3}{\frac{10x}{10} = \frac{3}{10}} \quad \frac{-1 \quad -1}{x = -1}$$

$$x = \frac{3}{10}$$

$$x = \frac{3}{10}$$

$$29) \frac{x-5}{x-9} (x-9)(x-3) + \frac{x+3}{x-3} (x-9)(x-3) = \frac{-4x^2}{x^2-12x+27} (x-9)(x-3)$$

$$LCD : (x-9)(x-3)$$

$$x-9 \neq 0 \quad x-3 \neq 0$$

$$\frac{+9}{+9} \frac{+9}{+9} \frac{+3}{+3} \frac{+3}{+3}$$

$$*x \neq 9 * *x \neq 3 *$$

$$x^2 - 3x - 5x + 15 + x^2 - 9x + 3x - 27 = -4x^2$$

$$2x^2 - 14x - 12 = -4x^2$$

$$\frac{+4x^2}{+4x^2} \quad \frac{+4x^2}{+4x^2}$$

$$6x^2 - 14x - 12 = 0$$

$$2(3x^2 - 7x - 6) = 0$$

$$2(3x+2)(x-3) = 0$$

$$3x+2=0 \quad x-3=0$$

$$\frac{-2}{-2} \frac{-2}{-2} \frac{+3}{+3} \frac{+3}{+3}$$

$$\frac{3x}{3} = \frac{-2}{3} \quad x = 3$$

$$x = -\frac{2}{3}$$

$$31) \frac{x-3}{x-6} (x-6)(x+3) + \frac{x+5}{x+3} (x-6)(x+3) = \frac{-2x^2}{x^2-3x-18} (x-6)(x+3)$$

$$LCD : (x-6)(x+3)$$

$$x-6 \neq 0 \quad x+3 \neq 0$$

$$\frac{+6}{+6} \frac{+6}{+6} \frac{-3}{-3} \frac{-3}{-3}$$

$$*x \neq 6 * \quad *x \neq -3 *$$

$$x^2 - 9 + x^2 - 6x + 5x - 30 = -2x^2$$

$$2x^2 - x - 90 = -2x^2$$

$$\frac{+2x^2}{+2x^2} \quad \frac{+2x^2}{+2x^2}$$

$$4x^2 - x - 90 = 0$$

$$(4x-13)(x+3) = 0$$

$$4x-13=0 \quad x+3=0$$

$$\frac{+13}{+13} \frac{+13}{+13} \frac{-3}{-3} \frac{-3}{-3}$$

$$\frac{4x}{4} = \frac{13}{4} \quad x = -3$$

$$x = \frac{13}{4}$$

$$33) \frac{4x+1}{x+3} (x+3)(x-1) + \frac{5x-3}{x-1} (x+3)(x-1) = \frac{8x^2}{x^2+2x-3} (x+3)(x-1)$$

$$(x+3)(x-1)$$

$$LCD : (x+3)(x-1)$$

$$x+3 \neq 0 \quad x-1 \neq 0$$

$$\frac{-3 \quad -3 \quad +1 \quad +1}{* x \neq -3 \quad ** x \neq 1 *$$

$$4x^2 - 4x + x - 1 + 5x^2 + 15x - 3x - 9 = 8x^2$$

$$9x^2 + 9x - 10 = 8x^2$$

$$\frac{-8x^2}{x^2 + 9x - 10} = \frac{-8x^2}{(x+10)(x-1)}$$

$$x^2 + 9x - 10 = 0$$

$$(x+10)(x-1) = 0$$

$$x+10 = 0 \quad x-1 = 0$$

$$\frac{-10 \quad -10 \quad +1 \quad +1}{x = -10 \quad \cancel{x = 1}}$$

7.8

1) 7 mi to yd

$$\left(\frac{7mi}{1}\right) \left(\frac{5280ft}{1mi}\right) \left(\frac{1yd}{3ft}\right) = \frac{36960yd}{3} = 12,320 yd$$

3) 11.2 mg to g

$$\left(\frac{11.2mg}{1}\right) \left(\frac{1g}{1000mg}\right) = \frac{11.29}{1000} = 0.0112 g$$

5) 9,800,000 mm to mi

$$\left(\frac{9,800,000mm}{1}\right) \left(\frac{1m}{1000mm}\right) \left(\frac{3.29ft}{1m}\right) \left(\frac{1mi}{5280ft}\right) = \frac{32,144,000mi}{5280000} = 6.088 mi$$

7) 435,000 m² to km²

$$\left(\frac{435,000m^2}{1}\right) \left(\frac{1km}{1,000m}\right)^2$$

$$\left(\frac{435,000m^2}{1}\right) \left(\frac{1km^2}{1,000,000m^2}\right) = \frac{435,000 km^2}{1,000,000} = 0.435 km^2$$

9) 0.0065 km³ to m³

$$\left(\frac{0.0065km^3}{1}\right) \left(\frac{1000m}{km}\right)^3$$

$$\left(\frac{0.0065km^3}{1}\right) \left(\frac{1,000,000,000m^3}{km^3}\right) = 6,500,000 m^3$$

11) 5,500 cm³ to yd³

$$\left(\frac{5,500cm^3}{1}\right) \left(\frac{1in}{2.54cm}\right)^3 \left(\frac{1yd}{36in}\right)^3$$

$$\left(\frac{5,500cm^3}{1}\right) \left(\frac{1in^3}{16.387064cm^3}\right) \left(\frac{1yd^3}{46656in^3}\right) = \frac{5,500 yd^3}{764554.858} = 0.00719 yd^3$$

13) $185 \text{ yd}/\text{min}$ to mi/hr

$$\left(\frac{185\cancel{\text{yd}}}{\cancel{\text{min}}}\right)\left(\frac{3\cancel{\text{ft}}}{1\cancel{\text{yd}}}\right)\left(\frac{1\cancel{\text{mi}}}{5280\cancel{\text{ft}}}\right)\left(\frac{60\cancel{\text{min}}}{1\cancel{\text{hr}}}\right) = \frac{33300\cancel{\text{mi}}}{5280\cancel{\text{hr}}} = 6.307 \text{ mi}/\text{hr}$$

15) $248 \text{ mi}/\text{hr}$ to m/sec

$$\left(\frac{248\cancel{\text{mi}}}{\cancel{\text{hr}}}\right)\left(\frac{1.61\cancel{\text{km}}}{1\cancel{\text{mi}}}\right)\left(\frac{1000\cancel{\text{m}}}{1\cancel{\text{km}}}\right)\left(\frac{1\cancel{\text{hr}}}{3600\cancel{\text{sec}}}\right) = \frac{399.280\cancel{\text{m}}}{3600\cancel{\text{sec}}} = 110.9 \text{ m}/\text{sec}$$

17) $7.5 \frac{\text{T}}{\text{yd}^2}$ to lbs/in^2

$$\left(\frac{7.5\cancel{\text{T}}}{\cancel{\text{yd}^2}}\right)\left(\frac{2000\cancel{\text{lbs}}}{1\cancel{\text{T}}}\right)\left(\frac{1\cancel{\text{yd}}}{36\cancel{\text{in}}}\right)^2$$

$$\left(\frac{7.5\cancel{\text{T}}}{\cancel{\text{yd}^2}}\right)\left(\frac{2000\cancel{\text{lbs}}}{1\cancel{\text{T}}}\right)\left(\frac{1\cancel{\text{yd}}^2}{1296\cancel{\text{in}}^2}\right) = \frac{15000\cancel{\text{lbs}}}{1296\cancel{\text{in}}^2} = 11.57 \text{ lbs}/\text{in}^2$$

19) On a recent trip, Jan traveled 260 miles using 8 gallons of gas. How many miles per 1-gallon did she travel? How many yards per 1-ounce?

$$\frac{260\cancel{\text{mi}}}{8\cancel{\text{gal}}} = 32.5 \text{ mi}/\text{gal}$$

$$\left(\frac{32.5\cancel{\text{mi}}}{\cancel{\text{gal}}}\right)\left(\frac{5280\cancel{\text{ft}}}{1\cancel{\text{mi}}}\right)\left(\frac{1\cancel{\text{yd}}}{3\cancel{\text{ft}}}\right)\left(\frac{1\cancel{\text{gal}}}{4\cancel{\text{qt}}}\right)\left(\frac{1\cancel{\text{qt}}}{2\cancel{\text{pt}}}\right)\left(\frac{1\cancel{\text{pt}}}{2\cancel{\text{sc}}}\right)\left(\frac{1\cancel{\text{sc}}}{8\cancel{\text{oz}}}\right) = \frac{171,600\cancel{\text{yd}}}{384\cancel{\text{oz}}} = 446.875 \text{ yd}/\text{oz}$$

21) A certain laser printer can print 12 pages per minute. Determine this printer's output in pages per day, and reams per month. (1 ream = 5000 pages)

$$\left(\frac{12\cancel{\text{pg}}}{1\cancel{\text{min}}}\right)\left(\frac{60\cancel{\text{min}}}{1\cancel{\text{hr}}}\right)\left(\frac{24\cancel{\text{hr}}}{1\cancel{\text{day}}}\right) = 17280 \text{ pg}/\text{day}$$

$$\left(\frac{17280\cancel{\text{pg}}}{\cancel{\text{day}}}\right)\left(\frac{30\cancel{\text{day}}}{\cancel{\text{mon}}}\right)\left(\frac{1\cancel{\text{ream}}}{5000\cancel{\text{pg}}}\right) = \frac{5184000\cancel{\text{reams}}}{5000\cancel{\text{months}}} = 103.68 \text{ reams}/\text{month}$$

23) Blood sugar levels are measured in milligrams of glucose per deciliter of blood volume. If a person's blood sugar level measured 128 mg/dL, how much is this in grams per liter?

$$\left(\frac{128\cancel{\text{mg}}}{\cancel{\text{dL}}}\right)\left(\frac{1\cancel{\text{g}}}{100\cancel{\text{mg}}}\right)\left(\frac{10\cancel{\text{dL}}}{1\cancel{\text{L}}}\right) = \frac{1280\cancel{\text{g}}}{1000\cancel{\text{L}}} = 1.28 \text{ g}/\text{L}$$

25) A car travels 14 miles in 15 minutes. How fast is it going in miles per hour? in meters per second?

$$\left(\frac{14\cancel{\text{mi}}}{15\cancel{\text{min}}}\right)\left(\frac{60\cancel{\text{min}}}{1\cancel{\text{hr}}}\right) = \frac{840\cancel{\text{mi}}}{15\cancel{\text{hr}}} = 56 \text{ mi}/\text{hr}$$

$$\left(\frac{14\cancel{\text{mi}}}{15\cancel{\text{min}}}\right)\left(\frac{1.61\cancel{\text{km}}}{1\cancel{\text{mi}}}\right)\left(\frac{1000\cancel{\text{m}}}{1\cancel{\text{km}}}\right)\left(\frac{1\cancel{\text{min}}}{60\cancel{\text{sec}}}\right) = \frac{22540\cancel{\text{m}}}{900\cancel{\text{sec}}} = 25.04 \text{ m}/\text{sec}$$

- 27) A local zoning ordinance says that a house's "footprint" (area of its ground floor) cannot occupy more than $\frac{1}{4}$ of the lot it is built on. Suppose you own a $\frac{1}{3}$ acre lot, what is the maximum allowed footprint for your house in square feet? in square inches? (1 acre = 43560 ft^2)

$$\left(\frac{1\cancel{acre}}{3}\right)\left(\frac{43560\cancel{ft^2}}{1\cancel{acre}}\right)\left(\frac{1}{4}\right) = \frac{43560ft^2}{12} = 3,630 ft^2$$

$$\left(\frac{3630ft^2}{1}\right)\left(\frac{12in}{1ft}\right)^2$$

$$\left(\frac{3630\cancel{ft^2}}{1}\right)\left(\frac{144\cancel{in^2}}{1\cancel{ft^2}}\right) = 522,720 in^2$$

- 29) In April 1996, the Department of the Interior released a "spike flood" from the Glen Canyon Dam on the Colorado River. Its purpose was to restore the river and the habitants along its bank. The release from the dam lasted a week at a rate of 25,800 cubic feet of water per second. About how much water was released during the 1-week flood

$$\left(\frac{25,800ft^3}{1\cancel{sec}}\right)\left(\frac{3600\cancel{sec}}{1\cancel{hr}}\right)\left(\frac{24\cancel{hr}}{1\cancel{day}}\right)\left(\frac{7\cancel{day}}{1\cancel{wk}}\right) = 15,603,840,000 ft^3/week$$

Chapter 8: Radicals

8.1

$$1) \frac{r\sqrt{245}}{\sqrt{5 \cdot 7^2}} \\ 7\sqrt{5}$$

$$4) \frac{\sqrt{36}}{\sqrt{2^2 \cdot 3^2}} \\ \frac{2 \cdot 3}{6}$$

$$5) \frac{\sqrt{12}}{\sqrt{2^2 \cdot 3}} \\ 2\sqrt{3}$$

$$7) \frac{3\sqrt{12}}{3\sqrt{2^2 \cdot 3}} \\ \frac{3 \cdot 2\sqrt{3}}{6\sqrt{3}}$$

$$9) \frac{6\sqrt{128}}{6\sqrt{2^7}} \\ \frac{6 \cdot 2^3 \sqrt{2}}{6 \cdot 8\sqrt{2}} \\ 48\sqrt{2}$$

$$11) \frac{-8\sqrt{392}}{-8\sqrt{7^2 \cdot 2^3}} \\ \frac{-8 \cdot 7 \cdot 2\sqrt{2}}{-112\sqrt{2}}$$

$$13) \frac{\sqrt{192n}}{\sqrt{2^6 \cdot 3n}} \\ \frac{2^3 \sqrt{3n}}{8\sqrt{3n}}$$

$$15) \frac{\sqrt{196v^2}}{\sqrt{2^2 \cdot 7^2 \cdot v^2}} \\ \frac{2 \cdot 7 \cdot v}{14v}$$

$$17) \frac{\sqrt{252x^2}}{\sqrt{2^2 \cdot 3^2 \cdot 7x^2}} \\ \frac{2 \cdot 3x\sqrt{7}}{6x\sqrt{7}}$$

$$19) \frac{-\sqrt{100k^4}}{-\sqrt{2^2 \cdot 5^2 k^4}} \\ \frac{-2 \cdot 5 \cdot k^2}{-10k^2}$$

$$21) \frac{-7\sqrt{64x^4}}{-7\sqrt{2^6 x^4}} \\ \frac{-7 \cdot 2^3 x^2}{-56x^2}$$

$$23) \frac{-5\sqrt{36m}}{-5\sqrt{2^2 \cdot 3^2 m}} \\ \frac{-5 \cdot 2 \cdot 3\sqrt{m}}{-30\sqrt{m}}$$

$$25) \frac{\sqrt{45x^2 y^2}}{\sqrt{3^2 \cdot 5 \cdot x^2 y^2}} \\ 3xy\sqrt{5}$$

$$27) \frac{\sqrt{16x^3 y^3}}{\sqrt{2^4 x^3 y^3}} \\ \frac{2^2 xy \sqrt{xy}}{4xy \sqrt{xy}}$$

$$29) \frac{\sqrt{320x^4 y^4}}{\sqrt{2^6 \cdot 5x^4 y^4}} \\ \frac{2^3 x^2 y^2 \sqrt{5}}{8x^2 y^2 \sqrt{5}}$$

$$31) \frac{6\sqrt{80xy^2}}{6\sqrt{2^4 \cdot 5xy^2}} \\ \frac{6 \cdot 2^2 y \sqrt{5x}}{6 \cdot 4y \sqrt{5x}} \\ 24y\sqrt{5x}$$

$$33) \frac{5\sqrt{245x^2 y^3}}{5\sqrt{5 \cdot 7^2 x^2 y^3}} \\ \frac{5 \cdot 7xy \sqrt{5y}}{35xy \sqrt{5y}}$$

$$35) \frac{-2\sqrt{180u^3 v}}{-2\sqrt{2^2 \cdot 3^2 \cdot 5u^3 v}} \\ \frac{-2 \cdot 2 \cdot 3u \sqrt{5uv}}{-12u \sqrt{5uv}}$$

$$37) \frac{-8\sqrt{180x^4 y^2 z^4}}{-8\sqrt{2^2 \cdot 3^2 \cdot 5x^4 y^2 z^4}} \\ \frac{-8 \cdot 2 \cdot 3x^2 yz^2 \sqrt{5}}{-48x^2 yz^2 \sqrt{5}}$$

$$39) \frac{2\sqrt{80hj^4 k}}{2\sqrt{2^4 \cdot 5hj^4 k}} \\ \frac{2 \cdot 2^2 j^2 \sqrt{5hk}}{2 \cdot 4j^2 \sqrt{5hk}} \\ 8j^2 \sqrt{5hk}$$

$$41) \frac{-4\sqrt{54mnp^2}}{-4\sqrt{2 \cdot 3^2 mnp^2}} \\ \frac{-4 \cdot 3p \sqrt{2 \cdot 3mn}}{-12p \sqrt{60n}}$$

8.2

$$1) \frac{\sqrt[3]{625}}{\sqrt[3]{5^4}} \\ 5^3\sqrt[3]{5}$$

$$3) \frac{r^3\sqrt[3]{750}}{\sqrt[3]{2 \cdot 3 \cdot 5^3}} \\ 5^3\sqrt[3]{2 \cdot 3} \\ 5^3\sqrt[3]{6}$$

$$5) \frac{\sqrt[3]{875}}{\sqrt[3]{5^3 \cdot 7}} \\ 5^3\sqrt[3]{7}$$

$$7) \frac{-4^4\sqrt[4]{96}}{-4^4\sqrt[4]{2^5 \cdot 3}} \\ -4 \cdot 2^4\sqrt[4]{2 \cdot 3} \\ -8^4\sqrt[4]{6}$$

$$9) \frac{6^4\sqrt[4]{112}}{6^4\sqrt[4]{2^4 \cdot 7}} \\ 6 \cdot 2^4\sqrt[4]{7} \\ 12^4\sqrt[4]{7}$$

$$11) \frac{-\sqrt[4]{112}}{-\sqrt[4]{2^4 \cdot 7}} \\ -2^4\sqrt[4]{7}$$

$$13) \frac{r^4\sqrt[4]{648a^2}}{\sqrt[4]{2^3 \cdot 3^4 a^2}} \\ 3^4\sqrt[4]{2^3 a^2} \\ 3^4\sqrt[4]{8a^2}$$

$$15) \frac{\sqrt[5]{224n^3}}{\sqrt[5]{2^5 \cdot 7n^3}} \\ 2^5\sqrt[5]{7n^3}$$

$$17) \frac{r^5\sqrt[5]{224p^5}}{\sqrt[5]{2^5 \cdot 7p^5}} \\ 2p^5\sqrt[5]{7}$$

$$19) \frac{-3^7\sqrt[7]{896r}}{-3^7\sqrt[7]{2^7 \cdot 7r}} \\ -3 \cdot 2^7\sqrt[7]{7r} \\ -6^7\sqrt[7]{7r}$$

$$21) \frac{r - 2^3\sqrt[3]{-48v^7}}{-2^3\sqrt[3]{-1^3 \cdot 2^4 \cdot 3v^7}} \\ -2 \cdot -1 \cdot 2v^2\sqrt[3]{2 \cdot 3v} \\ 4v^2\sqrt[3]{6v}$$

$$23) \frac{-7^3\sqrt[3]{320n^6}}{-7^3\sqrt[3]{2^6 \cdot 5n^6}} \\ -7 \cdot 2^2n^2\sqrt[3]{5} \\ -7 \cdot 4n^2\sqrt[3]{5} \\ -28n^2\sqrt[3]{5}$$

$$25) \frac{\sqrt[3]{-135x^5y^3}}{\sqrt[3]{-1^3 \cdot 3^3 \cdot 5x^5y^3}} \\ -1 \cdot 3xy^3\sqrt[3]{5x^2} \\ -3xy^3\sqrt[3]{5x^2}$$

$$27) \frac{\sqrt[3]{-32x^4y^4}}{\sqrt[3]{-1^3 \cdot 2^5x^4y^4}} \\ -1 \cdot 2xy^3\sqrt[3]{2^2xy} \\ -2xy^3\sqrt[3]{4xy}$$

$$29) \frac{r^3\sqrt[3]{256x^4y^6}}{\sqrt[3]{2^8x^4y^6}} \\ 2^2xy^2\sqrt[3]{2^2x} \\ 4xy^2\sqrt[3]{4x}$$

$$31) \frac{7^3\sqrt[3]{-81x^3y^7}}{7^3\sqrt[3]{-1^3 \cdot 3^4x^3y^7}} \\ 7 \cdot -1 \cdot 3xy^2\sqrt[3]{3y} \\ -21xy^2\sqrt[3]{3y}$$

$$33) \frac{2^3\sqrt[3]{375u^2v^8}}{2^3\sqrt[3]{3 \cdot 5^3u^2v^8}} \\ 2 \cdot 5v^2\sqrt[3]{3u^2v^2} \\ 10v^2\sqrt[3]{3u^2v^2}$$

$$35) \frac{-3^3\sqrt[3]{192ab^2}}{-3^3\sqrt[3]{2^6 \cdot 3ab^2}} \\ -3 \cdot 2^2\sqrt[3]{3ab^2} \\ -3 \cdot 4\sqrt[3]{3ab^2} \\ -12\sqrt[3]{3ab^2}$$

$$37) \frac{6^3\sqrt[3]{-54m^8n^3p^7}}{6^3\sqrt[3]{-1^3 \cdot 2 \cdot 3^3m^8n^3p^7}} \\ 6 \cdot -1 \cdot \\ 3m^2np^2\sqrt[3]{2m^2p} \\ -18m^2np^2\sqrt[3]{2m^2p}$$

$$39) \frac{6^4\sqrt[4]{648x^5y^7z^2}}{6^4\sqrt[4]{2^3 \cdot 3^4x^5y^7z^2}} \\ 6 \cdot 3xy^4\sqrt[4]{2^3xy^3z^2} \\ 18xy^4\sqrt[4]{8xy^3z^2}$$

$$41) \frac{7^4\sqrt[4]{128h^6j^8k^8}}{7^4\sqrt[4]{2^7h^6j^8k^8}} \\ 7 \cdot 2hj^2k^2\sqrt[4]{2^3h^2} \\ 14hj^2k^2\sqrt[4]{8h^2}$$

8.3

- 1) $2\sqrt{5} + 2\sqrt{5} + 2\sqrt{5}$
 $6\sqrt{5}$
- 3) $-3\sqrt{2} + 3\sqrt{5} + 3\sqrt{5}$
 $-3\sqrt{2} + 6\sqrt{5}$
- 6) $-2\sqrt{6} - 2\sqrt{6} - \sqrt{6}$
 $-5\sqrt{6}$
- 8) $3\sqrt{6} + 3\sqrt{5} + 2\sqrt{5}$
 $3\sqrt{6} + 5\sqrt{5}$
- 10) $2\sqrt{2} - 3\sqrt{18} - \sqrt{2}$
 $2\sqrt{2} - 3\sqrt{3^2 \cdot 2} - \sqrt{2}$
 $2\sqrt{2} - 3 \cdot 3\sqrt{2} - \sqrt{2}$
 $2\sqrt{2} - 9\sqrt{2} - \sqrt{2}$
 $-8\sqrt{2}$
- 12) $-3\sqrt{6} - \sqrt{12} + 3\sqrt{3}$
 $-3\sqrt{6} - \sqrt{2^2 \cdot 3} + 3\sqrt{3}$
 $-3\sqrt{6} - 2\sqrt{3} + 3\sqrt{3}$
 $-3\sqrt{6} + \sqrt{3}$
- 14) $3\sqrt{2} + 2\sqrt{8} - 3\sqrt{18}$
 $3\sqrt{2} + 2\sqrt{2^3} - 3\sqrt{2 \cdot 3^2}$
 $3\sqrt{2} + 2 \cdot 2\sqrt{2} - 3 \cdot 3\sqrt{2}$
 $3\sqrt{2} + 4\sqrt{2} - 9\sqrt{2}$
 $-2\sqrt{2}$
- 16) $3\sqrt{18} - \sqrt{2} - 3\sqrt{2}$
 $3\sqrt{2 \cdot 3^2} - \sqrt{2} - 3\sqrt{2}$
 $3 \cdot 3\sqrt{2} - \sqrt{2} - 3\sqrt{2}$
 $9\sqrt{2} - 2\sqrt{2} - 3\sqrt{2}$
 $5\sqrt{2}$
- 19) $-3\sqrt{6} - 3\sqrt{6} - \sqrt{3} + 3\sqrt{6}$
 $-3\sqrt{6} - \sqrt{3}$
- 20) $-2\sqrt{18} - 3\sqrt{8} - \sqrt{20} + 2\sqrt{20}$
 $-2\sqrt{2 \cdot 3^2} - 3\sqrt{2^3} - \sqrt{2^2 \cdot 5} + 2\sqrt{2^2 \cdot 5}$
 $-2 \cdot 3\sqrt{2} - 3 \cdot 2\sqrt{2} - 2\sqrt{5} + 2 \cdot 2\sqrt{5}$
 $-6\sqrt{2} - 6\sqrt{2} - 2\sqrt{5} + 4\sqrt{5}$
 $-12\sqrt{2} - 2\sqrt{5}$
- 21) $-2\sqrt{24} - 2\sqrt{6} + 2\sqrt{6} + 2\sqrt{20}$
 $-2\sqrt{2^3 \cdot 3} - 2\sqrt{6} + 2\sqrt{6} + 2\sqrt{2^2 \cdot 5}$
 $-2 \cdot 2\sqrt{2 \cdot 3} - 2\sqrt{6} + 2\sqrt{6} + 2 \cdot 2\sqrt{5}$
 $-4\sqrt{6} - 2\sqrt{6} + 2\sqrt{6} + 4\sqrt{5}$
 $-4\sqrt{6} + 4\sqrt{5}$
- 23) $3\sqrt{24} - 3\sqrt{27} + 2\sqrt{6} + 2\sqrt{8}$
 $3\sqrt{2^3 \cdot 3} - 3\sqrt{3^3} + 2\sqrt{6} + 2\sqrt{2^3}$
 $3 \cdot 2\sqrt{2 \cdot 3} - 3 \cdot 3\sqrt{3} + 2\sqrt{6} + 2 \cdot 2\sqrt{2}$
 $6\sqrt{6} - 9\sqrt{3} + 2\sqrt{6} + 4\sqrt{2}$
 $8\sqrt{6} - 9\sqrt{3} + 4\sqrt{2}$
- 25) $-2\sqrt[3]{16} + 2\sqrt[3]{16} + 2\sqrt[3]{2}$
 $-2\sqrt[3]{2^4} + 2\sqrt[3]{2^4} + 2\sqrt[3]{2}$
 $-2 \cdot 2\sqrt[3]{2} + 2 \cdot 2\sqrt[3]{2} + 2\sqrt[3]{2}$
 $-4\sqrt[3]{2} + 4\sqrt[3]{2} + 2\sqrt[3]{2}$
 $2\sqrt[3]{2}$
- 27) $2\sqrt[4]{243} - 2\sqrt[4]{253} - \sqrt[4]{3}$
 $2\sqrt[4]{3^5} - 2\sqrt[4]{3^5} - \sqrt[4]{3}$
 $2 \cdot 3\sqrt[4]{3} - 2 \cdot 3\sqrt[4]{3} - \sqrt[4]{3}$
 $6\sqrt[4]{3} - 6\sqrt[4]{3} - \sqrt[4]{3}$
 $-\sqrt[4]{3}$
- 29) $3\sqrt[4]{2} - 2\sqrt[4]{2} - \sqrt[4]{243}$
 $3\sqrt[4]{2} - 2\sqrt[4]{2} - \sqrt[4]{3^5}$
 $3\sqrt[4]{2} - 2\sqrt[4]{2} - 3\sqrt[4]{3}$
 $\sqrt[4]{2} - 3\sqrt[4]{3}$

$$\begin{aligned}
31) & -\sqrt[4]{324} + 3\sqrt[4]{324} - 3\sqrt[4]{4} \\
& -\sqrt[4]{3^5} + 3\sqrt[4]{3^5} - 3\sqrt[4]{4} \\
& -3\sqrt[4]{3} + 3 \cdot 3\sqrt[4]{3} - 3\sqrt[4]{4} \\
& -3\sqrt[4]{3} + 9\sqrt[4]{3} - 3\sqrt[4]{4} \\
& 6\sqrt[4]{3} - 3\sqrt[4]{4}
\end{aligned}$$

$$\begin{aligned}
33) & 2\sqrt[4]{2} + 2\sqrt[4]{3} + 3\sqrt[4]{64} - \sqrt[4]{3} \\
& 2\sqrt[4]{2} + 2\sqrt[4]{3} + 3\sqrt[4]{2^6} - \sqrt[4]{3} \\
& 2\sqrt[4]{2} + 2\sqrt[4]{3} + 3 \cdot 2\sqrt[4]{2^2} - \sqrt[4]{3} \\
& 2\sqrt[4]{2} + 2\sqrt[4]{3} + 6\sqrt[4]{4} - \sqrt[4]{3} \\
& 2\sqrt[4]{2} + \sqrt[4]{3} + 6\sqrt[4]{4}
\end{aligned}$$

$$\begin{aligned}
35) & -3\sqrt[5]{6} - \sqrt[5]{64} + 2\sqrt[5]{192} - 2\sqrt[5]{64} \\
& -3\sqrt[5]{6} - \sqrt[5]{2^6} + 2\sqrt[5]{2^6 \cdot 3} - 2\sqrt[5]{2^6} \\
& -3\sqrt[5]{6} - 2\sqrt[5]{2} + 2 \cdot 2\sqrt[5]{2 \cdot 3} - 2 \cdot 2\sqrt[5]{2} \\
& -3\sqrt[5]{6} - 2\sqrt[5]{2} + 4\sqrt[5]{6} - 4\sqrt[5]{2} \\
& \sqrt[5]{6} - 6\sqrt[5]{2}
\end{aligned}$$

$$\begin{aligned}
37) & 2\sqrt[5]{160} - 2\sqrt[5]{192} - \sqrt[5]{160} - \sqrt[5]{-160} \\
& 2\sqrt[5]{2^5 \cdot 5} - 2\sqrt[5]{2^6 \cdot 3} - \sqrt[5]{2^5 \cdot 5} - \sqrt[5]{-1^5 \cdot 2^5 \cdot 5} \\
& 2 \cdot 2\sqrt[5]{5} - 2 \cdot 2\sqrt[5]{2 \cdot 3} - 2\sqrt[5]{5} - (-1) \cdot 2\sqrt[5]{5} \\
& 4\sqrt[5]{5} - 4\sqrt[5]{6} - 2\sqrt[5]{5} + 2\sqrt[5]{5} \\
& 4\sqrt[5]{5} - 4\sqrt[5]{6}
\end{aligned}$$

$$\begin{aligned}
39) & -\sqrt[6]{256} - 2\sqrt[6]{4} - 3\sqrt[6]{320} - 2\sqrt[6]{128} \\
& -\sqrt[6]{2^8} - 2\sqrt[6]{4} - 3\sqrt[6]{2^6 \cdot 5} - 2\sqrt[6]{2^7} \\
& -2\sqrt[6]{2^2} - 2\sqrt[6]{4} - 3 \cdot 2\sqrt[6]{5} - 2 \cdot 2\sqrt[6]{2} \\
& -2\sqrt[6]{4} - 2\sqrt[6]{4} - 6\sqrt[6]{5} - 4\sqrt[6]{2} \\
& -4\sqrt[6]{4} - 6\sqrt[6]{5} - 4\sqrt[6]{2}
\end{aligned}$$

8.4

$$2) \begin{aligned} & 3\sqrt{5} \cdot -4\sqrt{16} \\ & -12\sqrt{80} \\ & -12\sqrt{2^4 \cdot 5} \\ & -12 \cdot 2^2\sqrt{5} \\ & -12 \cdot 4\sqrt{5} \\ & -48\sqrt{5} \end{aligned}$$

$$5) \begin{aligned} & \frac{\sqrt{12m}\sqrt{15m}}{\sqrt{180m^2}} \\ & \frac{\sqrt{2^2 \cdot 3^2 \cdot 5m^2}}{2 \cdot 3m\sqrt{5}} \\ & 6m\sqrt{5} \end{aligned}$$

$$7) \begin{aligned} & \frac{\sqrt[3]{4x^3}\sqrt[3]{2x^4}}{\sqrt[3]{8x^7}} \\ & \frac{\sqrt[3]{2^3x^7}}{2x^2\sqrt[3]{x}} \end{aligned}$$

$$9) \begin{aligned} & \sqrt{6}(\sqrt{2} + 2) \\ & \sqrt{12} + 2\sqrt{6} \\ & \sqrt{2^2 \cdot 3} + 2\sqrt{6} \\ & 2\sqrt{3} + 2\sqrt{6} \end{aligned}$$

$$11) \begin{aligned} & -5\sqrt{15}(3\sqrt{3} + 2) \\ & -15\sqrt{45} - 10\sqrt{15} \\ & -15\sqrt{3^2 \cdot 5} - 10\sqrt{15} \\ & -15 \cdot 3\sqrt{5} - 10\sqrt{15} \\ & -45\sqrt{5} - 10\sqrt{15} \end{aligned}$$

$$13) \begin{aligned} & 5\sqrt{10}(5n + \sqrt{2}) \\ & 25n\sqrt{10} + 5\sqrt{20} \\ & 25n\sqrt{10} + 5\sqrt{2^2 \cdot 5} \\ & 25n\sqrt{10} + 5 \cdot 2\sqrt{5} \\ & 25n\sqrt{10} + 10\sqrt{5} \end{aligned}$$

$$15) \begin{aligned} & (2 + 2\sqrt{2})(-3 + \sqrt{2}) \\ & -6 + 2\sqrt{2} - 6\sqrt{2} + 2\sqrt{4} \\ & -6 + 2\sqrt{2} - 6\sqrt{2} + 2\sqrt{2^2} \\ & -6 + 2\sqrt{2} - 6\sqrt{2} + 2 \cdot 2 \\ & -6 + 2\sqrt{2} - 6\sqrt{2} + 4 \\ & -2 - 4\sqrt{2} \end{aligned}$$

$$17) \begin{aligned} & (\sqrt{5} - 5)(2\sqrt{5} - 1) \\ & 2\sqrt{25} - \sqrt{5} - 10\sqrt{5} + 5 \\ & 2\sqrt{5^2} - \sqrt{5} - 10\sqrt{5} + 5 \\ & 2 \cdot 5 - \sqrt{5} - 10\sqrt{5} + 5 \\ & 10 - \sqrt{5} - 10\sqrt{5} + 5 \\ & 15 - 11\sqrt{5} \end{aligned}$$

$$20) \begin{aligned} & (\sqrt{2a} + 2\sqrt{3a})(3\sqrt{2a} + \sqrt{5a}) \\ & 3\sqrt{4a^2} + \sqrt{10a^2} + 6\sqrt{6a^2} + 2\sqrt{15a^2} \\ & 3\sqrt{2^2a^2} + \sqrt{10a^2} + 6\sqrt{6a^2} + 2\sqrt{15a^2} \\ & 3 \cdot 2a + a\sqrt{10} + 6a\sqrt{6} + 2a\sqrt{15} \\ & 6a + a\sqrt{10} + 6a\sqrt{6} + 2a\sqrt{15} \end{aligned}$$

$$21) \begin{aligned} & (-5 - 4\sqrt{3})(-3 - 4\sqrt{3}) \\ & 15 + 20\sqrt{3} + 12\sqrt{3} + 16\sqrt{9} \\ & 15 + 20\sqrt{3} + 12\sqrt{3} + 16\sqrt{3^2} \\ & 15 + 20\sqrt{3} + 12\sqrt{3} + 16 \cdot 3 \\ & 15 + 20\sqrt{3} + 12\sqrt{3} + 48 \\ & 63 + 32\sqrt{3} \end{aligned}$$

$$21) \frac{\sqrt{12}}{5\sqrt{100}} = \frac{\sqrt{3}}{5\sqrt{25}} = \frac{\sqrt{3}}{5\sqrt{5^2}} = \frac{\sqrt{3}}{5 \cdot 5} = \frac{\sqrt{3}}{25}$$

$$23) \frac{\sqrt{5}}{4\sqrt{125}} = \frac{1}{4\sqrt{25}} = \frac{1}{4\sqrt{5^2}} = \frac{1}{4 \cdot 5} = \frac{1}{20}$$

$$25) \frac{\sqrt{10}}{\sqrt{6}} = \frac{\sqrt{5}}{\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}} \right) = \frac{\sqrt{15}}{3}$$

$$27) \frac{2\sqrt{4}}{3\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}} \right) = \frac{2\sqrt{12}}{3 \cdot 3} = \frac{2\sqrt{2^2 \cdot 3}}{9} = \frac{2 \cdot 2\sqrt{3}}{9} = \frac{4\sqrt{3}}{9}$$

$$29) \frac{5x^2}{4\sqrt{3x^3y^3}} = \frac{5x^2}{4xy\sqrt{3xy}} = \frac{5x}{4y\sqrt{3xy}} \left(\frac{\sqrt{3xy}}{\sqrt{3xy}} \right) = \frac{5x\sqrt{3xy}}{4y \cdot 3xy} = \frac{5x\sqrt{3xy}}{12xy^2}$$

$$31) \frac{\sqrt{2p^2}}{\sqrt{3p}} = \frac{\sqrt{2p}}{\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}} \right) = \frac{\sqrt{6p}}{3}$$

$$33) \frac{3\sqrt[3]{10}}{5\sqrt[3]{27}} = \frac{3\sqrt[3]{10}}{5\sqrt[3]{3^2}} = \frac{3\sqrt[3]{10}}{5 \cdot 3} = \frac{3\sqrt[3]{10}}{15} = \frac{\sqrt[3]{10}}{5}$$

$$35) \frac{\sqrt[3]{5}}{4\sqrt[3]{4}} = \frac{\sqrt[3]{5}}{4\sqrt[3]{2^2}} \left(\frac{\sqrt[3]{2}}{\sqrt[3]{2}} \right) = \frac{3\sqrt[3]{10}}{4 \cdot 2} = \frac{3\sqrt[3]{10}}{8}$$

$$37) \frac{5\sqrt[4]{5r^4}}{\sqrt[4]{8r^2}} = \frac{5\sqrt[4]{5r^2}}{\sqrt[4]{8}} = \frac{5\sqrt[4]{5r^2}}{\sqrt[4]{2^3}} \left(\frac{\sqrt[4]{2}}{\sqrt[4]{2}} \right) = \frac{5\sqrt[4]{10r}}{2}$$

8.5

$$1) \frac{4+2\sqrt{3}}{\sqrt{9}} = \frac{4+2\sqrt{3}}{3}$$

$$3) \frac{4+2\sqrt{3}}{5\sqrt{4}} = \frac{4+2\sqrt{3}}{5 \cdot 2} = \frac{4+2\sqrt{3}}{10} = \frac{2(2+\sqrt{3})}{10} = \frac{2+\sqrt{3}}{5}$$

$$5) \frac{2-5\sqrt{5}}{4\sqrt{13}} \left(\frac{\sqrt{13}}{\sqrt{13}} \right) = \frac{2\sqrt{13}-5\sqrt{65}}{4 \cdot 13} = \frac{2\sqrt{13}-5\sqrt{65}}{52}$$

$$7) \frac{\sqrt{2}-3\sqrt{3}}{\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}} \right) = \frac{\sqrt{6}-3\sqrt{9}}{3} = \frac{\sqrt{6}-3 \cdot 3}{3} = \frac{\sqrt{6}-9}{3}$$

$$9) \frac{2p+3\sqrt{5p^4}}{5\sqrt{20p^2}} = \frac{2p+3\sqrt{5p^4}}{5\sqrt{2^2 \cdot 5p^2}} = \frac{2p+3p^2\sqrt{5}}{5 \cdot 2p\sqrt{5}} = \frac{2p+3p^2\sqrt{5}}{10p\sqrt{5}} \left(\frac{\sqrt{5}}{\sqrt{5}} \right) = \frac{2p\sqrt{5}+3p^2\sqrt{25}}{10p(5)} = \frac{2p\sqrt{5}+3p^2 \cdot 5}{50p} =$$

$$\frac{2p\sqrt{5}+15p^2}{50p} = \frac{p(2\sqrt{5}+15p)}{50p} = \frac{2\sqrt{5}+15p}{50}$$

$$11) \frac{\sqrt{3m^2-4\sqrt{2}m^4}}{5\sqrt{12m^4}} = \frac{\sqrt{3m^2-4\sqrt{2}m^4}}{5\sqrt{2^2 \cdot 3m^4}} = \frac{m\sqrt{3}-4m^2\sqrt{2}}{2 \cdot 5m^2\sqrt{3}} = \frac{m\sqrt{3}-4m^2\sqrt{2}}{10m^2\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}} \right) = \frac{m\sqrt{9}-4m^2\sqrt{6}}{10m^2 \cdot 3} =$$

$$\frac{3m-4m^2\sqrt{6}}{30m^2} = \frac{m(3-4m\sqrt{6})}{30m^2} = \frac{3-4m\sqrt{6}}{30m}$$

$$13) \frac{5}{3\sqrt{5}+\sqrt{2}} \left(\frac{3\sqrt{5}+2}{3\sqrt{5}+2} \right) = \frac{15\sqrt{5}-5\sqrt{2}}{9 \cdot 5 - 2} = \frac{15\sqrt{5}-5\sqrt{2}}{45-2} = \frac{15\sqrt{5}-5\sqrt{2}}{43}$$

$$15) \frac{2}{5+\sqrt{2}} \left(\frac{5-\sqrt{2}}{5-\sqrt{2}} \right) = \frac{10-2\sqrt{2}}{25-2} = \frac{10-2\sqrt{2}}{23}$$

$$17) \frac{3}{4-3\sqrt{3}} \left(\frac{4+3\sqrt{3}}{4+3\sqrt{3}} \right) = \frac{12+9\sqrt{3}}{16-9\cdot 3} = \frac{12+9\sqrt{3}}{16-27} = \frac{12+9\sqrt{3}}{-11}$$

$$19) \frac{4}{3+\sqrt{5}} \left(\frac{3-\sqrt{5}}{3-\sqrt{5}} \right) = \frac{12-4\sqrt{5}}{9-5} = \frac{12-4\sqrt{5}}{4} = \frac{4(3-\sqrt{5})}{4} = 3 - \sqrt{5}$$

$$21) \frac{-4}{4-4\sqrt{2}} \left(\frac{4+4\sqrt{2}}{4+4\sqrt{2}} \right) = \frac{-16-16\sqrt{2}}{16-16\cdot 2} = \frac{-16-16\sqrt{2}}{16-32} = \frac{-16-16\sqrt{2}}{-16} = \frac{-16(1+\sqrt{2})}{-16} = 1 + \sqrt{2}$$

$$23) \frac{5}{\sqrt{n^4-5}} = \frac{5}{n^2-5}$$

$$25) \frac{4p}{3-5\sqrt{p^4}} = \frac{4p}{3-5p^2}$$

$$27) \frac{4}{5+\sqrt{5x^2}} = \frac{4}{5+x\sqrt{5}} \left(\frac{5-x\sqrt{5}}{5-x\sqrt{5}} \right) = \frac{20-4x\sqrt{5}}{25+5x^2}$$

$$29) \frac{5}{2+\sqrt{5r^3}} = \frac{5}{2+r\sqrt{5r}} \left(\frac{2-r\sqrt{5r}}{2-r\sqrt{5r}} \right) = \frac{10-5r\sqrt{5r}}{4-r^2(5r)} = \frac{10-5r\sqrt{5r}}{4-5r^3}$$

$$31) \frac{5}{-5v-3\sqrt{v}} \left(\frac{-5v+3\sqrt{v}}{-5v+3\sqrt{v}} \right) = \frac{-25v+15\sqrt{v}}{25v^2-9v}$$

$$33) \frac{4\sqrt{2}+3}{3\sqrt{2}+\sqrt{3}} \left(\frac{3\sqrt{2}-\sqrt{3}}{3\sqrt{2}-\sqrt{3}} \right) = \frac{12\sqrt{4}-4\sqrt{6}+9\sqrt{2}-3\sqrt{3}}{9\cdot 2-3} = \frac{12\cdot 2-4\sqrt{6}+9\sqrt{2}-3\sqrt{3}}{18-3} = \frac{24-4\sqrt{6}+9\sqrt{2}-3\sqrt{3}}{15}$$

$$35) \frac{2-\sqrt{5}}{-3+\sqrt{5}} \left(\frac{-3-\sqrt{5}}{-3-\sqrt{5}} \right) = \frac{-6-2\sqrt{5}+3\sqrt{5}+\sqrt{25}}{9-5} = \frac{-6-2\sqrt{5}+3\sqrt{5}+5}{4} = \frac{-1+\sqrt{5}}{4}$$

$$37) \frac{5\sqrt{2}+\sqrt{3}}{5+5\sqrt{2}} \left(\frac{5-5\sqrt{2}}{5-5\sqrt{2}} \right) = \frac{25\sqrt{2}-25\sqrt{4}+5\sqrt{3}-5\sqrt{6}}{25-25\cdot 2} = \frac{25\sqrt{2}-25\cdot 2+5\sqrt{3}-5\sqrt{6}}{25-50} =$$

$$\frac{25\sqrt{2}-50+5\sqrt{3}-5\sqrt{6}}{-25} = \frac{5(5\sqrt{2}-10+\sqrt{3}-\sqrt{6})}{-25} = \frac{5\sqrt{2}-10+\sqrt{3}-\sqrt{6}}{-5}$$

$$39) \frac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{2}} \left(\frac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}+\sqrt{2}} \right) = \frac{2\sqrt{9}+\sqrt{6}+2\sqrt{6}+\sqrt{4}}{4\cdot 3-2} = \frac{2\cdot 3+\sqrt{6}+2\sqrt{6}+2}{12-2} = \frac{6+\sqrt{6}+2\sqrt{6}+2}{10} = \frac{8+3\sqrt{6}}{10}$$

$$41) \frac{\sqrt{3}-\sqrt{2}}{4+\sqrt{5}} \left(\frac{4-\sqrt{5}}{4-\sqrt{5}} \right) = \frac{4\sqrt{3}-\sqrt{15}-4\sqrt{2}+\sqrt{10}}{16-5} = \frac{4\sqrt{3}-\sqrt{15}-4\sqrt{2}+\sqrt{10}}{11}$$

$$43) \frac{4+2\sqrt{2x^2}}{5+2\sqrt{5x^3}} = \frac{4+2x\sqrt{2}}{5+2x\sqrt{5x}} \left(\frac{5-2x\sqrt{5x}}{5-2x\sqrt{5x}} \right) = \frac{20+8x\sqrt{5x}+10x\sqrt{2}-4x^2\sqrt{10x}}{25-4x^2(5x)} =$$

$$\frac{20+8x\sqrt{5x}+10x\sqrt{2}-4x^2\sqrt{10x}}{25-20x^3}$$

$$45) \frac{(2\sqrt{3m^2}-\sqrt{2m^4})}{5-\sqrt{3m^2}} = \frac{(2m\sqrt{3}-m^2\sqrt{2})}{5-m\sqrt{3}} \left(\frac{5+m\sqrt{3}}{5+m\sqrt{3}} \right) = \frac{10m\sqrt{3}+2m^2\sqrt{9}-5m^2\sqrt{2}-m^3\sqrt{6}}{25-3m^2} =$$

$$\frac{10m\sqrt{3}+2m^2(3)-5m^2\sqrt{2}-m^3\sqrt{6}}{25-3m^2} = \frac{10m\sqrt{3}+6m^2-5m^2\sqrt{2}-m^3\sqrt{6}}{25-3m^2}$$

$$47) \frac{2b-5\sqrt{2b}}{-1+\sqrt{2b^4}} = \frac{2b-5\sqrt{2b}}{-1+b^2\sqrt{2}} \left(\frac{-1-\sqrt{2}}{-1-\sqrt{2}} \right) = \frac{-2b-2b^3\sqrt{2}+5\sqrt{2b}-5b^2\sqrt{4b}}{1-2b^4} =$$

$$\frac{-2b-2b^3\sqrt{2}+5\sqrt{2b}-2\cdot 5b^2\sqrt{b}}{1-2b^4} = \frac{-2b-2b^3\sqrt{2}+5\sqrt{2b}-10b^2\sqrt{b}}{1-2b^4}$$

$$49) \frac{2-\sqrt{2x}}{4x-5\sqrt{3x^3}} = \frac{2-\sqrt{2x}}{4x-5x\sqrt{3x}} \left(\frac{4x+5x\sqrt{3x}}{4x+5x\sqrt{3x}} \right) = \frac{8x+10x\sqrt{3x}-4x\sqrt{2x}-5x\sqrt{6x^2}}{16x^2+25x^2(3x)} =$$

$$\frac{8x+10x\sqrt{3x}-4x\sqrt{2x}-5x^2\sqrt{6}}{16x^2+75x^2} = \frac{x(8+10\sqrt{3x}-4\sqrt{2x}-5x\sqrt{6})}{x(16x+75x)} = \frac{8+10\sqrt{3x}-4\sqrt{2x}-5x\sqrt{6}}{16x+75x}$$

$$51) \frac{-4p-\sqrt{p}}{-p-\sqrt{p^3}} = \frac{-4p-\sqrt{p}}{-p-p\sqrt{p}} \left(\frac{-p+p\sqrt{p}}{-p+p\sqrt{p}} \right) = \frac{4p^2-4p^2\sqrt{p}+p\sqrt{p}-p\sqrt{p^2}}{p^2-p^2\cdot p} = \frac{4p^2-4p^2\sqrt{p}+p\sqrt{p}-p^2}{p^2-p^3} =$$

$$\frac{3p^2-4p^2\sqrt{p}+p\sqrt{p}}{p^2-p^3} = \frac{p(3p-4p\sqrt{p}+\sqrt{p})}{p(p-p^2)} = \frac{3p-4p\sqrt{p}+\sqrt{p}}{p-p^2}$$

8.6

$$1) m^{\frac{3}{5}} = (\sqrt[5]{m})^3$$

$$3) r(7x)^{\frac{3}{2}} = (\sqrt{7x})^3$$

$$5) \frac{1}{(\sqrt{6x})^3} = (6x)^{-\frac{3}{2}}$$

$$7) \frac{1}{(\sqrt[4]{n})^7} = n^{-\frac{7}{4}}$$

$$9) 8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$$

$$11) 4^{\frac{3}{2}} = (\sqrt{4})^3 = 2^3 = 8$$

$$13) yx^{\frac{1}{3}} \cdot xy^{\frac{3}{2}}$$

$$y^{\frac{2}{2}}x^{\frac{1}{3}}x^{\frac{3}{3}}y^{\frac{3}{2}}$$

$$x^{\frac{4}{3}}y^{\frac{5}{2}}$$

$$15) (a^{\frac{1}{2}}b^{\frac{1}{2}})^{-1}$$

$$a^{-\frac{1}{2}}b^{-\frac{1}{2}}$$

$$\frac{1}{a^{\frac{1}{2}}b^{\frac{1}{2}}}$$

$$17) \frac{a^2b^0}{3a^4} = \frac{1}{3a^2}$$

$$19) uv \cdot u(v^{\frac{3}{2}})^3$$

$$uv \cdot uv^{\frac{9}{2}}$$

$$uv^2uv^{\frac{9}{2}}$$

$$u^2v^{\frac{11}{2}}$$

$$21) (x^0y^{\frac{1}{3}})^{\frac{3}{2}}x^0$$

$$y^{\frac{1}{2}}$$

$$23) \frac{a^{\frac{3}{4}}b^{-1}b^{\frac{7}{4}}}{3b^{-1}} = \frac{a^{\frac{3}{4}}b^{\frac{7}{4}}}{3}$$

$$25) \frac{3y^{\frac{5}{4}}}{y^{-1}2y^{-\frac{1}{3}}} = \frac{3yy^{\frac{1}{3}}}{2y^{\frac{5}{4}}} = \frac{3y^{\frac{3}{3}}y^{\frac{1}{3}}}{2y^{\frac{5}{4}}} = \frac{3y^{\frac{4}{3}}}{2y^{\frac{5}{4}}} = \frac{3y^{\frac{16}{12}}}{2y^{\frac{15}{12}}} = \frac{3y^{\frac{1}{12}}}{2}$$

$$27) \left(\frac{m^{\frac{3}{2}}n^{-2}}{(mn^{\frac{4}{3}})^{-1}} \right)^{\frac{7}{4}} = \left(\frac{m^{\frac{3}{2}}n^{-2}}{m^{-1}n^{-\frac{4}{3}}} \right)^{\frac{7}{4}} = \left(\frac{m^{\frac{3}{2}}mn^{\frac{4}{3}}}{n^2} \right)^{\frac{7}{4}} = \left(\frac{m^{\frac{3}{2}}m^{\frac{2}{2}}n^{\frac{4}{3}}}{n^{\frac{6}{3}}} \right)^{\frac{7}{4}} = \left(\frac{m^{\frac{5}{2}}}{n^{\frac{2}{3}}} \right)^{\frac{7}{4}} = \frac{m^{\frac{35}{8}}}{n^{\frac{7}{6}}}$$

$$29) r \frac{\left(m^2 n^{\frac{1}{2}}\right)^0}{n^{\frac{3}{4}}} = \frac{1}{n^{\frac{3}{4}}}$$

$$31) r \frac{\left(x^{-\frac{4}{3}} y^{-\frac{1}{3}}\right)^{-1}}{\frac{1}{x^{\frac{3}{3}} y^{-2}}} = \frac{\left(x^{-\frac{4}{3}} y^{-\frac{1}{3}}\right)^{-1}}{\frac{1}{x^{\frac{3}{3}} y^{-2}}} = \frac{\left(x^{-\frac{4}{3}} y^{\frac{2}{3}}\right)^{-1}}{\frac{1}{x^{\frac{3}{3}} y^{-2}}} = \frac{x^{\frac{4}{3}} y^{-\frac{2}{3}}}{\frac{1}{x^{\frac{3}{3}} y^{-2}}} = \frac{x^{\frac{4}{3}} y^2}{x^{\frac{3}{3}} y^{\frac{6}{3}}} = \frac{x^{\frac{4}{3}} y^{\frac{6}{3}}}{x^{\frac{3}{3}} y^{\frac{6}{3}}} = xy^{\frac{4}{3}}$$

$$33) \frac{(uv^2)^{\frac{1}{2}}}{v^{\frac{1}{4}v^2}} = \frac{u^{\frac{1}{2}}v}{v^{\frac{1}{4}v^2}} = \frac{u^{\frac{1}{2}}v^{\frac{1}{4}}}{v^2} = \frac{u^{\frac{1}{2}}v^{\frac{1}{4}}v^{\frac{3}{4}}}{v^2} = \frac{u^{\frac{1}{2}}v^1}{v^2} = \frac{u^{\frac{1}{2}}v^{\frac{5}{4}}}{v^2} = \frac{u^{\frac{1}{2}}v^{\frac{5}{4}}}{v^{\frac{8}{4}}} = \frac{u^{\frac{1}{2}}}{v^{\frac{3}{4}}}$$

8.7

$$1) \frac{\sqrt[8]{16x^4y^6}}{\sqrt[8]{2^4x^4y^6}} = \frac{\sqrt[4]{2^2x^2y^3}}{\sqrt[4]{4x^2y^3}}$$

$$3) \frac{\sqrt[12]{64x^4y^6z^8}}{\sqrt[12]{2^6x^4y^6z^8}} = \frac{\sqrt[6]{2^3x^2y^3z^4}}{\sqrt[6]{8x^2y^3z^4}}$$

$$5) \frac{\sqrt[6]{16x^2}}{\sqrt[6]{9y^4}} = \sqrt[6]{\frac{2^4x^2}{3^2y^4}} = \sqrt[3]{\frac{2^2x}{3y^2}} \left(\frac{\sqrt[3]{3^2y}}{\sqrt[3]{3^2y}}\right) = \frac{\sqrt[3]{36xy}}{3y}$$

$$7) \frac{\sqrt[12]{x^6y^9}}{\sqrt[4]{x^2y^3}}$$

$$9) \frac{\sqrt[8]{x^6y^4z^2}}{\sqrt[4]{x^3y^2z}}$$

$$11) \frac{\sqrt[9]{8x^3y^6}}{\sqrt[9]{2^3x^3y^6}} = \sqrt[3]{\frac{2xy^2}{2xy^2}}$$

$$13) \frac{\sqrt[3]{5}\sqrt{6}}{\sqrt[6]{5^2 \cdot 6^3}} = \frac{\sqrt[6]{25 \cdot 216}}{\sqrt[6]{5400}}$$

$$15) \frac{\sqrt{x^3}\sqrt{7y}}{\sqrt[6]{x^3 \cdot 7^2y^2}} = \frac{\sqrt[6]{49x^3y^2}}{\sqrt[6]{49x^3y^2}}$$

$$17) \frac{\sqrt{x} \sqrt[3]{x-2}}{\sqrt[6]{x^3(x-2)^2}}$$

$$19) \frac{\sqrt[5]{x^2y} \sqrt{xy}}{\sqrt[10]{x^4y^2 \cdot x^5y^5}} = \frac{\sqrt[10]{x^9y^7}}{\sqrt[10]{x^9y^7}}$$

$$21) \frac{\sqrt[4]{xy^2} \sqrt[3]{x^2y}}{\sqrt[12]{x^3y^6 \cdot x^8y^4}} = \frac{\sqrt[12]{x^{11}y^{10}}}{\sqrt[12]{x^{11}y^{10}}}$$

$$23) \sqrt[4]{a^2bc^2} \sqrt[5]{a^2b^3c}$$

$$\frac{\sqrt[20]{a^{10}b^5c^{10} \cdot a^8b^{12}c^4}}{\sqrt[20]{a^{18}b^{17}c^{14}}}$$

$$25) \sqrt{a} \sqrt[4]{a^3} \\ \sqrt[4]{a^2 \cdot a^3} \\ \sqrt[4]{a^5} \\ a^4 \sqrt{a}$$

$$27) \sqrt[5]{b^2} \sqrt{b^3} \\ \sqrt[10]{b^4 \cdot b^{15}} \\ \sqrt[10]{b^{19}} \\ b^{10} \sqrt{b^9}$$

$$29) \sqrt{xy^3} \sqrt[3]{x^2y} \\ \sqrt[6]{x^3y^9x^4y^2} \\ \sqrt[6]{x^7y^{11}} \\ xy^6 \sqrt{xy^5}$$

$$31) \sqrt[4]{9ab^3} \sqrt{3a^4b} \\ \sqrt[4]{3^2ab^3} \sqrt{3a^4b} \\ \sqrt[4]{3^2ab^3 \cdot 3^2a^8b^2} \\ \sqrt[4]{3^4a^9b^5} \\ 3a^2b^4 \sqrt{ab}$$

$$33) \sqrt[3]{3xy^2z} \sqrt[4]{9x^3yz^2} \\ \sqrt[3]{3xy^2z} \sqrt[4]{3^2x^3yz^2} \\ \sqrt[12]{3^4x^4y^8z^4 \cdot 3^6x^9y^3z^6} \\ \sqrt[12]{3^{10}x^{13}y^{11}z^{10}} \\ x^{12} \sqrt{59049xy^{11}z^{10}}$$

$$35) \sqrt{27a^5(b+1)} \sqrt[3]{81a(b+1)^4} \\ \sqrt{3^3a^5(b+1)} \sqrt[3]{3^4a(b+1)^4} \\ \sqrt[6]{3^9a^{15}(b+1)^3 \cdot 3^8a^2(b+1)^8} \\ \sqrt[6]{3^{17}a^{17}(b+1)^{11}} \\ 3^2a^2(b+1) \sqrt[6]{3^5a^5(b+1)^5} \\ 9a^2(b+1) \sqrt[6]{243a^5(b+1)^5}$$

$$37) \frac{\sqrt[3]{a^2}}{\sqrt[4]{a}} = \frac{12}{\sqrt[12]{a^8}} = \sqrt[2]{a^5}$$

$$39) \frac{\sqrt[4]{x^2y^3}}{\sqrt[3]{xy}} = \frac{12}{\sqrt[12]{x^6y^9}} = \sqrt[12]{x^2y^5}$$

$$41) \frac{\sqrt{ab^3c}}{\sqrt[5]{a^2b^3c^{-1}}} = \frac{10}{\sqrt[10]{a^5b^{15}c^5}} = \sqrt[10]{ab^9c^7}$$

$$43) \frac{\sqrt[4]{(3x-1)^3}}{\sqrt[5]{(3x-1)^3}} = \frac{20}{\sqrt[20]{(3x-1)^{15}}} = \sqrt[20]{(3x-1)^3}$$

$$45) \frac{\sqrt[3]{(2x+1)^2}}{\sqrt[5]{(2x+1)^2}} = \frac{15}{\sqrt[15]{(2x+1)^{10}}} = \sqrt[15]{(2x+1)^4}$$

8.8

$$\begin{aligned} 1) \quad & 3 - (-8 + 4i) \\ & 3 + 8 - 4i \\ & 11 - 4i \end{aligned}$$

$$\begin{aligned} 3) \quad & r7i - (3 - 2i) \\ & 7i - 3 + 2i \\ & -3 + 9i \end{aligned}$$

$$\begin{aligned} 5) \quad & -6i - (3 + 7i) \\ & -6i - 3 - 7i \\ & -3 - 13i \end{aligned}$$

$$\begin{aligned} 7) \quad & (3 - 3i) + (-7 - 8i) \\ & 3 - 3i - 7 - 8i \\ & -4 - 11i \end{aligned}$$

$$\begin{aligned} 9) \quad & i - (2 + 3i) - 6 \\ & i - 2 - 3i - 6 \\ & -8 - 2i \end{aligned}$$

$$\begin{aligned} 11) \quad & (6i)(-8i) \\ & -48i^2 \\ & -48(-1) \\ & 48 \end{aligned}$$

$$\begin{aligned} 13) \quad & (-5i)(8i) \\ & -40i^2 \\ & -40(-1) \\ & 40 \end{aligned}$$

$$\begin{aligned} 15) \quad & (-7i)^2 \\ & 49i^2 \\ & 49(-1) \\ & -49 \end{aligned}$$

$$\begin{aligned} 17) \quad & (6 + 5i)^2 \\ & 36 + 60i + 25i^2 \\ & 36 + 60i + 25(-1) \\ & 36 + 60i - 25 \\ & 11 + 60i \end{aligned}$$

$$\begin{aligned} 19) \quad & (-7 - 4i)(-8 + 6i) \\ & 56 - 42i + 32i - 24i^2 \\ & 56 - 42i + 32i - 24(-1) \\ & 56 - 42i + 32i + 24 \\ & 80 - 10i \end{aligned}$$

$$\begin{aligned} 21) \quad & (-4 + 5i)(2 - 7i) \\ & -8 + 28i + 10i - 35i^2 \\ & -8 + 28i + 10i - 35(-1) \\ & -8 + 28i + 10i + 35 \\ & 27 + 38i \end{aligned}$$

$$\begin{aligned} 23) \quad & (-8 - 6i)(-4 + 2i) \\ & 32 - 16i + 24i - 12i^2 \\ & 32 - 16i + 24i - 12(-1) \\ & 32 - 16i + 24i + 12 \\ & 44 + 8i \end{aligned}$$

$$\begin{aligned} 25) \quad & (1 + 5i)(2 + i) \\ & 2 + i + 10i + 5i^2 \\ & 2 + i + 10i + 5(-1) \\ & 2 + i + 10i - 5 \\ & -3 + 11i \end{aligned}$$

$$27) \quad \frac{(-9+5i)(i)}{i(i)} = \frac{-9i+5i^2}{i^2} = \frac{-9i+5(-1)}{-1} = \frac{-9i-5}{-1} = 9i+5$$

$$29) \quad \frac{(-10-9i)(i)}{6i(i)} = \frac{-10i-9i^2}{6i^2} = \frac{-10i-9(-1)}{6(-1)} = \frac{-10i+9}{-6}$$

$$31) \quad \frac{(-3-6i)(i)}{4i(i)} = \frac{-3i-6i^2}{4i^2} = \frac{-3i-6(-1)}{4(-1)} = \frac{-3i+6}{-4}$$

$$33) \frac{(10-i)(i)}{-i(i)} = \frac{10i-i^2}{-i^2} = \frac{10i-(-1)}{-(-1)} = \frac{10i+1}{1} = 10i + 1$$

$$35) \frac{4i(-10-i)}{-10+i(-10-i)} = \frac{-40i-4i^2}{100-i^2} = \frac{-40i-4(-1)}{100-(-1)} = \frac{-40i+4}{100+1} = \frac{-40i+4}{101}$$

$$37) \frac{8(7+6i)}{7-6i(7+6i)} = \frac{56+48i}{49-36i^2} = \frac{56+48i}{49-36(-1)} = \frac{56+48i}{49+36} = \frac{56+48i}{85}$$

$$39) \frac{7(10+7i)}{10-7i(10+7i)} = \frac{70+49i}{100-49i^2} = \frac{70+49i}{100-49(-1)} = \frac{70+49i}{100+49} = \frac{70+49i}{149}$$

$$41) \frac{5i(-6+i)}{-6-i(-6+i)} = \frac{-30i+5i^2}{36-i^2} = \frac{-30i+5(-1)}{36-1(-1)} = \frac{-30i+5(-1)}{36-1(-1)} = \frac{-30i-5}{36+1} = \frac{-30i-5}{37}$$

$$43) \sqrt{-81}$$

$$\sqrt{-1 \cdot 3^2}$$

$$3^2 i$$

$$9i$$

$$45) \sqrt{-10} \sqrt{-2}$$

$$\sqrt{-1 \cdot 10} \sqrt{-1 \cdot 2}$$

$$i\sqrt{10} \cdot i\sqrt{2}$$

$$i^2 \sqrt{20}$$

$$-1\sqrt{2^2 \cdot 5}$$

$$-1 \cdot 2\sqrt{5}$$

$$-2\sqrt{5}$$

$$47) \frac{3+\sqrt{-27}}{6} = \frac{3+\sqrt{-1 \cdot 3^3}}{6} = \frac{3+3i\sqrt{3}}{6} = \frac{3(1+i\sqrt{3})}{6} = \frac{1+i\sqrt{3}}{2}$$

$$49) \frac{8-\sqrt{-16}}{4} = \frac{8-\sqrt{-1 \cdot 2^4}}{4} = \frac{8-2^2 i}{4} = \frac{8-4i}{4} = \frac{4(2-i)}{4} = 2 - i$$

$$51) i^{73} = i^1 = i$$

$$53) i^{48} = i^0 = 1$$

$$55) i^{62} = i^2 = -1$$

$$57) i^{154} = i^2 = -1$$

Chapter 9: Quadratics

9.1

$$\begin{aligned}
 1) \quad & \sqrt{2x+3} - 3 = 0 \\
 & \quad \quad \quad \underline{+3 \quad +3} \\
 & (\sqrt{2x+3})^2 = 3^2 \\
 & 2x + 3 = 9 \\
 & \quad \quad \quad \underline{-3 \quad -3} \\
 & \quad \quad \quad \frac{2x}{2} = \frac{6}{2} \\
 & \quad \quad \quad x = 3
 \end{aligned}$$

$$\begin{aligned}
 \text{Check: } & \sqrt{2(3)+3} - 3 = 0 \\
 & \sqrt{6+3} - 3 = 0 \\
 & \sqrt{9} - 3 = 0 \\
 & 3 - 3 = 0 \\
 & 0 = 0 \checkmark \\
 & x = 3
 \end{aligned}$$

$$\begin{aligned}
 3) \quad & \sqrt{6x-5} - x = 0 \\
 & \quad \quad \quad \underline{+x \quad +x} \\
 & (\sqrt{6x-5})^2 = x^2 \\
 & 6x - 5 = x^2 \\
 & -6x + 5 - 6x + 5 \\
 & \quad \quad \quad 0 = x^2 - 6x + 5 \\
 & \quad \quad \quad 0 = (x-1)(x-5) \\
 & x - 1 = 0 \quad x - 5 = 0 \\
 & \quad \quad \quad \underline{+1 \quad +1} \quad \underline{+5 \quad +5} \\
 & \quad \quad \quad x = 1 \quad x = 5
 \end{aligned}$$

$$\begin{aligned}
 \text{Check: } & \sqrt{6(5)-5} - 5 = 0 \\
 & \sqrt{30-5} - 5 = 0 \\
 & \sqrt{25} - 5 = 0 \\
 & 5 - 5 = 0 \\
 & 0 = 0 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 \text{Check: } & \sqrt{6(1)-5} - 1 = 0 \\
 & \sqrt{6-5} - 1 = 0 \\
 & \sqrt{1} - 1 = 0 \\
 & 1 - 1 = 0 \\
 & 0 = 0 \checkmark \\
 & x = 5, 1
 \end{aligned}$$

$$\begin{aligned}
 5) \quad & (3+x)^2 = (\sqrt{6x+13})^2 \\
 & 9 + 6x + x^2 = 6x + 13 \\
 & \quad \quad \quad \underline{-13 \quad -6x} \quad \underline{-6x \quad -13} \\
 & x^2 - 4 = 0 \\
 & (x+2)(x-2) = 0 \\
 & x + 2 = 0 \quad x - 2 = 0 \\
 & \quad \quad \quad \underline{-2 \quad -2} \quad \underline{+2 \quad +2} \\
 & \quad \quad \quad x = -2 \quad x = 2
 \end{aligned}$$

$$\begin{aligned}
 \text{Check: } & 3 + (-2) = \sqrt{6(-2)+13} \\
 & 1 = \sqrt{-12+13} \\
 & 1 = \sqrt{1} \\
 & 1 = 1 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 \text{Check: } & 3 + (2) = \sqrt{6(2)+13} \\
 & 5 = \sqrt{12+13} \\
 & 5 = \sqrt{25} \\
 & 5 = 5 \checkmark \\
 & x = -2, 2
 \end{aligned}$$

$$\begin{aligned}
 7) \quad & \sqrt{3-3x} - 1 = 2x \\
 & \quad \quad \quad \underline{+1 \quad +1} \\
 & (\sqrt{3-3x})^2 = (2x+1)^2 \\
 & 3 - 3x = 4x^2 + 4x + 1 \\
 & \quad \quad \quad \underline{-3+3x} \quad \quad \underline{+3x-3} \\
 & \quad \quad \quad 0 = 4x^2 + 7x - 2 \\
 & \quad \quad \quad 0 = (4x-1)(x+2) \\
 & 4x - 1 = 0 \quad x + 2 = 0 \\
 & \quad \quad \quad \underline{+1 \quad +1} \quad \underline{-2 \quad -2} \\
 & \quad \quad \quad \frac{4x}{4} = \frac{1}{4} \quad x = -2 \\
 & \quad \quad \quad x = \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{Check: } & \sqrt{3-3\left(\frac{1}{4}\right)} - 1 = 2\left(\frac{1}{4}\right) \\
 & \sqrt{3-\frac{3}{4}} - 1 = \frac{1}{2} \\
 & \sqrt{\frac{9}{4}} - 1 = \frac{1}{2} \\
 & \frac{3}{2} - 1 = \frac{1}{2} \\
 & \frac{1}{2} = \frac{1}{2} \checkmark
 \end{aligned}$$

$$\text{Check: } \sqrt{3 - 3(-2)} - 1 = 2(-2)$$

$$\sqrt{3 + 6} - 1 = -4$$

$$\sqrt{9} - 1 = -4$$

$$3 - 1 = -4$$

$$2 = -4 \text{ No!}$$

$$x = \frac{1}{4}$$

$$9) \sqrt{4x + 5} - \sqrt{x + 4} = 2$$

$$\frac{+\sqrt{x+4} + \sqrt{x+4}}{(\sqrt{4x+5})^2 = (2 + \sqrt{x+4})^2}$$

$$4x + 5 = 4 + 4\sqrt{x+4} + x + 4$$

$$4x + 5 = 8 + x + 4\sqrt{x+4}$$

$$\frac{-x - 8 - 8 - x}{(3x - 3)^2 = (4\sqrt{x+4})^2}$$

$$9x^2 - 18x + 9 = 16(x + 4)$$

$$9x^2 - 18x + 9 = 16x + 64$$

$$\frac{-16x - 64 - 16x - 64}{9x^2 - 34x - 55 = 0}$$

$$9x^2 - 34x - 55 = 0$$

$$(9x + 11)(x - 5) = 0$$

$$9x + 11 = 0 \quad x - 5 = 0$$

$$\frac{-11 - 11}{9} = \frac{(-11)}{9} \quad \frac{+5 + 5}{9} \quad x = 5$$

$$\frac{9x}{9} = \frac{(-11)}{9} \quad x = 5$$

$$x = -\frac{11}{9}$$

$$\text{Check: } \sqrt{4\left(-\frac{11}{9}\right) + 5} - \sqrt{-\frac{11}{9} + 4} = 2$$

$$\sqrt{-\frac{44}{9} + 5} - \sqrt{\frac{25}{9}} = 2$$

$$\sqrt{\frac{1}{9} - \frac{5}{3}} = 2$$

$$\frac{1}{3} - \frac{5}{3} = 2$$

$$-2 = 2 \text{ No!}$$

$$\text{Check: } \sqrt{4(5) + 5} - \sqrt{(5) + 4} = 2$$

$$\sqrt{20 + 5} - \sqrt{9} = 2$$

$$\sqrt{25} - 3 = 2$$

$$5 - 3 = 2$$

$$2 = 2 \checkmark$$

$$x = 5$$

$$11) \sqrt{2x + 4} - \sqrt{x + 3} = 1$$

$$\frac{+\sqrt{x+3} + \sqrt{x+3}}{(\sqrt{2x+4})^2 = (1 + \sqrt{x+3})^2}$$

$$2x + 4 = 1 + 2\sqrt{x+3} + x + 3$$

$$2x + 4 = 4 + x + 2\sqrt{x+3}$$

$$\frac{-x - 4 - 4 - x}{(x)^2 = (2\sqrt{x+3})^2}$$

$$(x)^2 = (2\sqrt{x+3})^2$$

$$x^2 = 4(x + 3)$$

$$x^2 = 4x + 12$$

$$-4x - 12 - 4x - 12$$

$$x^2 - 4x - 12 = 0$$

$$(x - 6)(x + 2) = 0$$

$$x - 6 = 0 \quad x + 2 = 0$$

$$\frac{+6 + 6}{x = 6} \quad \frac{-2 - 2}{x = -2}$$

$$x = 6 \quad x = -2$$

$$\text{Check: } \sqrt{2(6) + 4} - \sqrt{(6) + 3} = 1$$

$$\sqrt{12 + 4} - \sqrt{9} = 1$$

$$\sqrt{16} - 3 = 1$$

$$4 - 3 = 1$$

$$1 = 1 \checkmark$$

$$\text{Check: } \sqrt{2(-2) + 4} - \sqrt{(-2) + 3} = 1$$

$$\sqrt{-4 + 4} - \sqrt{1} = 1$$

$$\sqrt{0} - 1 = 1$$

$$0 - 1 = 1$$

$$-1 = 1 \text{ No!}$$

$$x = 6$$

$$\begin{aligned}
13) \quad & \sqrt{2x+6} - \sqrt{x+4} = 1 \\
& \quad \quad \quad \frac{+\sqrt{x+4} \quad +\sqrt{x+4}}{(\sqrt{2x+6})^2 = (1 + \sqrt{x+4})^2} \\
& \quad \quad \quad 2x+6 = 1 + 2\sqrt{x+4} + x + 4 \\
& \quad \quad \quad 2x+6 = 5 + x + 2\sqrt{x+4} \\
& \quad \quad \quad \frac{-x-5 \quad -5-x}{(x+1)^2 = (2\sqrt{x+4})^2} \\
& \quad \quad \quad x^2 + 2x + 1 = 4(x+4) \\
& \quad \quad \quad x^2 + 2x + 1 = 4x + 16 \\
& \quad \quad \quad \frac{-4x-16 \quad -4x-16}{x^2 - 2x - 15 = 0} \\
& \quad \quad \quad (x-5)(x+3) = 0 \\
& \quad \quad \quad x-5 = 0 \quad x+3 = 0 \\
& \quad \quad \quad \frac{+5 \quad +5 \quad -3 \quad -3}{x = 5 \quad x = -3}
\end{aligned}$$

$$\begin{aligned}
\text{Check: } & \sqrt{2(5)+6} - \sqrt{(5)+4} = 1 \\
& \sqrt{10+6} - \sqrt{9} = 1 \\
& \sqrt{16} - 3 = 1 \\
& 4 - 3 = 1 \\
& 1 = 1 \checkmark \\
\text{Check: } & \sqrt{2(-3)+6} - \sqrt{(-3)+4} = 1 \\
& \sqrt{-6+6} - \sqrt{1} = 1 \\
& \sqrt{0} - 1 = 1 \\
& 0 - 1 = 1 \\
& -1 = 1 \quad \text{No!} \\
& x = 5
\end{aligned}$$

$$\begin{aligned}
15) \quad & \sqrt{6-2x} - \sqrt{2x+3} = 3 \\
& \quad \quad \quad \frac{+\sqrt{2x+3} \quad +\sqrt{2x+3}}{(\sqrt{6-2x})^2 = (3 + \sqrt{2x+3})^2} \\
& \quad \quad \quad 6-2x = 9 + 6\sqrt{2x+3} + 2x + 3 \\
& \quad \quad \quad 6-2x = 2x + 12 + 6\sqrt{2x+3} \\
& \quad \quad \quad \frac{-12-2x \quad -2x-12}{(-6-4x)^2 = (6\sqrt{2x+3})^2} \\
& \quad \quad \quad 36 + 48x + 16x^2 = 36(2x+3) \\
& \quad \quad \quad 16x^2 + 48x + 36 = 72x + 108 \\
& \quad \quad \quad \frac{-72x-108 \quad -72x-108}{16x^2 - 24x - 72 = 0} \\
& \quad \quad \quad 8(2x^2 - 3x - 9) = 0 \\
& \quad \quad \quad 8(2x+3)(x-3) = 0 \\
& \quad \quad \quad 2x+3 = 0 \quad x-3 = 0 \\
& \quad \quad \quad \frac{-3 \quad -3 \quad +3 \quad +3}{\frac{2x}{2} = \frac{-3}{2} \quad x = 3} \\
& \quad \quad \quad x = -\frac{3}{2}
\end{aligned}$$

$$\begin{aligned}
\text{Check: } & \sqrt{6-2(-\frac{3}{2})} - \sqrt{2(-\frac{3}{2})+3} = 3 \\
& \sqrt{6+3} - \sqrt{-3+3} = 3 \\
& \sqrt{9} - \sqrt{0} = 3 \\
& 3 - 0 = 3 \\
& 3 = 3 \checkmark \\
\text{Check: } & \sqrt{6-2(3)} - \sqrt{2(3)+3} = 3 \\
& \sqrt{6-6} - \sqrt{6+3} = 3 \\
& \sqrt{0} - \sqrt{9} = 3 \\
& 0 - 3 = 3 \\
& -3 = 3 \quad \text{No!} \\
& x = -\frac{3}{2}
\end{aligned}$$

9.2

$$\begin{aligned}
1) \quad & \sqrt{x^2} = \sqrt{75} \\
& x = \pm\sqrt{75} \\
& x = \pm\sqrt{5^2 \cdot 3} \\
& x = \pm 5\sqrt{3}
\end{aligned}$$

$$\begin{aligned}
3) \quad & x^2 + 5 = 13 \\
& \quad \quad \quad \frac{-5 \quad -5}{\sqrt{x^2} = \sqrt{8}} \\
& \quad \quad \quad x = \pm\sqrt{2^3} \\
& \quad \quad \quad x = \pm 2\sqrt{2}
\end{aligned}$$

$$5) 3x^2 + 1 = 73$$

$$\frac{-1 \quad -1}{\frac{3x^2}{3} = \frac{72}{3}}$$

$$\sqrt{x^2} = \sqrt{24}$$

$$x = \pm\sqrt{24}$$

$$x = \pm\sqrt{2^3 \cdot 3}$$

$$x = \pm 2\sqrt{2 \cdot 3}$$

$$x = \pm 2\sqrt{6}$$

$$7) \sqrt[5]{(x+2)^5} = \sqrt[5]{-243}$$

$$x+2 = -3$$

$$\frac{-2 \quad -2}{x = -5}$$

$$x = -5$$

$$9) (2x+5)^3 - 6 = 21$$

$$+6 \quad +6$$

$$\sqrt[3]{(2x+5)^3} = \sqrt[3]{27}$$

$$2x+5 = 3$$

$$\frac{-5 \quad -5}{\frac{2x}{2} = \frac{(-2)}{2}}$$

$$\frac{2x}{2} = \frac{(-2)}{2}$$

$$x = -1$$

$$11) (x-1)^{\frac{2}{3}} = 16$$

$$\sqrt{(\sqrt[3]{x-1})^2} = \sqrt{16}$$

$$(\sqrt[3]{x-1})^3 = (\pm 4)^3$$

$$x-1 = \pm 64$$

$$\frac{+1 \quad +1}{x = 1 \pm 64}$$

$$x = 65, -63$$

$$13) (2-x)^{\frac{3}{2}} = 27$$

$$\sqrt[3]{(\sqrt{2-x})^3} = \sqrt[3]{27}$$

$$(\sqrt{2-x})^2 = 3^2$$

$$2-x = 9$$

$$\frac{-2 \quad -2}{\frac{-x}{-1} = \frac{7}{-1}}$$

$$\frac{-x}{-1} = \frac{7}{-1}$$

$$x = -7$$

$$\text{Check: } (2 - (-7))^{\frac{3}{2}} = 27$$

$$9^{\frac{3}{2}} = 27$$

$$(\sqrt{9})^3 = 27$$

$$3^3 = 27$$

$$27 = 27 \checkmark$$

$$15) (2x-3)^{\frac{2}{3}} = 4$$

$$\sqrt{(\sqrt[3]{2x-3})^2} = \sqrt{4}$$

$$(\sqrt[3]{2x-3})^3 = (\pm 2)^3$$

$$2x-3 = \pm 8$$

$$\frac{+3 \quad +3}{\frac{2x}{2} = \frac{3 \pm 8}{2}}$$

$$\frac{2x}{2} = \frac{3 \pm 8}{2}$$

$$x = \frac{11}{2}, -\frac{5}{2}$$

$$17) \left(x + \frac{1}{2}\right)^{-\frac{2}{3}} = 4$$

$$\sqrt{\left(\sqrt[3]{\frac{1}{x+\frac{1}{2}}}\right)^2} = \sqrt{4}$$

$$\left(\sqrt[3]{\frac{1}{x+\frac{1}{2}}}\right)^3 = (\pm 2)^3$$

$$\frac{1}{x+\frac{1}{2}} = \pm 8$$

$$\left(x + \frac{1}{2}\right) \frac{1}{x+\frac{1}{2}} =$$

$$\pm 8 \left(x + \frac{1}{2}\right)$$

$$\frac{1}{\pm 8} = \frac{\pm 8(x+\frac{1}{2})}{\pm 8}$$

$$\pm \frac{1}{8} = x + \frac{1}{2}$$

$$\frac{-\frac{1}{2} \quad -\frac{1}{2}}{\frac{-\frac{1}{2} \pm \frac{1}{8}}{-\frac{1}{2}} = x}$$

$$\frac{-\frac{1}{2} \pm \frac{1}{8}}{-\frac{1}{2}} = x$$

$$x = -\frac{3}{8}, -\frac{5}{8}$$

$$19) r(x-1)^{-\frac{5}{2}} = 32$$

$$\left(\frac{1}{x-1}\right)^{\frac{5}{2}} = 32$$

$$\sqrt[5]{\left(\sqrt{\frac{1}{x-1}}\right)^5} = \sqrt[5]{32}$$

$$\left(\sqrt{\frac{1}{x-1}}\right)^2 = (2)^2$$

$$(x-1) \frac{1}{x-1} = 4(x-1)$$

$$1 = 4x - 4$$

$$\frac{+4 \quad +4}{\frac{5}{4} = \frac{4x}{4}}$$

$$\frac{5}{4} = \frac{4x}{4}$$

$$\frac{5}{4} = x$$

$$\text{Check: } \left(\frac{5}{4} - 1\right)^{-\frac{5}{2}} = 32$$

$$\left(\frac{1}{4}\right)^{-\frac{5}{2}} = 32$$

$$4^{\frac{5}{2}} = 32$$

$$(\sqrt{4})^5 = 32$$

$$2^5 = 32$$

$$32 = 32 \checkmark$$

$$x = \frac{5}{4}$$

$$21) (3x-2)^{\frac{4}{5}} = 16$$

$$\sqrt[4]{(\sqrt[5]{3x-2})^4} = \sqrt[4]{16}$$

$$(\sqrt[5]{3x-2})^5 = +2^5$$

$$3x-2 = \pm 32$$

$$\frac{+2 \quad +2}{\frac{3x}{3} = \frac{2 \pm 32}{3}}$$

$$x = \frac{34}{3}, -10$$

$$\begin{aligned}
 23) \quad (4x + 2)^{\frac{3}{5}} &= -8 \\
 \sqrt[3]{(\sqrt[5]{4x + 2})^3} &= \sqrt[3]{-8} \\
 (\sqrt[5]{4x + 2})^5 &= (-2)^5 \\
 4x + 2 &= -32 \\
 \frac{-2}{4} &= \frac{-34}{4} \\
 x &= -\frac{17}{2}
 \end{aligned}$$

9.3

$$\begin{aligned}
 1) \quad x^2 - 30x + \underline{\quad} \\
 (-30 \cdot \frac{1}{2})^2 \\
 (-15)^2 = 225 \\
 x^2 - 30x + 225 \\
 (x - 15)^2
 \end{aligned}$$

$$\begin{aligned}
 3) \quad m^2 - 36m + \underline{\quad} \\
 (-36 \cdot \frac{1}{2})^2 \\
 (-18)^2 = 324 \\
 m^2 - 36m + 324 \\
 (m - 18)^2
 \end{aligned}$$

$$\begin{aligned}
 5) \quad x^2 - 15x + \underline{\quad} \\
 (-15 \cdot \frac{1}{2})^2 \\
 (-\frac{15}{2})^2 = \frac{225}{4} \\
 x^2 - 15x + \frac{225}{4} \\
 (x - \frac{15}{2})^2
 \end{aligned}$$

$$\begin{aligned}
 7) \quad y^2 - y + \underline{\quad} \\
 (-1 \cdot \frac{1}{2})^2 \\
 (-\frac{1}{2})^2 = \frac{1}{4} \\
 y^2 - y + \frac{1}{4} \\
 (y - \frac{1}{2})^2
 \end{aligned}$$

$$\begin{aligned}
 9) \quad x^2 - 16x + 55 = 0 \\
 \frac{-55 - 55}{x^2 - 16x} = -55 \\
 (-16 \cdot \frac{1}{2})^2 \\
 (-8)^2 = 64 \\
 x^2 - 16x + 64 = -55 + 64 \\
 \sqrt{(x - 8)^2} = \sqrt{9} \\
 x - 8 = \pm 3 \\
 \frac{+8}{x} \quad \frac{+8}{+8} \\
 x = 8 \pm 3 \\
 x = 11, 5
 \end{aligned}$$

$$\begin{aligned}
 11) \quad v^2 - 8v + 45 = 0 \\
 \frac{-45 - 45}{v^2 - 8v} = -45 \\
 (-8 \cdot \frac{1}{2})^2 \\
 (-4)^2 = 16 \\
 v^2 - 8v + 16 = -45 + 16 \\
 \sqrt{(v - 4)^2} = \sqrt{-29} \\
 v - 4 = \pm i\sqrt{29} \\
 \frac{+4}{v} \quad \frac{+4}{+4} \\
 v = 4 \pm i\sqrt{29}
 \end{aligned}$$

$$13) 6x^2 + 12x + 63 = 0$$

$$\frac{6x^2}{6} + \frac{12x}{6} + \frac{-63}{6} = \frac{-63}{6}$$

$$x^2 + 2x = -\frac{21}{2}$$

$$\left(2 \cdot \frac{1}{2}\right)^2$$

$$(1)^2 = 1$$

$$x^2 + 2x + 1 = -\frac{21}{2} + 1$$

$$\sqrt{(x+1)^2} = \sqrt{-\frac{19}{2}} \left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$x+1 = \pm \frac{i\sqrt{38}}{2}$$

$$\frac{-1}{2} \quad \frac{-1}{2}$$

$$x = \frac{-2 \pm i\sqrt{38}}{2}$$

$$15) 5k^2 - 10k + 48 = 0$$

$$\frac{5k^2}{5} - \frac{10k}{5} + \frac{-48}{5} = \frac{-48}{5}$$

$$k^2 - 2k = \frac{-48}{5}$$

$$\left(-2 \cdot \frac{1}{2}\right)^2$$

$$(-1)^2 = 1$$

$$k^2 - 2k + 1 = -\frac{48}{5} + 1$$

$$\sqrt{(k-1)^2} = \sqrt{-\frac{43}{5}} \left(\frac{\sqrt{5}}{\sqrt{5}}\right)$$

$$k-1 = \pm \frac{i\sqrt{215}}{5}$$

$$\frac{+1}{5} \quad \frac{+1}{5}$$

$$k = \frac{5 \pm i\sqrt{215}}{5}$$

$$17) x^2 + 10x - 57 = 4$$

$$\frac{+57}{1} + \frac{+57}{1}$$

$$x^2 + 10x = 61$$

$$\left(10 \cdot \frac{1}{2}\right)^2$$

$$(5)^2 = 25$$

$$x^2 + 10x + 25 = 61 + 25$$

$$\sqrt{(x+5)^2} = \sqrt{86}$$

$$x+5 = \pm\sqrt{86}$$

$$\frac{-5}{1} \quad \frac{-5}{1}$$

$$x = -5 \pm \sqrt{86}$$

$$19) n^2 - 16n + 67 = 4$$

$$n^2 - 16n + \frac{-67}{1} = \frac{-67}{1}$$

$$\left(-16 \cdot \frac{1}{2}\right)^2 = (-8)^2 = 64$$

$$n^2 - 16n + 64 = 63 + 64$$

$$\sqrt{(n-8)^2} = \sqrt{1}$$

$$n-8 = \pm 1$$

$$\frac{+8}{1} \quad \frac{+8}{1}$$

$$n = 9, 7$$

$$21) 2x^2 + 4x + 38 = -6$$

$$\frac{2x^2}{2} + \frac{4x}{2} + \frac{-38}{2} = \frac{-44}{2}$$

$$x^2 + 2x = -22$$

$$\left(2 \cdot \frac{1}{2}\right)^2 = 1^2 = 1$$

$$x^2 + 2x + 1 = -22 + 1$$

$$\sqrt{(x+1)^2} = \sqrt{-21}$$

$$x+1 = \pm i\sqrt{21}$$

$$\frac{-1}{1} \quad \frac{-1}{1}$$

$$x = -1 \pm i\sqrt{21}$$

$$23) 8b^2 + 16b - 37 = 5$$

$$\frac{8b^2}{8} + \frac{16b}{8} - \frac{37}{8} = \frac{42}{8}$$

$$b^2 + 2b = \frac{21}{4}$$

$$\left(2 \cdot \frac{1}{2}\right)^2 = 1^2 = 1$$

$$b^2 + 2b + 1 = \frac{21}{4} + 1$$

$$\sqrt{(b+1)^2} = \sqrt{\frac{25}{4}}$$

$$b+1 = \pm \frac{5}{2}$$

$$\frac{-1}{1} \quad \frac{-1}{1}$$

$$b = -1 \pm \frac{5}{2}$$

$$b = \frac{3}{2}, -\frac{7}{2}$$

$$\begin{aligned}
25) \quad r \quad x^2 &= -10x - 29 \\
\frac{+10x + 10x}{x^2 + 10x} &= -29 \\
\left(10 \cdot \frac{1}{2}\right)^2 &= (5)^2 = 25 \\
x^2 + 10x + 25 &= -29 + 25 \\
\sqrt{(x+5)^2} &= \sqrt{-4} \\
x + 5 &= \pm 2i \\
\frac{-5 \quad -5}{x} &= -5 \pm 2i
\end{aligned}$$

$$\begin{aligned}
27) \quad n^2 &= -21 + 10n \\
\frac{-10n \quad -10n}{n^2 - 10n} &= -21 \\
\left(-10 \cdot \frac{1}{2}\right)^2 &= (-5)^2 = 25 \\
n^2 - 10n + 25 &= -21 + 25 \\
\sqrt{(n-5)^2} &= \sqrt{4} \\
n - 5 &= \pm 2 \\
\frac{+5 \quad +5}{n} &= 5 \pm 2 \\
n &= 7, 3
\end{aligned}$$

$$\begin{aligned}
29) \quad 3k^2 + 9 &= 6k \\
\frac{-6k - 9 \quad -6k - 9}{\frac{3k^2}{3} - \frac{6k}{3}} &= \frac{-9}{3} \\
k^2 - 2k &= -3 \\
\left(-2 \cdot \frac{1}{2}\right)^2 &= (-1)^2 = 1 \\
k^2 - 2k + 1 &= -3 + 1 \\
\sqrt{(k-1)^2} &= \sqrt{-2} \\
k - 1 &= \pm i\sqrt{2} \\
\frac{+1 \quad +1}{k} &= 1 \pm i\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
31) \quad 2x^2 + 63 &= 8x \\
\frac{-8x - 63 \quad -8x - 63}{\frac{2x^2}{2} - \frac{8x}{2}} &= -\frac{63}{2} \\
x^2 - 4x &= -\frac{63}{2} \\
\left(-4 \cdot \frac{1}{2}\right)^2 &= (-2)^2 = 4 \\
x^2 - 4x + 4 &= -\frac{63}{2} + 4 \\
\sqrt{(x-2)^2} &= \sqrt{-\frac{55}{2}} \left(\frac{\sqrt{2}}{\sqrt{2}}\right) \\
x - 2 &= \pm \frac{i\sqrt{110}}{2} \\
\frac{+2 \quad +2}{x} &= \frac{4 \pm i\sqrt{110}}{2}
\end{aligned}$$

$$\begin{aligned}
33) \quad p^2 - 8p &= -55 \\
\left(-8 \cdot \frac{1}{2}\right)^2 &= (-4)^2 = 16 \\
p^2 - 8p + 16 &= -55 + 16 \\
\sqrt{(p-4)^2} &= \sqrt{-39} \\
p - 4 &= \pm i\sqrt{39} \\
\frac{+4 \quad +4}{p} &= 4 \pm i\sqrt{39}
\end{aligned}$$

$$\begin{aligned}
35) \quad 7n^2 - n + 7 &= 7n + 6n^2 \\
\frac{-6n^2 - 7n - 7 \quad -6n^2 - 7n - 7}{n^2 - 8n} &= -7 \\
\left(-8 \cdot \frac{1}{2}\right)^2 &= (-4)^2 = 16 \\
n^2 - 8n + 16 &= -7 + 16 \\
\sqrt{(n-4)^2} &= \sqrt{9} \\
n - 4 &= \pm 3 \\
\frac{+4 \quad +4}{n} &= 4 \pm 3 \\
n &= 7, 1
\end{aligned}$$

$$37) 13b^2 + 15b + 44 = -5 + 7b^2 + 3b$$

$$\frac{-7b^2 - 3b - 44}{6} = \frac{-49}{6}$$

$$b^2 + 2b = -\frac{49}{6}$$

$$\left(2 \cdot \frac{1}{2}\right)^2 = 1^2 = 1$$

$$b^2 + 2b + 1 = -\frac{49}{6} + 1$$

$$\sqrt{(b+1)^2} = \sqrt{-\frac{43}{6}} \left(\frac{\sqrt{6}}{\sqrt{6}}\right)$$

$$b+1 = \pm \frac{i\sqrt{256}}{6}$$

$$\frac{-1 \quad -1}{b} = \frac{-6 \pm i\sqrt{256}}{6}$$

$$39) 5x^2 + 5x = -31 - 5x$$

$$\frac{+5x}{5} + \frac{+5x}{5} = \frac{-31}{5}$$

$$x^2 + 2x = -\frac{31}{5}$$

$$\left(2 \cdot \frac{1}{2}\right)^2 = 1^2 = 1$$

$$x^2 + 2x + 1 = -\frac{31}{5} + 1$$

$$\sqrt{(x+1)^2} = \sqrt{-\frac{26}{5}} \left(\frac{\sqrt{5}}{\sqrt{5}}\right)$$

$$x+1 = \pm \frac{i\sqrt{130}}{5}$$

$$\frac{-1 \quad -1}{x} = \frac{-5 \pm i\sqrt{130}}{5}$$

$$41) v^2 + 5v + 28 = 0$$

$$\frac{-28 \quad -28}{v^2 + 5v} = -28$$

$$\left(5 \cdot \frac{1}{2}\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

$$v^2 + 5v + \frac{25}{4} = -28 + \frac{25}{4}$$

$$\sqrt{\left(v + \frac{5}{2}\right)^2} = \sqrt{-\frac{87}{4}}$$

$$v + \frac{5}{2} = \pm \frac{i\sqrt{87}}{2}$$

$$\frac{-\frac{5}{2} \quad -\frac{5}{2}}{v} = \frac{-5 \pm i\sqrt{87}}{2}$$

$$43) 7x^2 - 6x + 40 = 0$$

$$\frac{-40 \quad -40}{7x^2 - 6x} = -\frac{40}{7}$$

$$x^2 - \frac{6}{7}x = -\frac{40}{7}$$

$$\left(-\frac{6}{7} \cdot \frac{1}{2}\right)^2 = \left(-\frac{3}{7}\right)^2 = \frac{9}{49}$$

$$x^2 - \frac{6}{7}x + \frac{9}{49} = -\frac{40}{7} + \frac{9}{49}$$

$$\sqrt{\left(x - \frac{3}{7}\right)^2} = \sqrt{-\frac{271}{49}}$$

$$x - \frac{3}{7} = \pm \frac{i\sqrt{271}}{7}$$

$$\frac{+\frac{3}{7} \quad +\frac{3}{7}}{x} = \frac{3 \pm i\sqrt{271}}{7}$$

$$45) k^2 - 7k + 50 = 3$$

$$\frac{-50 \quad -50}{k^2 - 7k} = -47$$

$$\left(-7 \cdot \frac{1}{2}\right)^2 = \left(-\frac{7}{2}\right)^2 = \frac{49}{4}$$

$$k^2 - 7k + \frac{49}{4} = -47 + \frac{49}{4}$$

$$\sqrt{\left(k - \frac{7}{2}\right)^2} = \sqrt{-\frac{139}{4}}$$

$$k - \frac{7}{2} = \pm \frac{i\sqrt{139}}{2}$$

$$\frac{+\frac{7}{2} \quad +\frac{7}{2}}{k} = \frac{7 \pm i\sqrt{139}}{2}$$

$$47) 5x^2 + 8x - 40 = 8$$

$$\frac{+40 \quad +40}{5x^2 + 8x} = \frac{48}{5}$$

$$x^2 + \frac{8}{5}x = \frac{48}{5}$$

$$\left(\frac{8}{5} \cdot \frac{1}{2}\right)^2 = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$

$$x^2 + \frac{8}{5}x + \frac{16}{25} = \frac{48}{5} + \frac{16}{25}$$

$$\sqrt{\left(x + \frac{4}{5}\right)^2} = \sqrt{\frac{256}{25}}$$

$$x + \frac{4}{5} = \pm \frac{16}{5}$$

$$\frac{-\frac{4}{5} \quad -\frac{4}{5}}{x} = \frac{-4 \pm 16}{5}$$

$$\begin{aligned}
49) \quad m^2 &= -15 + 9m \\
\frac{-9m}{m^2 - 9m} &= \frac{-9m}{-15} \\
\left(-9 \cdot \frac{1}{2}\right)^2 &= \left(-\frac{9}{2}\right)^2 = \frac{81}{4} \\
m^2 - 9m + \frac{81}{4} &= -15 + \frac{81}{4} \\
\sqrt{\left(m - \frac{9}{2}\right)^2} &= \sqrt{\frac{21}{4}} \\
m - \frac{9}{2} &= \pm \frac{\sqrt{21}}{2} \\
\frac{+ \frac{9}{2}}{m} &= \frac{+ \frac{9}{2}}{\pm \frac{\sqrt{21}}{2}} \\
m &= \frac{9 \pm \sqrt{21}}{2}
\end{aligned}$$

$$\begin{aligned}
51) \quad \frac{8r^2}{8} + \frac{10r}{8} &= -\frac{55}{8} \\
r^2 + \frac{5}{4}r &= -\frac{55}{8} \\
\left(\frac{5}{4} \cdot \frac{1}{2}\right)^2 &= \left(\frac{5}{8}\right)^2 = \frac{25}{64} \\
r^2 + \frac{5}{4}r + \frac{25}{64} &= -\frac{55}{8} + \frac{25}{64} \\
\sqrt{\left(r + \frac{5}{8}\right)^2} &= \sqrt{\frac{-415}{64}} \\
r + \frac{5}{8} &= \pm \frac{i\sqrt{415}}{8} \\
\frac{-\frac{5}{8}}{r} &= \frac{-\frac{5}{8}}{\pm \frac{i\sqrt{415}}{8}} \\
r &= \frac{-5 \pm i\sqrt{415}}{8}
\end{aligned}$$

$$\begin{aligned}
53) \quad 5n^2 - 8n + 60 &= -3n - 6 + 4n^2 \\
\frac{-4n^2 + 3n - 60 + 3n - 60 - 4n^2}{n^2 + 5n} &= \frac{-8n - 120}{-54} \\
\left(5 \cdot \frac{1}{2}\right)^2 &= \left(\frac{5}{2}\right)^2 = \frac{25}{4} \\
n^2 + 5n + \frac{25}{4} &= -54 + \frac{25}{4} \\
\sqrt{\left(n + \frac{5}{2}\right)^2} &= \sqrt{-\frac{191}{4}} \\
n + \frac{5}{2} &= \pm \frac{i\sqrt{191}}{2} \\
\frac{-\frac{5}{2}}{n} &= \frac{-\frac{5}{2}}{\pm \frac{i\sqrt{191}}{2}} \\
n &= \frac{-5 \pm i\sqrt{191}}{2}
\end{aligned}$$

$$\begin{aligned}
55) \quad 2x^2 + 3x - 5 &= -4x^2 \\
\frac{+4x^2}{6x^2 + 3x} + \frac{+5}{6} &= \frac{+4x^2 + 5}{6} \\
x^2 + \frac{1}{2}x &= \frac{5}{6} \\
\left(\frac{1}{2} \cdot \frac{1}{2}\right)^2 &= \left(\frac{1}{4}\right)^2 = \frac{1}{16} \\
x^2 + \frac{1}{2}x + \frac{1}{16} &= \frac{5}{6} + \frac{1}{16} \\
\sqrt{\left(x + \frac{1}{4}\right)^2} &= \sqrt{\frac{43}{48}} = \frac{\sqrt{43}}{4\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}}\right) \\
x + \frac{1}{4} &= \pm \frac{\sqrt{129}}{12} \\
\frac{-\frac{1}{4}}{x} &= \frac{-\frac{1}{4}}{\pm \frac{\sqrt{129}}{12}} \\
x &= \frac{(-3 \pm \sqrt{129})}{12}
\end{aligned}$$

$$\begin{aligned}
57) \quad -2x^2 + 3x - 5 &= -4x^2 \\
\frac{+4x^2}{2x^2 + 3x} + \frac{+5}{2} &= \frac{+4x^2 + 5}{2} \\
x^2 + \frac{3}{2}x &= \frac{5}{2} \\
\left(\frac{3}{2} \cdot \frac{1}{2}\right)^2 &= \left(\frac{3}{4}\right)^2 = \frac{9}{16} \\
x^2 + \frac{3}{2}x + \frac{9}{16} &= \frac{5}{2} + \frac{9}{16} \\
\sqrt{\left(x + \frac{3}{4}\right)^2} &= \sqrt{\frac{49}{16}} \\
x + \frac{3}{4} &= \pm \frac{7}{4} \\
\frac{-\frac{3}{4}}{x} &= \frac{-\frac{3}{4}}{\pm \frac{7}{4}} \\
x &= -\frac{3}{4} \pm \frac{7}{4} \\
x &= 1, -\frac{5}{2}
\end{aligned}$$

9.4

$$1) 4a^2 + 6 = 0$$

$$a = 4, b = 0, c = 6$$

$$\frac{-0 \pm \sqrt{0^2 - 4(4)(6)}}{2(4)} = \frac{\pm \sqrt{-96}}{8} = \frac{\pm \sqrt{-16 \cdot 6}}{8} = \frac{\pm 4i\sqrt{6}}{8} = \frac{\pm i\sqrt{6}}{2}$$

$$3) 2x^2 - 8x - 2 = 0$$

$$a = 2, b = -8, c = -2$$

$$\frac{8 \pm \sqrt{(-8)^2 - 4(2)(-2)}}{2(2)} = \frac{8 \pm \sqrt{64 + 16}}{4} = \frac{8 \pm \sqrt{80}}{4} = \frac{8 \pm \sqrt{16 \cdot 5}}{4} = \frac{8 \pm 4\sqrt{5}}{4} = 2 \pm \sqrt{5}$$

$$5) 2m^2 - 3 = 0$$

$$a = 2, b = 0, c = -3$$

$$\frac{-0 \pm \sqrt{(0)^2 - 4(2)(-3)}}{2(2)} = \frac{\pm \sqrt{24}}{4} = \frac{\pm \sqrt{4 \cdot 6}}{4} = \frac{(\pm 2\sqrt{6})}{4} = \frac{\pm \sqrt{6}}{2}$$

$$7) 3r^2 - 2r - 1 = 0$$

$$a = 3, b = -2, c = -1$$

$$\frac{2 \pm \sqrt{(-2)^2 - 4(3)(-1)}}{2(3)} = \frac{2 \pm \sqrt{4 + 12}}{6} = \frac{2 \pm \sqrt{16}}{6} = \frac{2 \pm 4}{6} = 1, -\frac{1}{3}$$

$$9) 4n^2 - 36 = 0$$

$$a = 4, b = 0, c = -36$$

$$\frac{-0 \pm \sqrt{0^2 - 4(4)(-36)}}{2(4)} = \frac{\pm \sqrt{576}}{8} = \frac{\pm 24}{8} = \pm 3$$

$$11) v^2 - 4v - 5 = -8$$

$$\frac{\quad +8 \quad +8}{\quad \quad \quad}$$

$$v^2 - 4v + 3 = 0$$

$$a = 1, b = -4, c = 3$$

$$\frac{4 \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)} = \frac{4 \pm \sqrt{16 - 12}}{2} = \frac{4 \pm \sqrt{4}}{2} = \frac{4 \pm 2}{2} = 3, 1$$

$$13) 2a^2 + 3a + 14 = 6$$

$$\frac{\quad -14 \quad -14}{\quad \quad \quad}$$

$$2a^2 + 3a + 8 = 0$$

$$a = 2, b = 3, c = 8$$

$$\frac{-3 \pm \sqrt{(3)^2 - 4(2)(8)}}{2(2)} = \frac{-3 \pm \sqrt{9 - 64}}{4} = \frac{-3 \pm \sqrt{-55}}{4} = \frac{-3 \pm i\sqrt{55}}{4}$$

$$15) 3k^2 + 3k - 4 = 7$$

$$\frac{-7 \quad -7}{\quad \quad}$$

$$3k^2 + 3k - 11 = 0$$

$$a = 3, b = 3, c = -11$$

$$\frac{-3 \pm \sqrt{3^2 - 4(3)(-11)}}{2(3)} = \frac{-3 \pm \sqrt{9 + 132}}{6} = \frac{-3 \pm \sqrt{141}}{6}$$

$$17) 7x^2 + 3x - 16 = -2$$

$$\frac{+2 \quad +2}{\quad \quad}$$

$$7x^2 + 3x - 14 = 0$$

$$a = 7, b = 3, c = -14$$

$$\frac{-3 \pm \sqrt{3^2 - 4(7)(-14)}}{2(7)} = \frac{-3 \pm \sqrt{9 + 392}}{14} = \frac{-3 \pm \sqrt{401}}{14}$$

$$19) 2p^2 + 6p - 16 = 4$$

$$\frac{-4 \quad -4}{\quad \quad}$$

$$2p^2 + 6p - 20 = 0$$

$$a = 2, b = 6, c = -20$$

$$\frac{-6 \pm \sqrt{6^2 - 4(2)(-20)}}{2(2)} = \frac{-6 \pm \sqrt{36 + 160}}{4} = \frac{-6 \pm \sqrt{196}}{4} = \frac{-6 \pm 14}{4} = 2, -5$$

$$21) 3n^2 + 3n = -3$$

$$\frac{+3 \quad +3}{\quad \quad}$$

$$3n^2 + 3n + 3 = 0$$

$$a = 3, b = 3, c = 3$$

$$\frac{-3 \pm \sqrt{3^2 - 4(3)(3)}}{2(3)} = \frac{-3 \pm \sqrt{9 - 36}}{6} = \frac{-3 \pm \sqrt{-9 \cdot 3}}{6} = \frac{-3 \pm 3i\sqrt{3}}{6} = \frac{-1 \pm i\sqrt{3}}{2}$$

$$23) 2x^2 = -7x + 49$$

$$\frac{+7x - 49 + 7x - 49}{\quad \quad}$$

$$2x^2 + 7x - 49 = 0$$

$$a = 2, b = 7, c = -49$$

$$\frac{-7 \pm \sqrt{7^2 - 4(2)(-49)}}{2(2)} = \frac{-7 \pm \sqrt{49 + 392}}{4} = \frac{-7 \pm \sqrt{441}}{4}$$

$$25) 5x^2 = 7x + 7$$

$$\frac{-7x - 7 - 7x - 7}{\quad \quad}$$

$$5x^2 - 7x - 7 = 0$$

$$a = 5, b = -7, c = -7$$

$$\frac{7 \pm \sqrt{(-7)^2 - 4(5)(-7)}}{2(5)} = \frac{7 \pm \sqrt{49 + 140}}{10} = \frac{7 \pm \sqrt{189}}{10} = \frac{7 \pm \sqrt{9 \cdot 21}}{10} = \frac{7 \pm 3\sqrt{21}}{10}$$

$$\begin{aligned}
27) \quad & 8n^2 \quad = -3n - 8 \\
& \frac{+3n + 8 \quad + 3n + 8}{8n^2 + 3n + 8 = 0} \\
& a = 8, b = 3, c = 8 \\
& \frac{-3 \pm \sqrt{3^2 - 4(8)(8)}}{2(8)} = \frac{-3 \pm \sqrt{9 - 256}}{16} = \frac{-3 \pm \sqrt{-247}}{16} = \frac{-3 \pm i\sqrt{247}}{16}
\end{aligned}$$

$$\begin{aligned}
29) \quad & 2x^2 + 5x = -3 \\
& \frac{+3 \quad + 3}{2x^2 + 5x + 3 = 0} \\
& a = 2, b = 5, c = 3 \\
& \frac{-5 \pm \sqrt{5^2 - 4(2)(3)}}{2(2)} = \frac{-5 \pm \sqrt{25 - 24}}{4} = \frac{-5 \pm \sqrt{1}}{4} = \frac{-5 \pm 1}{4} = -1, -\frac{3}{2}
\end{aligned}$$

$$\begin{aligned}
31) \quad & 4a^2 - 64 = 0 \\
& a = 4, b = 0, c = -64 \\
& \frac{-0 \pm \sqrt{0^2 - 4(4)(-64)}}{2(4)} = \frac{\pm \sqrt{1024}}{8} = \frac{\pm 32}{8} = \pm 4
\end{aligned}$$

$$\begin{aligned}
33) \quad & 4p^2 + 5p - 36 = 3p^2 \\
& \frac{-3p^2 \quad - 3p^2}{p^2 + 5p - 36 = 0} \\
& a = 1, b = 5, c = -36 \\
& \frac{-5 \pm \sqrt{5^2 - 4(1)(-36)}}{2(1)} = \frac{-5 \pm \sqrt{25 + 144}}{2} = \frac{-5 \pm \sqrt{169}}{2} = \frac{-5 \pm 13}{2} = 4, -9
\end{aligned}$$

$$\begin{aligned}
35) \quad & -5n^2 - 3n - 52 = 2 - 7n^2 \\
& \frac{+7n^2 \quad - 2 \quad - 2 + 7n^2}{2n^2 - 3n - 54 = 0} \\
& a = 2, b = -3, c = -54 \\
& \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-54)}}{2(2)} = \frac{3 \pm \sqrt{9 + 432}}{4} = \frac{3 \pm \sqrt{441}}{4} = \frac{3 \pm 21}{4} = 6, -\frac{9}{2}
\end{aligned}$$

$$\begin{aligned}
37) \quad & 7r^2 - 12 = -3r \\
& \frac{+3r \quad + 3r}{7r^2 + 3r - 12 = 0} \\
& a = 7, b = 3, c = -12 \\
& \frac{-3 \pm \sqrt{3^2 - 4(7)(-12)}}{2(7)} = \frac{-3 \pm \sqrt{9 + 336}}{14} = \frac{-3 \pm \sqrt{345}}{14}
\end{aligned}$$

$$39) 2n^2 - 9 = 4$$

$$\begin{array}{r} -4 \quad -4 \\ \hline 2n^2 - 13 = 0 \end{array}$$

$$a = 2, b = 0, c = -13$$

$$\frac{-0 \pm \sqrt{0^2 - 4(2)(-13)}}{2(2)} = \frac{\pm \sqrt{104}}{4} = \frac{\pm \sqrt{4 \cdot 26}}{4} = \frac{\pm 2\sqrt{26}}{4} = \frac{\pm \sqrt{26}}{2}$$

9.5

$$1) 2, 5$$

$$x = 2, \quad x = 5$$

$$\begin{array}{r} -2 \quad -2 \quad -5 \quad -5 \\ \hline \end{array}$$

$$x - 2 = 0 \quad x - 5 = 0$$

$$(x - 2)(x - 5) = 0$$

$$x^2 - 5x - 2x + 10 = 0$$

$$x^2 - 7x + 10 = 0$$

$$9) -4, 11$$

$$x = -4 \quad x = 11$$

$$\begin{array}{r} +4 \quad +4 \quad -11 \quad -11 \\ \hline \end{array}$$

$$x + 4 = 0 \quad x - 11 = 0$$

$$(x + 4)(x - 11) = 0$$

$$x^2 + 4x - 11x - 44 = 0$$

$$x^2 - 7x - 44 = 0$$

$$15) \frac{3}{7}, 4$$

$$(7)x = \frac{3}{7}(x) \quad x = 4$$

$$7x = 3 \quad \begin{array}{r} -4 \quad -4 \\ \hline \end{array}$$

$$\begin{array}{r} -3 \quad -3 \\ \hline \end{array} \quad x - 4 = 0$$

$$7x - 3 = 0$$

$$(7x - 3)(x - 4) = 0$$

$$7x^2 - 3x - 28x + 12 = 0$$

$$7x^2 - 31x + 12 = 0$$

$$3) 20, 2$$

$$x = 20 \quad x = 2$$

$$\begin{array}{r} -20 \quad -20 \quad -2 \quad -2 \\ \hline \end{array}$$

$$x - 20 = 0 \quad x - 2 = 0$$

$$(x - 20)(x - 2) = 0$$

$$x^2 - 20x - 2x + 40 = 0$$

$$x^2 - 22x + 40 = 0$$

$$11) \frac{3}{4}, \frac{1}{4}$$

$$4(x) = \left(\frac{3}{4}\right)4, \quad 4(x) = \left(\frac{1}{4}\right)4$$

$$4x = 3 \quad 4x = 1$$

$$\begin{array}{r} -3 \quad -3 \quad -1 \quad -1 \\ \hline \end{array}$$

$$4x - 3 = 0 \quad 4x - 1 = 0$$

$$(4x - 3)(4x - 1) = 0$$

$$16x^2 - 12x - 4x + 3 = 0$$

$$16x^2 - 8x + 3 = 0$$

$$17) -\frac{1}{3}, \frac{5}{6}$$

$$(3)x = -\frac{1}{3}(3) \quad 6(x) = \frac{5}{6}(6)$$

$$3x = -1 \quad 6x = 5$$

$$\begin{array}{r} +1 \quad +1 \quad -5 \quad -5 \\ \hline \end{array}$$

$$3x + 1 = 0 \quad 6x - 5 = 0$$

$$(3x + 1)(6x - 5) = 0$$

$$18x^2 + 6x - 15x - 5 = 0$$

$$18x^2 - 9x - 5 = 0$$

$$5) 4, 4$$

$$x = 4 \quad x = 4$$

$$\begin{array}{r} -4 \quad -4 \quad -4 \quad -4 \\ \hline \end{array}$$

$$x - 4 = 0 \quad x - 4 = 0$$

$$(x - 4)(x - 4) = 0$$

$$x^2 - 4x - 4x + 16 = 0$$

$$x^2 - 8x + 16 = 0$$

$$13) \frac{1}{2}, \frac{1}{3}$$

$$(2)x = \frac{1}{2}(2) \quad (3)x = \frac{1}{3}(3)$$

$$2x = 1 \quad 3x = 1$$

$$\begin{array}{r} -1 \quad -1 \quad -1 \quad -1 \\ \hline \end{array}$$

$$2x - 1 = 0 \quad 3x - 1 = 0$$

$$(2x - 1)(3x - 1) = 0$$

$$6x^2 - 3x - 2x + 1 = 0$$

$$6x^2 - 5x + 1 = 0$$

$$19) -6, \frac{1}{9}$$

$$x = -6 \quad (9)x = \frac{1}{9}(9)$$

$$+6 \quad +6 \quad 9x = 1$$

$$\begin{array}{r} -1 \quad -1 \\ \hline \end{array} \quad x + 6 = 0$$

$$9x - 1 = 0$$

$$(x + 6)(9x - 1) = 0$$

$$9x^2 + 54x - x - 6 = 0$$

$$9x^2 - 53x - 6 = 0$$

$$7) 0, 0$$

$$x = 0, x = 0$$

$$xx = 0$$

$$x^2 = 0$$

$$21) \pm 5$$

$$x^2 = (\pm 5)^2$$

$$x^2 = 25$$

$$\begin{array}{r} -25 \quad -25 \\ \hline \end{array}$$

$$x^2 - 25 = 0$$

$$\begin{aligned}
 23) \quad & \pm \frac{1}{5} \\
 & x^2 = \left(\pm \frac{1}{5}\right)^2 \\
 & (25)x^2 = \frac{1}{25}(25) \\
 & 25x^2 = 1 \\
 & \frac{-1 \quad -1}{25x^2 - 1} = 0
 \end{aligned}$$

$$\begin{aligned}
 25) \quad & \pm\sqrt{11} \\
 & x^2 = (\pm\sqrt{11})^2 \\
 & x^2 = 11 \\
 & \frac{-11 \quad -11}{x^2 - 11} = 0
 \end{aligned}$$

$$\begin{aligned}
 27) \quad & \pm \frac{\sqrt{3}}{4} \\
 & 4x = \pm \frac{\sqrt{3}}{4} (4) \\
 & (4x)^2 = (\pm\sqrt{3})^2 \\
 & 16x^2 = 3 \\
 & \frac{-3 \quad -3}{16x^2 - 3} = 0
 \end{aligned}$$

$$\begin{aligned}
 29) \quad & \pm i\sqrt{13} \\
 & x^2 = (\pm i\sqrt{13})^2 \\
 & x^2 = -13 \\
 & \frac{+13 \quad +13}{x^2 + 13} = 0
 \end{aligned}$$

$$\begin{aligned}
 31) \quad & 2 \pm \sqrt{6} \\
 & x = 2 \pm \sqrt{6} \\
 & \frac{-2 \quad -2}{(x-2)^2 = (\pm\sqrt{6})^2} \\
 & x^2 - 4x + 4 = 6 \\
 & \frac{-6 \quad -6}{x^2 - 4x - 2} = 0
 \end{aligned}$$

$$\begin{aligned}
 33) \quad & 1 \pm 3i \\
 & x = 1 \pm 3i \\
 & \frac{-1 \quad -1}{(x-1)^2 = (\pm 3i)^2} \\
 & x^2 - 2x + 1 = -9 \\
 & \frac{+9 \quad +9}{x^2 - 2x + 10} = 0
 \end{aligned}$$

$$\begin{aligned}
 35) \quad & 6 \pm i\sqrt{3} \\
 & x = 6 \pm i\sqrt{3} \\
 & \frac{-6 \quad -6}{(x-6)^2 = (\pm i\sqrt{3})^2} \\
 & x^2 - 12x + 36 = -3 \\
 & \frac{+3 \quad +3}{x^2 - 12x + 39} = 0
 \end{aligned}$$

$$\begin{aligned}
 37) \quad & \frac{-1 \pm \sqrt{6}}{2} \\
 & (2)x = \frac{-1 \pm \sqrt{6}}{2} (2) \\
 & 2x = -1 \pm \sqrt{6} \\
 & \frac{+1 \quad +1}{(2x+1)^2 = (\pm\sqrt{6})^2} \\
 & 4x^2 + 4x + 1 = 6 \\
 & \frac{-6 \quad -6}{4x^2 + 4x - 5} = 0
 \end{aligned}$$

$$\begin{aligned}
 39) \quad & \frac{6 \pm i\sqrt{2}}{8} \\
 & (8)x = \frac{6 \pm i\sqrt{2}}{8} (8) \\
 & 8x = 6 \pm i\sqrt{2} \\
 & \frac{-6 \quad -6}{(8x-6)^2 = (\pm i\sqrt{2})^2} \\
 & 64x^2 - 96x + 36 = -2 \\
 & \frac{+2 \quad +2}{64x^2 - 96x + 38} = 0
 \end{aligned}$$

9.6

1) $x^4 - 5x^2 + 4 = 0$

$y = x^2, y^2 = x^4$

$y^2 - 5y + 4 = 0$

$(y - 4)(y - 1) = 0$

$y - 4 = 0 \quad y - 1 = 0$

$\frac{+4 \quad +4}{\quad} \quad \frac{+1 \quad +1}{\quad}$

$y = 4 \quad y = 1$

$\sqrt{x^2} = \sqrt{4} \quad x^2 = \sqrt{1}$

$x = \pm 2, \pm 1$

3) $m^4 - 7m^2 - 8 = 0$

$y = m^2 \quad y^2 = m^4$

$y^2 - 7y - 8 = 0$

$(y - 8)(y + 1) = 0$

$y - 8 = 0 \quad y + 1 = 0$

$\frac{+8 \quad +8}{\quad} \quad \frac{-1 \quad -1}{\quad}$

$y = 8 \quad y = -1$

$\sqrt{m^2} = \sqrt{8} \quad \sqrt{m^2} = \sqrt{(-1)^2}$

$m = \pm 2\sqrt{2}, \pm i$

5) $a^4 - 50a^2 + 49 = 0$

$y = a^2 \quad y^2 = a^4$

$y^2 - 50y + 49 = 0$

$(y - 49)(y - 1) = 0$

$y - 49 = 0 \quad y - 1 = 0$

$\frac{+49 \quad +49}{\quad} \quad \frac{+1 \quad +1}{\quad}$

$y = 49 \quad y = 1$

$\sqrt{a^2} = \sqrt{49} \quad \sqrt{a^2} = \sqrt{1}$

$a = \pm 7, \pm 1$

7) $x^4 - 25x^2 + 144 = 0$

$y = x^2, y^2 = x^4$

$y^2 - 25y + 144 = 0$

$(y - 9)(y - 16) = 0$

$y - 9 = 0 \quad y - 16 = 0$

$\frac{+9 \quad +9}{\quad} \quad \frac{+16 \quad +16}{\quad}$

$y = 9 \quad y = 16$

$\sqrt{x^2} = \sqrt{9} \quad \sqrt{x^2} = \sqrt{16}$

$x = \pm 3, \pm 4$

9) $m^4 - 20m^2 + 64 = 0$

$y = m^2 \quad y^2 = m^4$

$y^2 - 20y + 64 = 0$

$(y - 4)(y - 16) = 0$

$y - 4 = 0 \quad y - 16 = 0$

$\frac{+4 \quad +4}{\quad} \quad \frac{+16 \quad +16}{\quad}$

$y = 4 \quad y = 16$

$\sqrt{m^2} = \sqrt{4} \quad \sqrt{m^2} = \sqrt{16}$

$m = \pm 2, \pm 4$

$$11) z^6 - 216 = 19z^3$$

$$y = z^3, y^2 = z^6$$

$$y^2 - 216 = 19y$$

$$\frac{-19y}{-19y} \quad \frac{-19y}{-19y}$$

$$y^2 - 19y - 216 = 0$$

$$(y - 27)(y + 8) = 0$$

$$y - 27 = 0 \quad y + 8 = 0$$

$$\frac{+27}{+27} \quad \frac{+27}{+27} \quad \frac{-8}{-8} \quad \frac{-8}{-8}$$

$$y = 27 \quad y = -8$$

$$z^3 = 27 \quad z^3 = -8$$

$$\frac{-27}{-27} \quad \frac{-27}{-27} \quad \frac{+8}{+8} \quad \frac{+8}{+8}$$

$$z^3 - 27 = 0 \quad z^3 - 8 = 0$$

$$(z - 3)(z^2 + 3z + 9) = 0$$

$$z - 3 = 0 \quad z^2 + 3z + 9 = 0$$

$$\frac{+3}{+3} \quad \frac{+3}{+3} \quad \frac{-3 \pm \sqrt{3^2 - 4(1)(9)}}{2(1)} = \frac{2 \pm \sqrt{-27}}{2} = \frac{-3 \pm 3i\sqrt{3}}{2}$$

$$z = 3$$

$$(z + 2)(z^2 - 2z + 4) = 0$$

$$z + 2 = 0 \quad z^2 - 2z + 4 = 0$$

$$\frac{-2}{-2} \quad \frac{-2}{-2} \quad \frac{2 \pm \sqrt{(-2)^2 - 4(1)(4)}}{2} = \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm 2i\sqrt{3}}{2} = 1 \pm i\sqrt{3}$$

$$z = -2$$

$$z = 3, \frac{-3 \pm 3i\sqrt{3}}{2}, -2, 1 \pm i\sqrt{3}$$

$$13) 6z^4 - z^2 = 12$$

$$y = z^2, y^2 = z^4$$

$$6y^2 - y = 12$$

$$\frac{-12}{-12} \quad \frac{-12}{-12}$$

$$6y^2 - y - 12 = 0$$

$$(3y + 4)(2y - 3) = 0$$

$$3y + 4 = 0 \quad 2y - 3 = 0$$

$$\frac{-4}{-4} \quad \frac{-4}{-4} \quad \frac{+3}{+3} \quad \frac{+3}{+3}$$

$$\frac{3y}{3} = -\frac{4}{3} \quad \frac{2y}{2} = \frac{3}{2}$$

$$y = -\frac{4}{3}, \frac{3}{2}$$

$$\sqrt{z^2} = \sqrt{-\frac{4}{3} \left(\frac{\sqrt{3}}{\sqrt{3}}\right)} \quad \sqrt{z^2} = \sqrt{\frac{3}{2} \left(\frac{\sqrt{2}}{\sqrt{2}}\right)}$$

$$z = \frac{\pm 2i\sqrt{3}}{3}, \frac{\pm\sqrt{6}}{2}$$

$$15) x^{\frac{2}{3}} - 35 = 2x^{\frac{1}{3}}$$

$$y = x^{\frac{1}{3}}, y^2 = x^{\frac{2}{3}}$$

$$y^2 - 35 = 2y$$

$$\frac{-2y}{-2y} \quad \frac{-2y}{-2y}$$

$$y^2 - 2y - 35 = 0$$

$$(y - 7)(y + 5) = 0$$

$$y - 7 = 0 \quad y + 5 = 0$$

$$\frac{+7}{+7} \quad \frac{+7}{+7} \quad \frac{-5}{-5} \quad \frac{-5}{-5}$$

$$y = 7, y = -5$$

$$x^{\frac{1}{3}} = -5 \quad x^{\frac{1}{3}} = 7$$

$$(\sqrt[3]{x})^3 = (-5)^3 (\sqrt[3]{x})^3 = 7^3$$

$$x = -125, 343$$

$$17) y^{-6} + 7y^{-3} = 8$$

$$z = y^{-3} \quad z^2 = y^{-6}$$

$$z^2 + 7z = 8$$

$$\frac{-8 \quad -8}{z^2 + 7z - 8 = 0}$$

$$(z + 8)(z - 1) = 0$$

$$z + 8 = 0 \quad z - 1 = 0$$

$$\frac{-8 \quad -8}{z = -8, \quad z = 1} \quad \frac{+1 \quad +1}{z = -8, \quad z = 1}$$

$$z = -8, \quad z = 1$$

$$y^{-3} = -8, \quad y^{-3} = 1$$

$$(y^3) \left(\frac{1}{y^3} \right) = -8(y^3) \quad (y^3) \frac{1}{y^3} = 1(y^3)$$

$$1 = -8y^3 \quad 1 = y^3$$

$$\frac{+8y^3 \quad +8y^3}{8y^3 + 1 = 0} \quad \frac{-1 \quad -1}{0 = y^3 - 1}$$

$$(2y + 1)(4y^2 - 2y + 1) = 0$$

$$2y + 1 = 0 \quad 4y^2 - 2y + 1 = 0$$

$$\frac{-1 \quad -1}{2} \quad \frac{2 \pm \sqrt{(-2)^2 - 4(4)(1)}}{2(4)}$$

$$\frac{2y}{2} = \frac{-1}{2} \quad \frac{2 \pm \sqrt{-12}}{8}$$

$$y = -\frac{1}{2} \quad \frac{2 \pm 2i\sqrt{3}}{8} = \frac{1 \pm i\sqrt{3}}{4}$$

$$y = -\frac{1}{2}, \frac{1 \pm i\sqrt{3}}{4}, 1, \frac{-1 \pm i\sqrt{3}}{2}$$

$$0 = (y - 1)(y^2 + y + 1)$$

$$y - 1 = 0 \quad y^2 + y + 1 = 0$$

$$\frac{+1 \quad +1}{2} \quad \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$$

$$y = 1 \quad \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

$$19) x^4 - 2x^2 - 3 = 0$$

$$y = x^2 \quad y^2 = x^4$$

$$y^2 - 2y - 3 = 0$$

$$(y - 3)(y + 1) = 0$$

$$y - 3 = 0 \quad y + 1 = 0$$

$$\frac{+3 \quad +3}{y = 3} \quad \frac{-1 \quad -1}{y = -1}$$

$$\sqrt{x^2} = \sqrt{3} \quad \sqrt{x^2} = \sqrt{-1}$$

$$x = \pm\sqrt{3}, \pm i$$

$$21) 2x^4 - 5x^2 + 2 = 0$$

$$y = x^2, y^2 = x^4$$

$$2y^2 - 5y + 2 = 0$$

$$(2y - 1)(y - 2) = 0$$

$$2y - 1 = 0 \quad y - 2 = 0$$

$$\frac{+1 \quad +1}{2y} = \frac{1}{2} \quad \frac{+2 \quad +2}{y} = 2$$

$$y = \frac{1}{2}$$

$$\sqrt{x^2} = \sqrt{\frac{1}{2}} \left(\frac{\sqrt{2}}{\sqrt{2}} \right) \quad \sqrt{x^2} = \sqrt{2}$$

$$x = \pm \frac{\sqrt{2}}{2}, \pm \sqrt{2}$$

$$23) x^4 - 9x^2 + 8 = 0$$

$$y = x^2, y^2 = x^4$$

$$y^2 - 9y + 8 = 0$$

$$(y - 8)(y - 1) = 0$$

$$y - 8 = 0 \quad y - 1 = 0$$

$$\frac{+8 \quad +8}{y} = 8 \quad \frac{+1 \quad +1}{y} = 1$$

$$\sqrt{x^2} = \sqrt{8} \quad \sqrt{x^2} = \sqrt{1}$$

$$x = \pm 2\sqrt{2}, \pm 1$$

$$25) 8x^6 - x^3 + 1 = 0$$

$$y = x^3, y^2 = x^6$$

$$8y^2 - y + 1 = 0$$

$$(8y - 1)(y - 1) = 0$$

$$8y - 1 = 0 \quad y - 1 = 0$$

$$\frac{+1 \quad +1}{8y} = \frac{1}{8} \quad \frac{+1 \quad +1}{y} = 1$$

$$y = \frac{1}{8}$$

$$(8)x^3 = \frac{1}{8}(8)$$

$$8x^3 = 1$$

$$\frac{-1 \quad -1}{8x^3} = 1$$

$$8x^3 - 1 = 0$$

$$(2x - 1)(4x^2 + 2x + 1) = 0$$

$$2x - 1 = 0 \quad 4x^2 + 2x + 1 = 0$$

$$\frac{+1 \quad +1}{2x} = \frac{1}{2} \quad \frac{-2 \pm \sqrt{2^2 - 4(4)(1)}}{2(4)}$$

$$\frac{2x}{2} = \frac{1}{2} \quad \frac{-2 \pm \sqrt{-12}}{8}$$

$$x^3 = 1$$

$$\frac{-1 \quad -1}{x^3} = 1$$

$$x^3 - 1 = 0$$

$$(x - 1)(x^2 + x + 1) = 0$$

$$x - 1 = 0 \quad x^2 + x + 1 = 0$$

$$\frac{+1 \quad +1}{x} = 1 \quad \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$$

$$x = 1 \quad \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

$$x = \frac{1}{2} \frac{-2 \pm 2i\sqrt{3}}{8} = \frac{-1 \pm i\sqrt{3}}{4}$$

$$x = \frac{1}{2}, \frac{-1 \pm i\sqrt{3}}{4}, 1, \frac{-1 \pm i\sqrt{3}}{2}$$

$$27) x^8 - 17x^4 + 16 = 0$$

$$y = x^4, y^2 = x^8$$

$$y^2 - 17y + 16 = 0$$

$$(y - 16)(y - 1) = 0$$

$$y - 16 = 0 \quad y - 1 = 0$$

$$\frac{+16 + 16}{y = 16} \quad \frac{+1 \quad +1}{y = 1}$$

$$x^4 = 16$$

$$-16 \quad -16$$

$$x^4 - 16 = 0$$

$$(x^2 + 4)(x^2 - 4) = 0$$

$$x^2 + 4 = 0 \quad x^2 - 4 = 0$$

$$\frac{-4 \quad -4}{\sqrt{x^2} = \sqrt{-4}} \quad \frac{+4 \quad +4}{\sqrt{x^2} = \sqrt{4}}$$

$$x = \pm 2i \quad x = \pm 2$$

$$x = \pm 2i, \pm 2, \pm i, \pm 1$$

$$x^4 = 1$$

$$-1 \quad -1$$

$$x^4 - 1 = 0$$

$$(x^2 - 1)(x^2 + 1) = 0$$

$$x^2 - 1 = 0 \quad x^2 + 1 = 0$$

$$\frac{+1 \quad +1}{\sqrt{x^2} = \sqrt{1}} \quad \frac{-1 \quad -1}{\sqrt{x^2} = \sqrt{-1}}$$

$$x = \pm 1 \quad x = \pm i$$

$$29) (y + b)^2 - 4(y + b) = 21$$

$$z = (y + b), z^2 = (y + b)^2$$

$$z^2 - 4z = 21$$

$$\frac{-21 \quad -21}{z^2 - 4z - 21 = 0}$$

$$(z - 7)(z + 3) = 0$$

$$z - 7 = 0 \quad z + 3 = 0$$

$$\frac{+7 \quad +7}{z = 7} \quad \frac{-3 \quad -3}{z = -3}$$

$$y + b = 7 \quad y + b = -3$$

$$\frac{-b \quad -b}{y = 7 - b} \quad \frac{-b \quad -b}{y = -3 - b}$$

$$31) (y + 2)^2 - 6(y + 2) = 16$$

$$z = y + 2, \quad z^2 = (y + 2)^2$$

$$z^2 - 6z = 16$$

$$\begin{array}{r} -16 \quad -16 \\ z^2 - 6z - 16 = 0 \end{array}$$

$$(z - 8)(z + 2) = 0$$

$$z - 8 = 0 \quad z + 2 = 0$$

$$\begin{array}{r} +8 \quad +8 \quad -2 \quad -2 \end{array}$$

$$z = 8 \quad z = -2$$

$$y + 2 = 8 \quad y + 2 = -2$$

$$\begin{array}{r} -2 \quad -2 \quad -2 \quad -2 \\ y = 6 \quad y = -4 \end{array}$$

$$33) (x - 3)^2 - 2(x - 3) = 35$$

$$y = (x - 3), \quad y^2 = (x - 3)^2$$

$$y^2 - 2y = 35$$

$$\begin{array}{r} -35 \quad -35 \\ y^2 - 2y - 35 = 0 \end{array}$$

$$(y - 7)(y + 5) = 0$$

$$y - 7 = 0 \quad y + 5 = 0$$

$$\begin{array}{r} +7 \quad +7 \quad -5 \quad -5 \\ y = 7 \quad y = -5 \end{array}$$

$$x - 3 = 7 \quad x - 3 = -5$$

$$\begin{array}{r} +3 \quad +3 \quad +3 \quad +3 \\ x = 10, -2 \end{array}$$

$$35) (r - 1)^2 - 8(r - 1) = 20$$

$$y = (r - 1), \quad y^2 = (r - 1)^2$$

$$y^2 - 8y = 20$$

$$\begin{array}{r} -20 \quad -20 \\ y^2 - 8y - 20 = 0 \end{array}$$

$$(y - 10)(y + 2) = 0$$

$$y - 10 = 0 \quad y + 2 = 0$$

$$\begin{array}{r} +10 \quad +10 \quad -2 \quad -2 \\ y = 10 \quad y = -2 \end{array}$$

$$r - 1 = 10 \quad r - 1 = -2$$

$$\begin{array}{r} +1 \quad +1 \quad +1 \quad +1 \\ r = 11, -1 \end{array}$$

$$37) 3(y + 1)^2 - 14(y + 1) = 5$$

$$z = (y + 1), \quad z^2 = (y + 1)^2$$

$$3z^2 - 14z = 5$$

$$\begin{array}{r} -5 \quad -5 \\ 3z^2 - 14z - 5 = 0 \end{array}$$

$$(3z + 1)(z - 5) = 0$$

$$3z + 1 = 0 \quad z - 5 = 0$$

$$\begin{array}{r} -1 \quad -1 \quad +5 \quad +5 \\ \frac{3z}{3} = \frac{-1}{3} \quad z = 5 \end{array}$$

$$z = -\frac{1}{3}$$

$$y + 1 = -\frac{1}{3} \quad y + 1 = 5$$

$$\begin{array}{r} -1 \quad -1 \quad -1 \quad -1 \\ y = -\frac{4}{3}, 4 \end{array}$$

$$39) (3x^2 - 2x)^2 + 5 = 6(3x^2 - 2x)$$

$$y = (3x^2 - 2x), \quad y^2 = (3x^2 - 2x)^2$$

$$\begin{aligned}
y^2 + 5 &= 6y \\
-6y &\quad -6y \\
y^2 - 6y + 5 &= 0 \\
(y-1)(y-5) &= 0 \\
y-1=0 &\quad y-5=0 \\
\frac{+1}{y=1} &\quad \frac{+5}{y=5} \\
3x^2 - 2x &= 1 & 3x^2 - 2x &= 5 \\
-1 &\quad -1 & -5 &\quad -5 \\
3x^2 - 2x - 1 &= 0 & 3x^2 - 2x - 5 &= 0 \\
(3x-5)(x+1) &= 0 & (3x+1)(x-1) &= 0 \\
3x-5=0 &\quad x+1=0 & 3x+1=0 &\quad x-1=0 \\
\frac{+5}{\frac{3x}{3}=\frac{5}{3}} &\quad \frac{-1}{x=-1} & \frac{-1}{\frac{3x}{3}=\frac{-1}{3}} &\quad \frac{+1}{x=1} \\
x &= \frac{5}{3}, -1, -\frac{1}{3}, 1
\end{aligned}$$

$$\begin{aligned}
41) \quad 2(3x+1)^{\frac{2}{3}} - 5(3x+1)^{\frac{1}{3}} &= 88 \\
y &= (3x+1)^{\frac{1}{3}}, \quad y^2 = (3x+1)^{\frac{2}{3}} \\
2y^2 - 5y &= 88 \\
-88 &\quad -88 \\
2y^2 - 5y - 88 &= 0 \\
(2y+11)(y-8) &= 0 \\
2y+11=0 &\quad y-8=0 \\
\frac{-11}{\frac{2y}{2}=\frac{-11}{2}} &\quad \frac{+8}{y=8} \\
y &= -\frac{11}{2} \\
(3x+1)^{\frac{1}{3}} &= -\frac{11}{2} & (3x+1)^{\frac{1}{3}} &= 8 \\
(\sqrt[3]{3x+1})^3 &= \left(-\frac{11}{2}\right)^3 & (\sqrt[3]{3x+1})^3 &= 8^3 \\
3x+1 &= -\frac{1331}{8} & 3x+1 &= 512 \\
-1 &\quad -1 & -1 &\quad -1
\end{aligned}$$

$$\frac{3x}{3} = \left(-\frac{1329}{8}\right) \quad \frac{3x}{3} = \frac{511}{3}$$

$$x = \frac{1329}{24}, \frac{511}{3}$$

$$43) (x^2 + 2x)^2 - 2(x^2 + 2x) = 3$$

$$y = (x^2 + 2x), y^2 = (x^2 + 2x)^2$$

$$y^2 - 2y = 3$$

$$\underline{-3 \quad -3}$$

$$y^2 - 2y - 3 = 0$$

$$(y - 3)(y + 1) = 0$$

$$y - 3 = 0 \quad y + 1 = 0$$

$$\underline{+3 \quad +3} \quad \underline{-1 \quad -1}$$

$$y = 3 \quad y = -1$$

$$x^2 + 2x = 3 \quad x^2 + 2x = -1$$

$$\underline{-3 \quad -3} \quad \underline{+1 \quad +1}$$

$$x^2 + 2x - 3 = 0 \quad x^2 + 2x + 1 = 0$$

$$(x + 3)(x - 1) = 0 \quad (\sqrt{(x + 1)^2} = \sqrt{0})$$

$$x + 3 = 0 \quad x - 1 = 0 \quad x + 1 = 0$$

$$\underline{-3 \quad -3} \quad \underline{+1 \quad +1} \quad \underline{-1 \quad -1}$$

$$x = -3, 1, -1$$

$$45) (2x^2 - x)^2 - 4(2x^2 - x) + 3 = 0$$

$$y = (2x^2 - x), y^2 = (2x^2 - x)^2$$

$$y^2 - 4y + 3 = 0$$

$$(y - 3)(y - 1) = 0$$

$$y - 3 = 0 \quad y - 1 = 0$$

$$\underline{+3 \quad +3} \quad \underline{+1 \quad +1}$$

$$y = 3 \quad y = 1$$

$$2x^2 - x = 3$$

$$\underline{-3 \quad -3}$$

$$2x^2 - x - 3 = 0$$

$$(2x - 3)(x + 1) = 0$$

$$2x - 3 = 0 \quad x + 1 = 0$$

$$\underline{+3 \quad +3} \quad \underline{-1 \quad -1}$$

$$\frac{2x}{2} = \frac{3}{2} \quad x = -1$$

$$2x^2 - x = 1$$

$$\underline{-1 \quad -1}$$

$$2x^2 - x - 1 = 0$$

$$(2x + 1)(x - 1) = 0$$

$$2x + 1 = 0 \quad x - 1 = 0$$

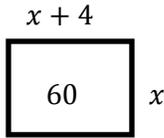
$$\underline{-1 \quad -1} \quad \underline{+1 \quad +1}$$

$$\frac{2x}{2} = \frac{-1}{2} \quad x = 1$$

$$x = \frac{3}{2}, -1, -\frac{1}{2}, 1$$

9.7

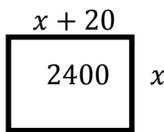
- 1) In a landscape plan, a rectangular flowerbed is designed to be 4 meters longer than it is wide. If 60 square meters are needed for the plants in the bed, what should the dimensions of the rectangular bed be?



6m by 10m

$$\begin{aligned}
 x(x + 4) &= 60 \\
 x^2 + 4x &= 60 \\
 &\quad -60 \quad -60 \\
 x^2 + 4x - 60 &= 0 \\
 (x - 6)(x + 10) &= 0 \\
 x - 6 = 0 \quad x + 10 = 0 \\
 \frac{+6 \quad +6}{x = 6} \quad \frac{-10 \quad -10}{x = -10}
 \end{aligned}$$

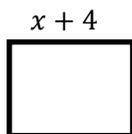
- 3) A rectangular lot is 20 yards longer than it is wide and its area is 2400 square yards. Find the dimensions of the lot



40yds x 60 yds

$$\begin{aligned}
 x(x + 20) &= 2400 \\
 x^2 + 20x &= 2400 \\
 \left(20 \cdot \frac{1}{2}\right)^2 &= 10^2 = 100 \\
 x^2 + 20x + 100 &= 2400 + 100 \\
 \sqrt{(x + 10)^2} &= \sqrt{2500} \\
 x + 10 &= \pm 50 \\
 \frac{-10 \quad -10}{x = -10 \pm 50} \\
 x &= 40, -60
 \end{aligned}$$

- 5) The length of a rectangular lot is 4 rods greater than its width, and its area is 60 square rods. Find the dimensions of the lot.



$$\begin{aligned}
 x(x + 4) &= 60 \\
 x^2 + 4x &= 60 \\
 &\quad -60 \quad -60
 \end{aligned}$$

$$\begin{aligned}
 60 \quad x \quad x^2 + 4x - 60 &= 0 \\
 (x + 10)(x - 6) &= 0 \\
 x + 10 = 0 \quad x - 6 &= 0 \\
 \frac{-10 \quad -10 \quad +6 \quad +6}{\cancel{x - 10} \quad x = 6}
 \end{aligned}$$

6 rods x 10 rods

- 7) A rectangular piece of paper is twice as long as a square piece and 3 inches wider. The area of the rectangular piece is 108 in^2 . Find the dimensions of the square piece.

$2x$

108

$x + 3$

$$\begin{aligned}
 2x(x + 3) &= 108 \\
 2x^2 + 6x &= 108 \\
 -108 \quad -108 & \\
 \frac{2x^2 + 6x - 108}{2} &= \frac{0}{2} \\
 x^2 + 3x - 54 &= 0 \\
 (x + 9)(x - 6) &= 0 \\
 x + 9 = 0 \quad x - 6 &= 0 \\
 \frac{-9 \quad -9 \quad +6 \quad +6}{\cancel{x - 9} \quad x = 6}
 \end{aligned}$$

6in x 6in

- 9) The area of a rectangle is 48 ft^2 and its perimeter is 32 ft. Find its length and width.

x

48

y

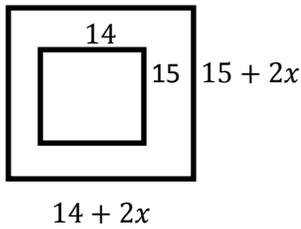
$p = 32$

$$\begin{aligned}
 2x + 2y &= 32 \\
 -2x \quad -2x & \\
 \frac{2y}{2} &= \frac{32}{2} - \frac{2x}{2} \\
 y &= 16 - x \\
 xy &= 48 \\
 x(16 - x) &= 48 \\
 16x - x^2 &= 48 \\
 \frac{-16x + x^2 - 16x + x^2}{0} &= \frac{x^2 - 16x + 48}{0} \\
 0 &= (x - 12)(x - 4) \\
 x - 12 = 0 \quad x - 4 &= 0 \\
 \frac{+12 \quad +12 \quad +4 \quad +4}{x = 12 \quad x = 4} \\
 y = 16 - 12 = 4 \quad y = 16 - 4 = 12
 \end{aligned}$$

12ft x 4 ft

- 11) A mirror 14 inches by 15 inches has a frame of uniform width. If the area of the frame equals that of the mirror, what is the width of the frame?

$$(15 + 2x)(14 + 2x) = 420$$



$$A = 2(14 \cdot 15) = 420$$

3 in

$$210 + 30x + 28 + 4x^2 = 420$$

$$4x^2 + 58x + 210 = 420$$

$$\frac{-420 - 210}{4}$$

$$\frac{4x^2}{2} + \frac{58x}{2} - \frac{210}{2} = \frac{0}{2}$$

$$2x^2 + 29x - 105 = 0$$

$$(2x + 35)(x - 3) = 0$$

$$2x + 35 = 0$$

$$x - 3 = 0$$

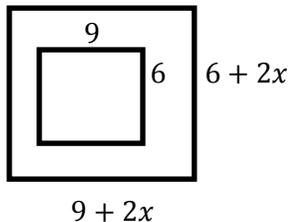
$$\frac{-35}{2} = \frac{-35}{2}$$

$$\frac{+3}{+3}$$

$$\frac{2x}{2} = \frac{-35}{2} = x = \frac{-35}{2}$$

$$x = 3$$

- 13) A grass plot 9 yards long and 6 yards wide has a path of uniform width around it. If the area of the path is equal to the area of the plot, determine the width of the path.



$$A = 2(9 \cdot 6) = 108$$

$$(6 + 2x)(9 + 2x) = 108$$

$$54 + 12x + 18x + 4x^2 = 108$$

$$4x^2 + 30x + 54 = 108$$

$$\frac{-108 - 54}{4}$$

$$\frac{4x^2}{2} + \frac{30x}{2} - \frac{54}{2} = \frac{0}{2}$$

$$2x^2 + 15x - 27 = 0$$

$$(2x - 3)(x + 9) = 0$$

$$2x - 3 = 0 \quad x + 9 = 0$$

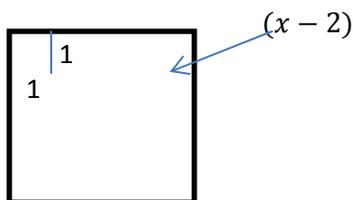
$$\frac{+3}{2} = \frac{+3}{2} \quad \frac{-9}{-9}$$

$$\frac{2x}{2} = \frac{3}{2} \quad x = -9$$

$$x = \frac{3}{2} = 1.5$$

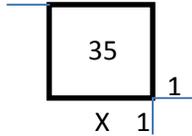
1.5 yds

- 15) A page is to have a margin of 1 inch, and is to contain 35 in^2 of painting. How large must the page be if the length is to exceed the width by 2 inches?



$$x(x - 2) = 35$$

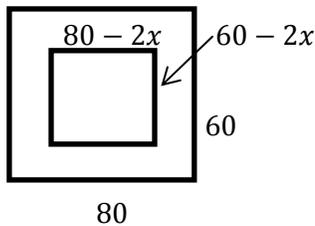
$$x^2 - 2x = 35$$



7in x 9in

$$\begin{aligned} & \frac{-35 - 35}{x^2 - 2x - 35 = 0} \\ & (x - 7)(x + 5) = 0 \\ & x - 7 = 0 \quad x + 5 = 0 \\ & \frac{+7 + 7}{x = 7} \quad \frac{-5 - 5}{\cancel{x = -5}} \end{aligned}$$

- 17) A rectangular wheat field is 80 rods long by 60 rods wide. A strip of uniform width is cut around the field, so that half the grain is left standing in the form of a rectangular plot. How wide is the strip that is cut?

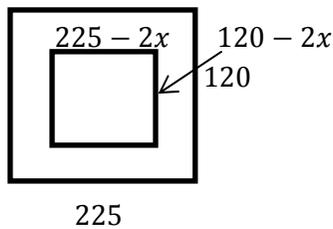


$$A = \frac{1}{2}(60 \cdot 80) = 2400$$

10 rods

$$\begin{aligned} & (80 - 2x)(60 - 2x) = 2400 \\ & 48000 - 160x - 120x + 4x^2 = 2400 \\ & 4x^2 - 280x + 4800 = 2400 \\ & \frac{-2400 - 2400}{4x^2 - 280x + 4800 = 2400} \\ & \frac{4x^2}{4} - \frac{280x}{4} + \frac{2400}{4} = \frac{0}{4} \\ & x^2 - 70x + 600 = 0 \\ & (x - 10)(x - 60) = 0 \\ & x - 10 = 0 \quad x - 60 = 0 \\ & \frac{+10 + 10}{x = 10} \quad \frac{+60 + 60}{\cancel{x = 60}} \end{aligned}$$

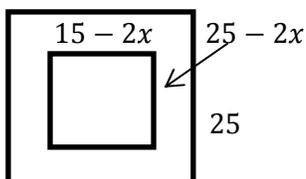
- 19) A rectangular field 225 ft by 120 ft has a ring of uniform width cut around the outside edge. The ring leaves 65% of the field uncut in the center. What is the width of the ring?



$$A = 0.65(120 \cdot 225) = 17500$$

$$\begin{aligned} & (225 - 2x)(120 - 2x) = 17500 \\ & 27000 - 450x - 240x + 4x^2 = 17500 \\ & 4x^2 - 690x + 27000 = 17500 \\ & \frac{-17500 - 17500}{4x^2 - 690x + 27000 = 17500} \\ & \frac{4x^2}{2} - \frac{690x}{2} + \frac{9450}{2} = \frac{0}{2} \\ & 2x^2 - 345x + 4725 = 0 \\ & \frac{345 \pm \sqrt{345^2 - 4(2)(4725)}}{2(2)} = \frac{345 \pm \sqrt{81225}}{4} = \frac{345 \pm 285}{4} = \frac{\cancel{815}}{2} \& 15 \\ & 15ft \end{aligned}$$

- 21) A frame is 15 in by 25 in and is of uniform width. The inside of the frame leaves 75% of the total area available for the picture. What is the width of the frame?



$$\begin{aligned} & (15 - 2x)(25 - 2x) = 281.25 \\ & 375 - 30x - 50x + 4x^2 = 281.25 \\ & 4x^2 - 80x + 375 = 281.25 \\ & \frac{-281.25 - 281.25}{4x^2 - 80x + 375 = 281.25} \\ & \frac{4x^2}{4} - \frac{80x}{4} + \frac{93.75}{4} = \frac{0}{4} \end{aligned}$$

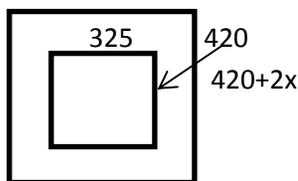
15

$$A = .75(25 \cdot 15) = 281.25$$

1.25 in

$$\begin{aligned} x^2 - 20x + 23.4375 &= 0 \\ x^2 - 20x &= \frac{-23.4375 - 23.4375}{-23.4375} \\ x^2 - 20x + 100 &= -23.4375 + 100 \\ \sqrt{(x - 10)^2} &= \sqrt{(76.5625)} \\ x - 10 &= \pm 8.75 \\ \frac{+10}{+10} & \quad \frac{+10}{+10} \\ x &= 10 \pm 8.75 \\ x &= \cancel{18.75}, 1.25 \end{aligned}$$

- 23) The farmer in the previous problem has a neighbor who has a field 325 ft by 420 ft. His neighbor wants to increase the size of his field by 20% by cultivating a band of uniform width around the outside of his lot. How wide a band should his neighbor cultivate?

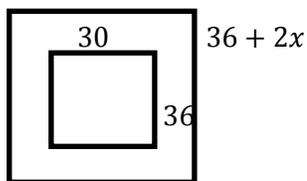


$$325 + 2x$$

$$A = 1.2(325)(420) = 163800$$

$$\begin{aligned} (420 + 2x)(325 + 2x) &= 163800 \\ 136500 + 840x + 650x + 4x^2 &= 163800 \\ 4x^2 + 1490x + 136500 &= 163800 \\ & \quad \frac{-163800 - 163800}{-163800 - 163800} \\ \frac{4x^2}{4} + \frac{1490x}{4} - \frac{27300}{4} &= \frac{0}{4} \\ x^2 + 745x - 13650 &= 0 \\ \frac{-745 \pm \sqrt{(745)^2 - 4(2)(-13650)}}{2(2)} &= \frac{-745 \pm \sqrt{664225}}{4} \\ \frac{-745 \pm 815}{4} &= 17.5, -390 \\ 17.5 \text{ ft} \end{aligned}$$

- 25) Donna has a garden that is 30 ft by 36 ft. She wants to increase the size of the garden by 40%. How wide a ring around the outside should she cultivate?



$$\begin{aligned} (36 + 2x)(30 + 2x) &= 1512 \\ 1080 + 72x + 60x + 4x^2 &= 1512 \\ 4x^2 + 132x + 1080 &= 1512 \\ & \quad \frac{-1512 - 1512}{-1512 - 1512} \end{aligned}$$

$$30 + 2x$$

$$A = 1.4(30 \cdot 36) = 1512$$

$$\frac{4x^2}{4} + \frac{132x}{4} - \frac{432}{4} = \frac{0}{4}$$

$$x^2 + 33x - 108 = 0$$

$$\frac{-33 \pm \sqrt{33^2 - 4(1)(-108)}}{2(1)} = \frac{-33 \pm \sqrt{1521}}{2} = \frac{-33 \pm 39}{2} = 3, \cancel{-36}$$

3 ft.

9.8

- 1) Bills father can paint a room in two hours less than Bill can paint it. Working together they can complete the job in two hours and 24 minutes. How much time would each require working alone?

Father: $x - 2$

Bill: x

Team: $2 \frac{24}{60} = 2 \frac{2}{5} = \frac{12}{5}$

$$\frac{1}{x-2} 12x(x-2) + \frac{1}{x} 12x(x-2) = \frac{5}{12} 12x(x-2)$$

LCD: $12x(x-2)$

$$12x + 12(x-2) = 5x(x-2)$$

$$12x + 12x - 24 = 5x^2 - 10x$$

$$24x - 24 = 5x^2 - 10$$

$$\frac{-24x + 24 \quad -24x + 24}{0 = 5x^2 - 34x + 24}$$

$$0 = 5x^2 - 34x + 24$$

$$0 = (5x - 4)(x - 6)$$

$$5x - 4 = 0 \quad x - 6 = 0$$

$$\frac{+4 \quad +4}{\frac{5x}{5} = \frac{4}{5}} \quad \frac{+6 \quad +6}{x = 6}$$

$$\frac{5x}{5} = \frac{4}{5}$$

$$\cancel{x = .8}$$

Bill: 6hr, Father: 4

- 3) Jack can wash and wax the family car in one hour less than Bob can. The two working together can complete the job in $1\frac{1}{5}$ hours. How much time would each require if they worked alone?

Jack: $x - 1$

Bob: x

Team: $1\frac{1}{5} = \frac{6}{5}$

$$\frac{1}{x-1}(6x(x-1)) + \frac{1}{x}(6x(x-1)) = \frac{5}{6}(6x(x-1))$$

LCD: $6x(x-1)$

$$6x + 6(x-1) = 5x(x-1)$$

$$6x + 6x - 6 = 5x^2 - 5x$$

$$12x - 6 = 5x^2 - 5x$$

$$\begin{array}{r} -12x + 6 \\ -12x + 6 \\ \hline \end{array}$$

$$0 = 5x^2 - 17x + 6$$

$$0 = (5x - 2)(x - 3)$$

$$5x - 2 = 0 \quad x - 3 = 0$$

$$\begin{array}{r} +2 \quad +2 \quad +3 \quad +3 \\ \hline \end{array}$$

$$\frac{5x}{5} = \frac{2}{5} \quad x = 3$$

~~$$x = 4$$~~

Bob: 3hr, Jack: 2hr

- 5) Working alone it takes John 8 hours longer than Carlos to do a job. Working together they can do the job in 3 hours. How long will it take each to do the job working alone

John: $x + 8$

Carlos: x

Team: 3

$$\frac{1}{x+8} 3x(x+8) + \frac{1}{x} 3x(x+8) = \frac{1}{3} 3x(x+8)$$

LCD: $3x(x+8)$

$$3x + 3(x+8) = x(x+8)$$

$$3x + 3x + 24 = x^2 + 8x$$

$$6x + 24 = x^2 + 8x$$

$$\begin{array}{r} -6x - 24 \\ -6x - 24 \\ \hline \end{array}$$

$$0 = x^2 + 2x - 24$$

$$0 = (x + 6)(x - 4)$$

$$x + 6 = 0 \quad x - 4 = 0$$

$$\begin{array}{r} -6 \quad -6 \quad +4 \quad +4 \\ \hline \end{array}$$

~~$$x = -6$$~~

$$x = 4$$

Carlos: 4hr, John: 12hr

- 7) A can do a piece of work in 4 days and B can do it in half the time. How long will it take them to do the work together?

A: 4

B: 2

Team: x

$$\frac{1}{4}(4x) + \frac{1}{2}(4x) = \frac{1}{x}(4x)$$

LCD: $4x$

$$x + 2x = 4$$

- 19) A sink has two faucets, one for hot water and one for cold water. The sink can be filled by a cold-water faucet in 3.5 minutes. If both faucets are open, the sink is filled in 2.1 minutes. How long does it take to fill the sink with just the hot-water faucet open?

$$\begin{array}{l}
 \text{Hot: } x \\
 \text{Cold: } 3.5 = \frac{35}{10} = \frac{7}{2} \\
 \text{Team: } 2.1 = \frac{21}{10}
 \end{array}
 \qquad
 \begin{array}{l}
 \frac{1}{x}(21x) + \frac{2}{7}(21x) = \frac{10}{21}(21x) \\
 \text{LCD: } 21x \\
 21 + 6x = 10x \\
 \quad -6x \quad -6x \\
 \quad \quad \frac{21}{4} = \frac{4x}{4} \\
 \boxed{5.25 \text{ hr}} = x
 \end{array}$$

- 21) A tank can be emptied by any one of three caps. The first can empty the tank in 20 minutes while the second takes 32 minutes. If all three working together could empty the tank in $8\frac{8}{59}$ minutes, how long would the third take to empty the tank?

$$\begin{array}{l}
 \text{First: } 20 \\
 \text{Second: } 32 \\
 \text{Third: } x \\
 \text{Team: } 8\frac{8}{59} = \frac{480}{59}
 \end{array}
 \qquad
 \begin{array}{l}
 \frac{1}{20}(480x) + \frac{1}{32}(480x) + \frac{1}{x}(480x) = \frac{59}{480}(480x) \\
 \text{LCD: } 480x \\
 24x + 15x + 480 = 59x \\
 39x + 480 = 59x \\
 \quad -39x \quad \quad -39x \\
 \quad \quad \frac{480}{20} = \frac{20x}{20} \\
 \quad \quad 24 = x
 \end{array}
 \qquad
 \boxed{24 \text{ min.}}$$

- 23) Sam takes 6 hours longer than Susan to wax a floor. Working together they can wax the floor in 4 hours. How long will it take each of them working alone to wax the floor?

$$\begin{array}{l}
 \text{Sam: } x + 6 \\
 \text{Susan: } x \\
 \text{Team: } 4
 \end{array}
 \qquad
 \begin{array}{l}
 \frac{1}{x+6}(4x(x+6)) + \frac{1}{x}(4x(x+6)) = \frac{1}{4}(4x(x+6)) \\
 \text{LCD: } (4x(x+6)) \\
 4x + 4(x+6) = x(x+6) \\
 4x + 4x + 24 = x^2 + 6x \\
 8x + 24 = x^2 + 6x \\
 \quad -8x - 24 \quad \quad -8x - 24
 \end{array}$$

$$\boxed{\text{Susan} = 6\text{hr}, \text{Sam} = 12\text{hr}}$$

$$\begin{aligned} 0 &= x^2 - 2x - 24 \\ 0 &= (x - 6)(x + 4) \\ x - 6 &= 0 & x + 4 &= 0 \\ \frac{+6}{x} &= \frac{+6}{x} & \frac{-4}{x} &= \frac{-4}{x} \end{aligned}$$

- 25) It takes Sally $10\frac{1}{2}$ minutes longer than Patricia to clean up their dorm room. If they work together they can clean it in 5 minutes. How long will it take each of them if they work alone?

$$\text{Sally: } x + 10.5 = x + \frac{21}{2} = \frac{2x+21}{2} \quad \frac{2}{2x+21} (5x(2x+21)) + \frac{1}{x} (5x(2x+21)) = \frac{1}{5} (5x(2x+21))$$

$$\text{Patricia: } x$$

$$\text{Team: } 5$$

$$\begin{aligned} \text{LCD: } &(5x(2x+21)) \\ 10x + 5(2x+21) &= x(2x+21) \\ 10x + 10x + 105 &= 2x^2 + 21x \\ 20x + 105 &= 2x^2 + 21x \\ \frac{-20x - 105}{0} &= \frac{-20x - 105}{2x^2 + x - 105} \\ 0 &= 2x^2 + x - 105 \\ 0 &= (2x+15)(x-7) \\ 2x + 15 &= 0 & x - 7 &= 0 \\ \frac{-15}{2} &= \frac{-15}{2} & \frac{+7}{x} &= \frac{+7}{x} \\ \frac{2x}{2} &= \frac{-15}{2} & x &= 7 \end{aligned}$$

$$\boxed{\text{Patricia} = 7\text{min.}, \text{Sally} = 17.5\text{min.}}$$

- 27) Secretary A takes 6 minutes longer than Secretary B to type 10 pages of manuscript. If they divide the job and work together it will take them $8\frac{3}{4}$ minutes to type 10 pages. How long will it take each working alone to type the 10 pages?

$$A: x + 6$$

$$B: x$$

$$\text{Team: } 8\frac{3}{4} = \frac{35}{4}$$

$$\frac{1}{x+6} (35x(x+6)) + \frac{1}{x} (35x(x+6)) = \frac{4}{35} (35x(x+6))$$

$$\text{LCD: } (35x(x+6))$$

$$35x + 35(x+6) = 4x(x+6)$$

$$35x + 35x + 210 = 4x^2 + 24x$$

$$70x + 210 = 4x^2 + 24x$$

$$\frac{-70x - 210}{0} = \frac{-70x - 210}{4x^2 + 24x}$$

$$\frac{0}{2} = \frac{4x^2}{2} - \frac{46x}{2} - \frac{210}{2}$$

$$0 = 2x^2 - 23x - 105$$

$$0 = (2x+7)(x-15)$$

$$2x + 7 = 0 & x - 15 = 0$$

$$\frac{-7}{2} = \frac{-7}{2} & \frac{+15}{x} = \frac{+15}{x}$$

$$\frac{2x}{2} = \frac{-7}{2} & x = 15$$

$$\frac{x}{x} = \frac{-7}{2}$$

$$\boxed{B: 15\text{hr}, A: 21\text{hr}}$$

9.9

$$\begin{aligned}
 1) \quad \frac{xy}{y} &= \frac{72}{y} & x &= \frac{72}{y} \\
 \frac{(x+2)(y-4)}{y-4} &= \frac{128}{y-4} \\
 x + 2 &= \frac{128}{y-4} \\
 y(y-4)\frac{72}{y} + 2y(y-4) &= \frac{128}{y-4}y(y-4) \\
 LCD: y(y-4) \\
 72(y-4) + 2y(y-4) &= 128y \\
 72y - 288 + 2y^2 - 8y &= 128y \\
 2y^2 + 64y - 288 &= 128y \\
 \frac{-128y}{-128y} & \quad \frac{-128y}{-128y} \\
 \frac{2y^2}{2} - \frac{64y}{2} - \frac{288}{2} &= 0 \\
 y^2 - 32y - 144 &= 0 \\
 \frac{+144}{+144} & \quad \frac{+144}{+144} \\
 y^2 - 32y &= 144 \\
 \left(32 \cdot \frac{1}{2}\right)^2 &= 16^2 = 256 \\
 y^2 - 32y + 256 &= 144 + 256 \\
 \sqrt{(y-16)^2} &= \sqrt{400} \\
 y - 16 &= \pm 20 \\
 \frac{+16}{+16} & \quad \frac{+16}{+16} \\
 y &= 36, -4 \\
 x = \frac{72}{36} = 2 & \quad x = \frac{72}{-4} = -18 \\
 (2, 36), (48, -4) &
 \end{aligned}$$

$$\begin{aligned}
 3) \quad \frac{xy}{y} &= \frac{150}{y} & x &= \frac{150}{y} \\
 \frac{(x-6)(y+1)}{y+1} &= \frac{64}{y+1} \\
 x - 6 &= \frac{64}{y+1} \\
 \frac{150y}{y} (y(y+1)) - 6(y(y+1)) &= \frac{64}{y+1} (y(y+1)) \\
 LCD: (y(y+1)) \\
 150(y+1) - 6y(y+1) &= 64y \\
 150y + 150 - 6y^2 + 6y &= 64y \\
 -6y^2 + 156y + 150 &= 64y \\
 \frac{+6y^2 - 156y - 150}{+6y^2 - 156y - 150} & \quad \frac{+6y^2 - 156y - 150}{+6y^2 - 156y - 150} \\
 \frac{0}{2} = \frac{6y^2}{2} - \frac{80y}{2} - \frac{150}{2} & \\
 0 = 3y^2 - 40y - 75 & \\
 0 = (3y + 5)(y - 15) & \\
 3y + 5 = 0 & \quad y - 15 = 0 \\
 \frac{-5}{-5} & \quad \frac{-5}{-5} \quad \frac{+15}{+15} \quad \frac{+15}{+15} \\
 \frac{3y}{3} = -\frac{5}{3} & \quad y = 15 \\
 y = -\frac{5}{3} & \quad x = \frac{150}{15} = 10 \\
 x = \frac{150}{-\frac{5}{3}} = 150 \left(-\frac{3}{5}\right) &= -90 \\
 \left(-90, -\frac{5}{3}\right), (10, 15) &
 \end{aligned}$$

$$5) \frac{xy}{y} = \frac{45}{y} \quad x = \frac{45}{y}$$

$$\frac{(x+2)(y+1)}{y+1} = \frac{70}{y+1}$$

$$x + 2 = \frac{70}{y+1}$$

$$\frac{45}{y} (y(y+1)) + 2(y(y+1)) = \frac{70}{y+1} (y(y+1))$$

LCD: $(y(y+1))$

$$45(y+1) + 2y(y+1) = 70y$$

$$45y + 45 + 2y^2 + 2y = 70y$$

$$2y^2 + 47y + 2y = 70y$$

$$\begin{array}{r} -70y \quad -70y \\ \hline 2y^2 - 23y + 45 = 0 \end{array}$$

$$(2y-5)(y-9) = 0$$

$$2y-5 = 0 \quad y-9 = 0$$

$$\begin{array}{r} +5 \quad +5 \quad +9 \quad +9 \\ \hline \frac{2y}{2} = \frac{5}{2} \quad y = 9 \end{array}$$

$$y = \frac{5}{2} \quad x = \frac{45}{9} = 5$$

$$x = \frac{45}{\frac{5}{2}} = 45 \cdot \frac{2}{5} = 18$$

$(18, \frac{5}{2}), (5, 9)$

$$7) \frac{xy}{y} = \frac{90}{y} \quad x = \frac{90}{y}$$

$$\frac{(x-5)(y+1)}{y+1} = \frac{120}{y+1}$$

$$x - 5 = \frac{120}{y+1}$$

$$\frac{90}{y} (y(y+1)) - 5(y(y+1)) = \frac{120}{y+1} (y(y+1))$$

LCD: $(y(y+1))$

$$90(y+1) - 5y(y+1) = 120y$$

$$90y + 90 - 5y^2 - 5y = 120y$$

$$-5y^2 + 85y + 90 = 120y$$

$$\begin{array}{r} +5y^2 - 85y - 90 \quad +5y^2 - 85y - 90 \\ \hline 0 = \frac{5y^2}{5} + \frac{35y}{5} - \frac{90}{5} \end{array}$$

$$0 = y^2 + 7y - 18$$

$$0 = (y+9)(y-2)$$

$$y+9 = 0 \quad y-2 = 0$$

$$\begin{array}{r} -9 \quad -9 \quad +2 \quad +2 \\ \hline y = -9 \quad y = 2 \end{array}$$

$$x = \frac{90}{-9} = -10 \quad x = \frac{90}{2} = 45$$

$$9) \frac{xy}{y} = \frac{12}{y} \quad x = \frac{12}{y}$$

$$\frac{(x+1)(y-4)}{y-4} = \frac{16}{y-4}$$

$$x + 1 = \frac{16}{y-4}$$

$$\frac{12}{y} (y(y-4)) + 1(y(y-4)) = \frac{16}{y-4} (y(y-4))$$

LCD: $(y(y-4))$

$$12(y-4) + (y(y-4)) = 16y$$

$$12y - 48 + y^2 - 4y = 16y$$

$$y^2 + 8y - 48 = 16y$$

$$\begin{array}{r} -16y \quad -16y \\ \hline y^2 - 8y - 48 = 0 \end{array}$$

$$(y-12)(y+4) = 0$$

$$y-12 = 0 \quad y+4 = 0$$

$$\begin{array}{r} +12 \quad +12 \quad -4 \quad -4 \\ \hline y = 12 \quad y = -4 \end{array}$$

$$x = \frac{12}{12} = 1 \quad x = \frac{12}{-4} = -3$$

$(1, 12), (-3, -4)$

$$11) \frac{xy}{y} = \frac{45}{y} \quad x = \frac{45}{y}$$

$$\frac{(x-5)(y+3)}{y+3} = \frac{160}{y+3}$$

$$x - 5 = \frac{160}{y+3}$$

$$\frac{45}{y} (y(y+3)) - 5(y(y+3)) = \frac{160}{y+3} (y(y+3))$$

$$45(y+3) - 5y(y+3) = 160y$$

$$45y + 135 - 5y^2 - 15y = 160y$$

$$-5y^2 + 30y + 135 = 160y$$

$$\begin{array}{r} +5y^2 - 30y - 135 \quad +5y^2 - 30y - 135 \\ \hline 0 = \frac{5y^2}{5} + \frac{130y}{5} - \frac{135}{5} \end{array}$$

$$0 = y^2 + 26y - 27$$

$$0 = (y+27)(y-1)$$

$$y+27 = 0 \quad y-1 = 0$$

$$\begin{array}{r} -27 \quad -27 \quad +1 \quad +1 \\ \hline y = -27 \quad y = 1 \end{array}$$

$$x = \frac{45}{-27} = -\frac{5}{3} \quad x = \frac{45}{1}$$

$(-\frac{5}{3}, -27), (45, 1)$

9.10

- 1) A merchant bought some pieces of silk for \$900. Had he bought 3 pieces more for the same money, he would have paid \$15 less for each piece. Find the number of pieces purchased.

N	P	T
N	P	900
$n + 3$	$p - 15$	900

$$\frac{np}{n} = \frac{900}{n} \quad p = 900n$$

$$\frac{(n+3)(p-15)}{n+3} = \frac{900}{n+3}$$

$$n + 3 = \frac{900}{n+3}$$

$$\frac{900}{n}(n(n+3)) - 15(n(n+3)) = \frac{900}{n+3}n(n+3)$$

LCD: $n(n+3)$

$$900(n+3) - 15n(n+3) = 900n$$

$$900n + 2700 - 15n^2 - 45n = 900n$$

$$-15n^2 + 855n + 2700 = 900n$$

$$\begin{array}{r} +15n^2 - 855n - 2700 \\ +15n^2 - 855n - 2700 \\ \hline 0 = \frac{15n^2}{15} + \frac{45n}{15} - \frac{2700}{15} \\ 0 = n^2 + 3n - 180 \\ 0 = (n+15)(n-12) \\ n+15 = 0 \quad n-12 = 0 \\ \begin{array}{r} -15 \quad -15 \quad +12 \quad +12 \\ \hline n = -15 \quad n = 12 \end{array} \end{array}$$

- 3) A merchant bought a number of barrels of apples for \$120. He kept two barrels and sold the remainder at a profit of \$2 per barrel making a total profit of \$34. How many barrels did he originally buy?

N	P	T
N	P	120
$n - 2$	$p + 2$	900

$$\frac{np}{n} = \frac{120}{n} \quad p = \frac{120}{n}$$

$$\frac{(n-2)(p+2)}{n-2} = \frac{154}{n-2}$$

$$p + 2 = \frac{154}{n-2}$$

$$\frac{120}{n}(n(n-2)) + 2(n(n-2)) = \frac{154}{n-2}(n(n-2))$$

LCD: $(n(n-2))$

$$120(n-2) + 2n(n-2) = 154n$$

$$120n - 240 + 2n^2 - 4n = 154n$$

$$2n^2 + 116n - 240 = 154n$$

$$\begin{array}{r} -154n \quad -154n \\ \hline \frac{2n^2}{2} - \frac{38n}{2} - \frac{240}{2} = \frac{0}{2} \\ n^2 - 17n - 120 = 0 \\ (n-24)(n+5) = 0 \\ n-24 = 0 \quad n+5 = 0 \\ \begin{array}{r} +24 \quad +24 \quad -5 \quad -5 \\ \hline n = 24 \quad n = -5 \end{array} \end{array}$$

- 5) A man bought a number of articles at equal cost for \$500. He sold all but two for \$540 at a profit of \$5 for each item. How many articles did he buy?

N	P	T
N	P	500
$n - 2$	$p + 5$	540

$$\frac{np}{n} = \frac{500}{n} \quad p = \frac{500}{n}$$

$$\frac{(n-2)(p+5)}{n-2} = \frac{540}{n-2}$$

$$p + 5 = \frac{540}{n-2}$$

$$\frac{500}{n} (n(n+2)) + 5(n(n+2)) = \frac{540}{n-2} (n(n+2))$$

LCD: (n(n+2))

$$500(n-2) + 5n(n-2) = 540n$$

$$500n - 1000 + 5n^2 - 10n = 540n$$

$$5n^2 + 490n - 1000 = 540n$$

$$\begin{array}{r} -540n \qquad \qquad \qquad -540n \\ \hline \frac{5n^2}{5} - \frac{50n}{5} - \frac{1000}{5} = \frac{0}{5} \\ n^2 - 10n - 200 = 0 \\ (n-20)(n+40) = 0 \\ n-20 = 0 \quad n+40 = 0 \\ \begin{array}{r} +20 \quad +20 \quad -40 \quad -40 \\ \hline n = 20 \qquad \qquad \qquad n = \cancel{-40} \end{array} \end{array}$$

- 7) A group of boys bought a boat for \$450. Five boys failed to pay their share, hence each remaining boys were compelled to pay \$4.50 more. How many boys were in the original group and how much had each agreed to pay?

N	P	T
N	P	450
$n - 5$	$p + 4.5$	450

$$\frac{np}{n} = \frac{450}{n} \quad p = \frac{450}{n}$$

$$\frac{(n-5)(p+4.5)}{n-5} = \frac{450}{n-5}$$

$$p + 4.5 = \frac{450}{n-5}$$

$$\frac{450}{n} (2n(n-5)) + \frac{9}{2} (2n(n-5)) = \frac{450}{n-5} (2n(n-5))$$

LCD: (2n(n-5))

$$900(n-5) + 9n(n-5) = 900n$$

$$900n - 4500 + 9n^2 - 45n = 900n$$

$$9n^2 + 855n - 4500 = 900n$$

$$\begin{array}{r} -900n \qquad \qquad \qquad -900n \\ \hline \frac{9n^2}{9} - \frac{45n}{9} - \frac{4500}{9} = \frac{0}{9} \\ n^2 - 5n - 500 = 0 \\ (n-25)(n+20) = 0 \\ n-25 = 0 \quad n+20 = 0 \\ \begin{array}{r} +25 \quad +25 \quad -20 \quad -20 \\ \hline n = 25 \qquad \qquad \qquad n = \cancel{-20} \end{array} \end{array}$$

- 9) A factory tests the road performance of new model cars by driving them at two different rates of speed for at least 100 kilometers at each rate. The speed rates range from 50 to 70 km/hr in the lower range and from 70 to 90 km/hr in the higher range. A driver plans to test a car on an available speedway by driving it for 120 kilometers at a speed in the lower range and then driving 120 kilometers at a rate that is 20 km/hr faster. At what rates should he drive if he plans to complete the test in $3\frac{1}{2}$ hours?

r	t	d
r	t	120
$r + 20$	$35 - t$	120

$$\frac{rt}{r} = \frac{120}{r} \quad t = \frac{120}{r}$$

$$\frac{(r+20)(35-t)}{r+20} = \frac{120}{r+20}$$

$$\frac{7}{2} (2r(r+20)) - \frac{120}{r} (2r(r+20)) = \frac{120}{r+20} (2r(r+20))$$

LCD: $(2r(r+20))$

$$7r(r+20) - 240(r+20) = 240r$$

$$7r^2 + 140r - 240r - 480 = 240r$$

$$7r^2 - 100r - 480 = 240r$$

$$\frac{-240r}{-240r} \quad \frac{-240r}{-240r}$$

$$7r^2 - 340r - 4800 = 0$$

$$\frac{340 \pm \sqrt{(340)^2 - 4(7)(-4800)}}{2(7)} = \frac{340 \pm \sqrt{25000}}{14} = \frac{340 \pm 500}{14} = 60, \frac{80}{7}$$

60mph & 80mph

- 11) The rate of the current in a stream is 3 km/hr. A man rowed upstream for 3 kilometers and then returned. The round trip required 1 hour and 20 minutes. How fast was he rowing?

r	t	d
r	t	3
$r - 3$	$\frac{4}{3} - t$	3

$$1\frac{1}{3} = \frac{4}{3}$$

$$\frac{(r+3)t}{r+3} = \frac{3}{r+3} \quad t = \frac{3}{r+3}$$

$$\frac{(r-3)(\frac{4}{3}-t)}{r-3} = \frac{3}{r-3}$$

$$\frac{4}{3} - t = \frac{3}{r-3}$$

$$\frac{4}{3} 3(r+3)(r-3) - \frac{3}{r+3} 3(r+3)(r-3) = \frac{3}{r-3} 3(r+3)(r-3)$$

LCD: $3(r+3)(r-3)$

$$4(r^2 - 9) - 9(r-3) = 9(r+3)$$

$$4r^2 - 36 - 9r + 27 = 9r + 27$$

$$4r^2 - 9r - 9 = 9r + 27$$

$$\frac{-9r - 27}{-9r - 27} \quad \frac{-9r - 27}{-9r - 27}$$

$$\frac{4r^2}{2} - \frac{18r}{2} - \frac{36}{2} = 0$$

$$2r^2 - 9r - 18 = 0$$

$$(2r+3)(r-6) = 0$$

$$2r+3 = 0 \quad r-6 = 0$$

$$\frac{-3}{-3} \quad \frac{-3}{-3} \quad \frac{+6}{+6} \quad \frac{+6}{+6}$$

$$\frac{2r}{2} = -\frac{3}{2} \quad r = 6$$

~~$r = -\frac{3}{2}$~~

- 13) Two drivers are testing the same model car at speeds that differ by 20 km/hr. The one driving at the slower rate drives 70 kilometers down a speedway and returns by the same route. The one driving at the faster rate drives 76 kilometers down the speedway and returns by the same route. Both drivers leave at the same time, and the faster car returns $\frac{1}{2}$ hour earlier than the slower car.

At what rates were the cars driven?

r	t	d
$r + 20$	$t - \frac{1}{2}$	152
r	t	140

$$\frac{rt}{r} = \frac{140}{r} \quad t = \frac{140}{r}$$

$$\frac{(r+20)(t-\frac{1}{2})}{r+20} = \frac{152}{r+20}$$

$$t - \frac{1}{2} = \frac{152}{r+20}$$

$$\frac{140}{r}(2r(r+20)) - \frac{1}{2}(2r(r+20)) = \frac{152}{r+20}(2r(r+20))$$

LCD: (2r(r+20))

$$280(r+20) - r(r+20) = 304r$$

$$280r + 560 - r^2 - 20r = 304r$$

$$-r^2 + 260r + 560 = 304r$$

$$\underline{+r^2 - 260r - 560} \quad \underline{+r^2 - 260r - 560}$$

$$0 = r^2 + 44r - 5600$$

$$0 = (r - 56)(r + 100)$$

$$r - 56 = 0 \quad r + 100 = 0$$

$$\underline{+56 \quad +56 \quad -100 \quad -100}$$

$$r = 56 \quad \cancel{r = -100}$$

56km/hr & 76km/hr

- 15) An automobile goes to a place 72 miles away and then returns, the round trip occupying 9 hours. His speed in returning is 12 miles per hour faster than his speed in going. Find the rate of speed in both going and returning.

r	t	d
r	t	72
$r + 12$	$9 - t$	72

$$\frac{rt}{r} = \frac{72}{r} \quad t = \frac{72}{r}$$

$$\frac{(r+12)(9-t)}{r+12} = \frac{72}{r+12}$$

$$9 - t = \frac{72}{r+12}$$

$$9(r(r+12)) - \frac{72}{r}(r(r+12)) = \frac{72}{r+12}(r(r+12))$$

LCD: r(r+12)

$$9r(r+12) - 72(r+12) = 72r$$

$$9r^2 + 108r - 72r - 864 = 72r$$

$$9r^2 + 36r - 864 = 72r$$

$$\underline{-72r} \quad \underline{-72r}$$

$$\frac{9r^2}{9} - \frac{36r}{9} - \frac{864}{9} = \frac{0}{9}$$

$$r^2 - 4r - 96 = 0$$

$$(r - 12)(r + 8) = 0$$

$$r - 12 = 0 \quad r + 8 = 0$$

$$\underline{+12 \quad +12 \quad -8 \quad -8}$$

$$r = 12 \quad \cancel{r = -8}$$

12 mph & 24mph

- 17) The rate of a stream is 3 miles an hour. If a crew rows downstream for a distance of 8 miles and then back again, the round trip occupying 5 hours, what is the rate of the crew in still water?

r	t	d
$r + 3$	t	8
$r - 3$	$5 - t$	8

$$\frac{(r+3)t}{r+3} = \frac{8}{r+3} \quad t = \frac{8}{r+3}$$

$$\frac{(r-3)(5-t)}{r-3} = \frac{8}{r-3}$$

$$5 - t = \frac{8}{r-3}$$

$$5(r+3)(r-3) - \frac{8}{r+3}(r+3)(r-3) = \frac{8}{r-3}(r+3)(r-3)$$

$$5(r^2 - 9) - 8(r-3) = 8(r+3)$$

$$5r^2 - 45 - 8r + 24 = 8r + 24$$

$$5r^2 - 8r - 21 = 8r + 24$$

$$\frac{-8r - 24 - 8r - 24}{-8r - 24 - 8r - 24}$$

$$5r^2 - 16r - 46 = 0$$

$$(5r + 9)(r - 5) = 0$$

$$5r + 9 = 0 \quad r - 5 = 0$$

$$\frac{-9 \quad -9 \quad +5 \quad +5}{\frac{5r}{5} = -\frac{9}{5} \quad r = 5mph}$$

$$\cancel{r = -\frac{9}{5}}$$

- 19) By going 15 miles per hour faster, a train would have required 1 hour less to travel 180 miles. How fast did it travel?

r	t	d
r	t	180
$r + 15$	$t - 1$	180

$$\frac{rt}{r} = \frac{180}{r} \quad t = \frac{180}{r}$$

$$\frac{(r+15)(t-1)}{r+15} = \frac{180}{r+15}$$

$$t - 1 = \frac{180}{r+15}$$

$$\frac{180}{r}(r(r+15)) - 1(r(r+15)) = \frac{180}{r+15}(r(r+15))$$

LCD: $r(r+15)$

$$180(r+15) - r(r+15) = 180r$$

$$180r + 2700 - r^2 - 15r = 180r$$

$$-r^2 + 165r + 2700 = 180r$$

$$\frac{+r^2 - 165 - 2700 \quad +r^2 - 165 - 2700}{0 = r^2 + 15r - 2700}$$

$$0 = (r + 60)(r - 45)$$

$$r + 60 = 0 \quad r - 45 = 0$$

$$\frac{-60 \quad -60 \quad +45 \quad +45}{\cancel{r = -60} \quad r = 45mph}$$

- 21) If a train had traveled 5 miles an hour faster, it would have needed $1\frac{1}{2}$ hours less time to travel 150 miles. Find the rate of the train.

r	t	d
r	t	150
$r + 5$	$t - \frac{3}{2}$	150

$$\frac{rt}{r} = \frac{150}{r} \quad t = \frac{150}{r}$$

$$\frac{(r+5)(t-\frac{3}{2})}{r+5} = \frac{150}{r+5}$$

$$t - \frac{3}{2} = \frac{150}{r+5}$$

$$\frac{150}{r}(2r(r+5)) - \frac{3}{2}(2r(r+5)) = \frac{150}{r+5}(2r(r+5))$$

LCD: $(2r(r+5))$

$$300(r+5) - 3r(r+5) = 300r$$

$$300r + 1500 - 3r^2 - 15r = 300r$$

$$-3r^2 + 285r + 1500 = 300r$$

$$\frac{+3r^2 - 285r - 1500}{+3r^2 - 285r - 1500}$$

$$0 = \frac{3r^2}{3} + \frac{15r}{3} - \frac{1500}{3}$$

$$0 = (r+25)(r-20)$$

$$r+25 = 0 \quad r-20 = 0$$

$$\frac{-25 \quad -25 \quad +20 \quad +20}{\cancel{r = -25} \quad r = 20\text{mph}}$$

9.11

1) $y = x^2 - 2x - 8$

y -inter: $(0, -8)$

x -inter: $0 = x^2 - 2x - 8$

$$0 = (x-4)(x+2)$$

$$x-4 = 0 \quad x+2 = 0$$

$$\frac{+4 \quad +4 \quad -2 \quad -2}{x = 4 \quad x = -2}$$

$(4, 0), (-2, 0)$

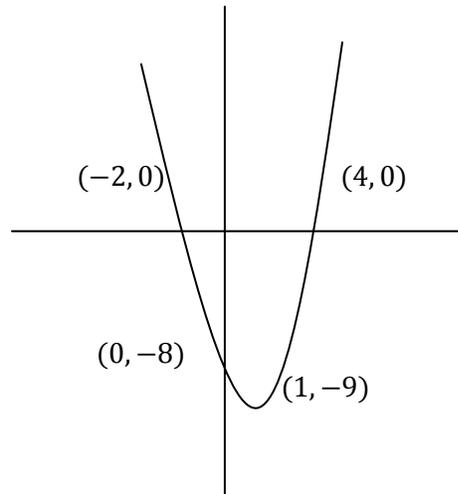
vertex: $x = \frac{2}{2(1)} = \frac{2}{2} = 1$

$$y = (1)^2 - 2(1) - 8$$

$$y = 1 - 2 - 8$$

$$y = -9$$

$(1, -9)$



3) $y = 2x^2 - 12x + 10$

y - *inter*: $(0, 10)$

x - *inter*: $0 = 2x^2 - 12x + 10$

$0 = 2(x^2 - 6x + 5)$

$0 = 2(x - 5)(x - 1)$

$x - 5 = 0$ $x - 1 = 0$

$$\frac{+5 \quad +5 \quad +1 \quad +1}{x = 5 \quad x = 1}$$

$(5, 0)$ $(1, 0)$

vertex: $x = \frac{12}{2(2)} = \frac{12}{4} = 3$

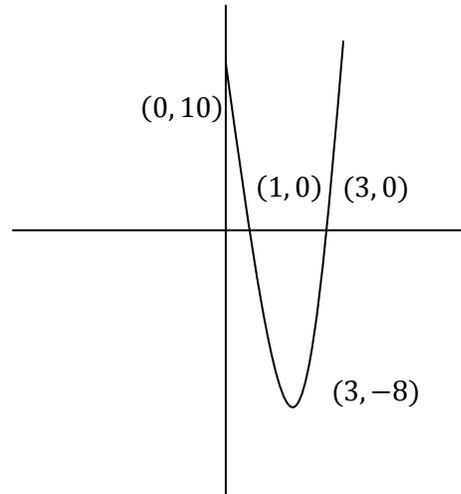
$y = 2(3)^2 - 12(3) + 10$

$y = 2(9) - 36 + 10$

$y = 18 - 36 + 10$

$y = -8$

$(3, -8)$



5) $y = -2x^2 + 12x - 18$

y - *inter*: $(0, 10)$

x - *inter*: $0 = -2x^2 + 12x - 18$

$0 = -2(x^2 - 6x + 9)$

$0 = -2(x - 3)^2$

$x - 3 = 0$

$$\frac{+3 \quad +3}{x = 3}$$

$x = 3$

$(3, 0)$

vertex: $x = \frac{-12}{2(-2)} = \frac{-12}{-4} = 3$

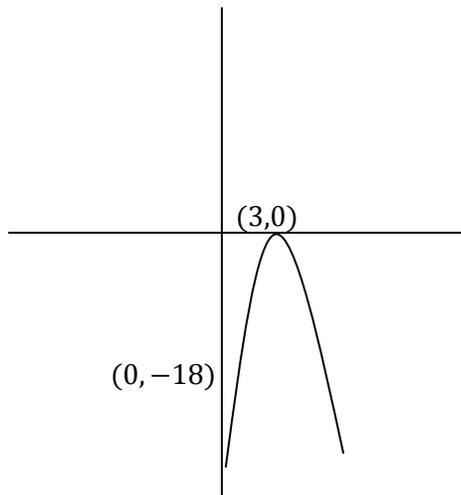
$y = -2(3)^2 + 12(3) - 18$

$y = -2(9) + 36 - 18$

$y = -18 + 36 - 18$

$y = 0$

$(3, 0)$



7) $y = -3x^2 + 24x - 45$

y - *inter*: $(0, -45)$

x - *inter*: $0 = -3x^2 + 24x - 45$

$$0 = -3(x^2 - 8x + 15)$$

$$0 = -3(x - 5)(x - 3)$$

$$x - 5 = 0 \quad x - 3 = 0$$

$$\begin{array}{r} +5 \quad +5 \quad +3 \quad +3 \\ \hline \end{array}$$

$$x = 5 \quad x = 3$$

$$(5, 0) \quad (3, 0)$$

vertex: $x = \frac{-24}{2(-3)} = \frac{-24}{-6} = 4$

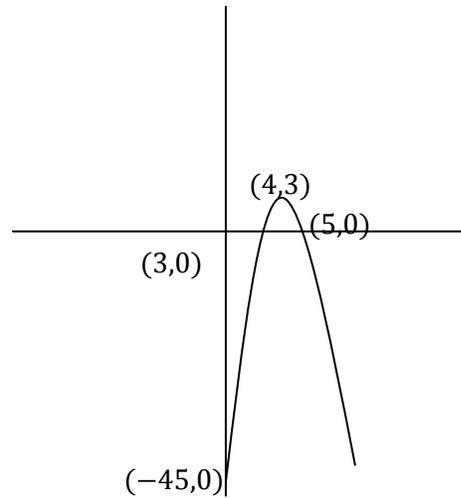
$$y = -3(4)^2 + 24(4) - 45$$

$$y = -3(16) + 96 - 45$$

$$y = -48 + 96 - 45$$

$$y = 3$$

$$(4, 3)$$



9) $y = -x^2 + 4x + 5$

y - *inter*: $(0, 5)$

x - *inter*: $0 = -x^2 + 4x + 5$

$$0 = -1(x^2 - 4x - 5)$$

$$0 = -1(x - 5)(x + 1)$$

$$x - 5 = 0 \quad x + 1 = 0$$

$$\begin{array}{r} +5 \quad +5 \quad -1 \quad -1 \\ \hline \end{array}$$

$$x = 5 \quad x = -1$$

$$(5, 0) \quad (-1, 0)$$

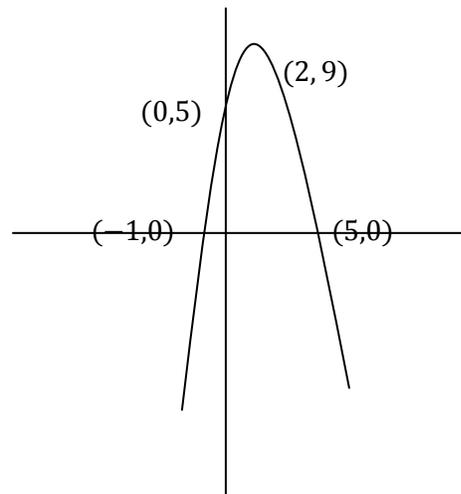
vertex: $x = \frac{-4}{2(-1)} = \frac{-4}{-2} = 2$

$$y = -(2)^2 + 4(2) + 5$$

$$y = -4 + 8 + 5$$

$$y = 9$$

$$(2, 9)$$



$$11) y = -x^2 + 6x - 5$$

$$y - \text{inter: } (0, -5)$$

$$x - \text{inter: } 0 = -x^2 + 6x - 5$$

$$0 = -1(x^2 - 6x + 5)$$

$$0 = -1(x - 1)(x - 5)$$

$$x - 1 = 0 \quad x - 5 = 0$$

$$\begin{array}{r} +1 \quad +1 \quad +5 \quad +5 \\ \hline \end{array}$$

$$x = 1 \quad x = 5$$

$$(1, 0) \quad (5, 0)$$

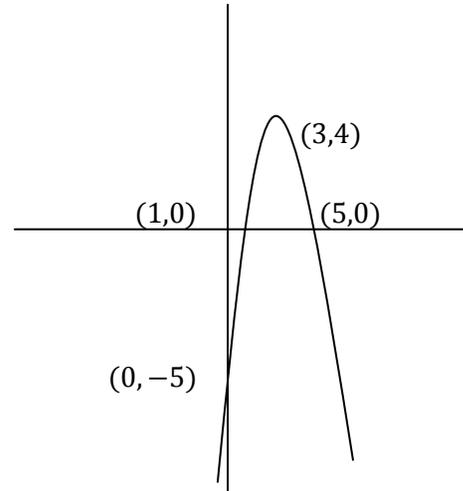
$$\text{vertex: } x = \frac{-6}{2(-1)} = \frac{-6}{-2} = 3$$

$$y = -(3)^2 + 6(3) - 5$$

$$y = -9 + 18 - 5$$

$$y = 4$$

$$(3, 4)$$



$$13) y = -2x^2 + 16x - 24$$

$$y - \text{inter: } (0, -24)$$

$$x - \text{inter: } 0 = -2x^2 + 16x - 24$$

$$0 = -2(x^2 - 8x + 12)$$

$$0 = -2(x - 2)(x - 6)$$

$$x - 2 = 0 \quad x - 6 = 0$$

$$\begin{array}{r} +2 \quad +2 \quad +6 \quad +6 \\ \hline \end{array}$$

$$x = 2 \quad x = 6$$

$$(2, 0) \quad (6, 0)$$

$$\text{vertex: } x = \frac{-16}{2(-2)} = \frac{-16}{-4} = 4$$

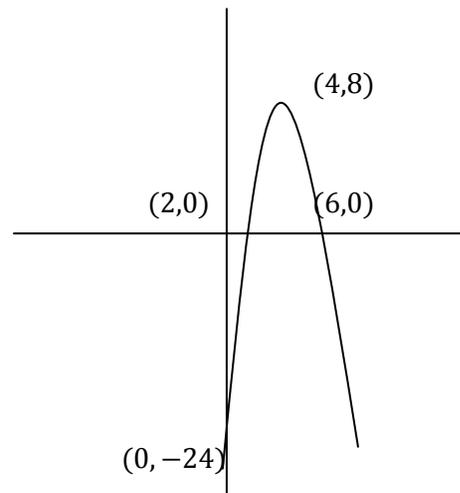
$$y = -2(4)^2 + 16(4) - 24$$

$$y = -2(16) + 64 - 24$$

$$y = -32 + 64 - 24$$

$$y = 8$$

$$(4, 8)$$



$$15) y = 3x^2 + 12x + 9$$

$$y - \text{inter: } (0, 9)$$

$$x - \text{inter: } 0 = 3x^2 + 12x + 9$$

$$0 = 3(x^2 + 4x + 3)$$

$$0 = 3(x + 1)(x + 3)$$

$$x + 1 = 0 \quad x + 3 = 0$$

$$\begin{array}{r} -1 \quad -1 \quad -3 \quad -3 \\ \hline x = -1 \quad x = -3 \end{array}$$

$$(-1, 0) \quad (-3, 0)$$

$$\text{vertex: } x = \frac{-12}{2(3)} = \frac{-12}{6} = -2$$

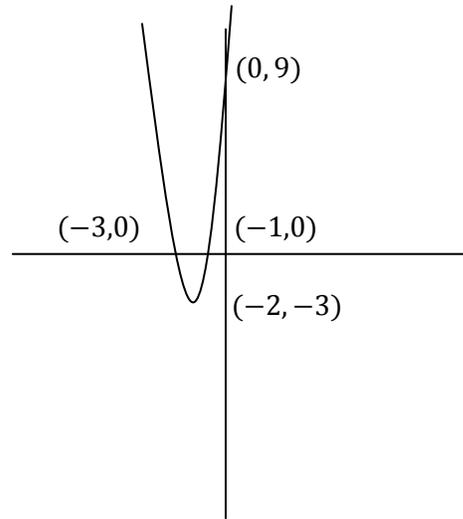
$$y = 3(-2)^2 + 12(-2) + 9$$

$$y = 3(4) - 24 + 9$$

$$y = 12 - 24 + 9$$

$$y = -3$$

$$(-2, -3)$$



$$17) y = 5x^2 - 40x + 75$$

$$y - \text{inter: } (0, 75)$$

$$x - \text{inter: } 0 = 5x^2 - 40x + 75$$

$$0 = 5(x^2 - 8x + 15)$$

$$0 = 5(x - 3)(x - 5)$$

$$x - 3 = 0 \quad x - 5 = 0$$

$$\begin{array}{r} +3 \quad +3 \quad +5 \quad +5 \\ \hline x = 3 \quad x = 5 \end{array}$$

$$\text{vertex: } \frac{40}{2(5)} = \frac{40}{10} = 4$$

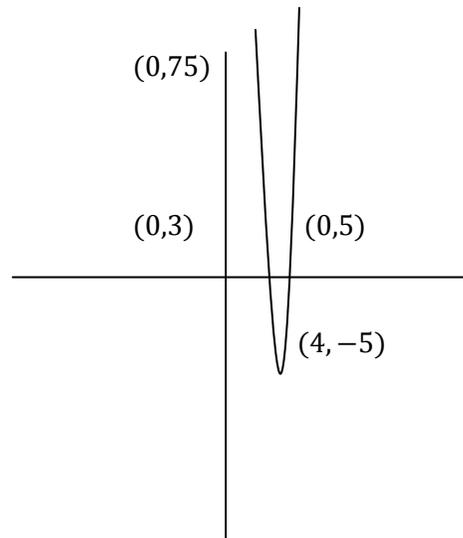
$$y = 5(4)^2 - 40(4) + 75$$

$$y = 5(16) - 160 + 75$$

$$y = 80 - 160 + 75$$

$$y = -5$$

$$(4, -5)$$



$$19) y = -5x^2 - 60x - 175$$

$$y - \text{inter: } (0, -175)$$

$$x - \text{inter: } 0 = -5x^2 - 60x - 175$$

$$0 = -5(x^2 + 12x + 35)$$

$$0 = -5(x + 5)(x + 7)$$

$$x + 5 = 0 \quad x + 7 = 0$$

$$\begin{array}{cc} -5 & -5 & -7 & -7 \\ \hline \end{array}$$

$$x = -5 \quad x = -7$$

$$(-5, 0) \quad (-7, 0)$$

$$\text{vertex: } x = \frac{60}{2(-5)} = \frac{60}{-10} = -6$$

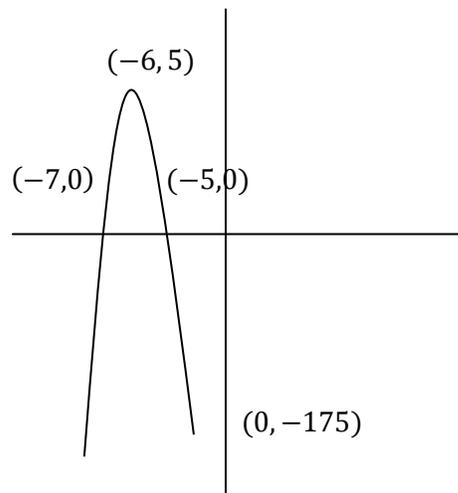
$$y = -5(-6)^2 - 60(-6) - 175$$

$$y = -5(36) + 360 - 175$$

$$y = -180 + 360 - 175$$

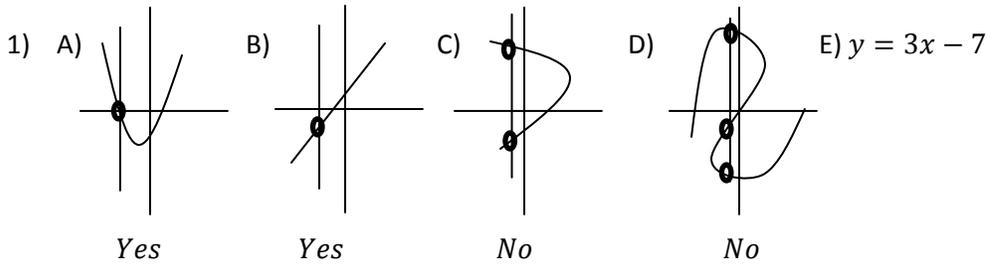
$$y = 5$$

$$(-6, 5)$$



Chapter 10: Functions

10.1



F) $y^2 - x^2 = 1$
 $\frac{+x^2 + x^2}{\sqrt{y^2} = \sqrt{(x^2 + 1)}}$
 $y = \pm\sqrt{x^2 + 1}$
 No

G) $\sqrt{y} + x = 2$
 $\frac{-x - x}{(\sqrt{y})^2 = (2 - x)^2}$
 $y = (2 - x)^2$
 Yes

H) $x^2 + y^2 = 1$
 $\frac{-x^2 - x^2}{\sqrt{y^2} = \sqrt{(1 - x^2)}}$
 $y = \pm\sqrt{1 - x^2}$
 No

3) $f(x) = \sqrt{5 - 4x}$
 $5 - 4x \geq 0$
 $\frac{-5 - 5}{-\frac{4x}{-4} \geq \frac{-5}{-4}}$
 $x \leq \frac{5}{4}$

5) $f(x) = x^2 - 3x - 4$
 All Real Numbers \mathbb{R}

7) $f(x) = \sqrt{x - 16}$
 $x - 16 \geq 0$
 $\frac{+16 + 16}{x \geq 16}$

9) $h(x) = \frac{\sqrt{3x-12}}{x^2-25}$
 $x^2 - 25 \neq 0$
 $(x - 5)(x + 5) \neq 0$
 $x - 5 = 0 \quad x + 5 = 0$
 $\frac{+5 + 5}{x = 5} \quad \frac{-5 - 5}{x = -5}$
 $x \geq 4, x \neq 5$

$3x - 12 \geq 0$
 $+ 12 + 12$
 $\frac{3x}{3} \geq \frac{12}{3}$
 $x \geq 4$

11) $g(x) = 4x - 4 \quad g(0)$
 $g(0) = 4(0) - 4$
 $g(0) = 0 - 4$
 $g(0) = -4$

13) $f(x) = |3x + 1| + 1 \quad f(0)$
 $f(0) = |3(0) + 1| + 1$
 $f(0) = |0 + 1| + 1$
 $f(0) = |1| + 1$
 $f(0) = 1 + 1$
 $f(0) = 2$

15) $f(n) = -2|-n - 2| + 1 \quad f(-6)$
 $f(-6) = -2|-(-6) - 2| + 1$
 $f(-6) = -2|6 - 2| + 1$
 $f(-6) = -2|4| + 1$
 $f(-6) = -2(4) + 1$
 $f(-6) = -8 + 1$
 $f(-6) = -7$

17) $f(rt) = 3^t - 2 \quad f(-2)$
 $f(-2) = 3^{-2} - 2$
 $f(-2) = \frac{1}{3^2} - 2$
 $f(-2) = \frac{1}{9} - 2$
 $f(-2) = \frac{1}{9} - \frac{18}{9}$
 $f(-2) = -\frac{17}{9}$

$$19) f(t) = |t + 3| \quad f(10)$$

$$f(10) = |10 + 3|$$

$$f(10) = |13|$$

$$f(10) = 13$$

$$29) k(n) = |n - 1| \quad k(3)$$

$$k(3) = |3 - 1|$$

$$k(3) = |2|$$

$$k(3) = 2$$

$$21) w(n) = 4n + 3 \quad w(2)$$

$$w(2) = 4(2) + 3$$

$$w(2) = 8 + 3$$

$$w(2) = 11$$

$$31) h(x) = x^3 + 2 \quad h(-4x)$$

$$h(-4x) = (-4x)^3 + 2$$

$$h(-4x) = -64x^3 + 2$$

$$23) w(n) = 2^{n+2} \quad w(-2)$$

$$w(-2) = 2^{-2+2}$$

$$w(-2) = 2^0$$

$$w(-2) = 1$$

$$33) h(x) = 3x + 2 \quad h(-1 + x)$$

$$h(-1 + x) = 3(-1 + x) + 2$$

$$h(-1 + x) = -3 + 3x + 2$$

$$h(-1 + x) = 3x - 1$$

$$25) p(n) = -3|n| \quad p(7)$$

$$p(7) = -3|7|$$

$$p(7) = -3(7)$$

$$p(7) = -21$$

$$35) h(t) = 2|-3t - 1| + 2 \quad h(n^2)$$

$$h(n^2) = 2|-3n^2 - 1| + 2$$

$$37) g(x) = x + 1 \quad g(3x)$$

$$g(3x) = 3x + 1$$

$$27) p(t) = -t^3 + t \quad p(4)$$

$$p(4) = -4^3 + 4$$

$$p(4) = -64 + 4$$

$$p(4) = -60$$

$$39) g(x) = 5^x \quad g(-3 - x)$$

$$g(-3 - x) = 5^{-3-x}$$

10.2

$$1) g(a) = a^3 + 5a^2$$

$$f(a) = 2a + 4$$

$$g(3) + f(3)$$

$$g(3) = 3^3 + 5(3)^2$$

$$g(3) = 27 + 5(9)$$

$$g(3) = 27 + 45$$

$$g(3) = 72$$

$$f(3) = 2(3) + 4$$

$$f(3) = 6 + 4$$

$$f(3) = 10$$

$$g(3) + f(3) = 72 + 10 = 82$$

$$3) g(a) = 3a + 3$$

$$f(a) = 2a - 2$$

$$(g + f)(a) = g(9) + f(9)$$

$$g(9) = 3(9) + 3$$

$$g(9) = 27 + 3$$

$$g(9) = 30$$

$$f(9) = 2(9) - 2$$

$$f(9) = 18 - 2$$

$$f(9) = 16$$

$$g(9) + f(9) = 30 + 16 = 46$$

$$\begin{aligned}
5) \quad & g(x) = x + 3 \\
& f(x) = -x + 4 \\
& (g - f)(3) = g(3) - f(3) \\
& g(3) = 3 + 3 \\
& g(3) = 6 \\
& f(3) = -3 + 4 \\
& f(3) = -1 \\
& g(3) + f(3) = 6 - 1 = 5
\end{aligned}$$

$$\begin{aligned}
7) \quad & g(x) = x^2 + 2 \\
& f(x) = 2x + 5 \\
& (g - f)(0) = g(0) - f(0) \\
& g(0) = 0^2 + 2 \\
& g(0) = 0 + 2 \\
& g(0) = 2 \\
& f(0) = 2(0) + 5 \\
& f(0) = 0 + 5 \\
& f(0) = 5 \\
& g(0) - f(0) = 2 - 5 = -3
\end{aligned}$$

$$\begin{aligned}
9) \quad & g(t) = t - 3 \\
& h(t) = -3t^3 + 6t \\
& g(1) + h(1) \\
& g(1) = 1 - 3 \\
& g(1) = -2 \\
& h(1) = -3(1)^3 + 6(1) \\
& h(1) = -3(1) + 6 \\
& h(1) = -3 + 6 \\
& h(1) = 3 \\
& g(1) + h(1) = -2 + 3 = 1
\end{aligned}$$

$$\begin{aligned}
11) \quad & h(t) = t + 5 \\
& g(t) = 3t - 5 \\
& (h \cdot g)(5) = h(5) \cdot g(5) \\
& h(5) = 5 + 5 \\
& h(5) = 10 \\
& g(5) = 3(5) - 5 \\
& g(5) = 15 - 5 \\
& g(5) = 10 \\
& h(5) \cdot g(5) = 10 \cdot 10 = 100
\end{aligned}$$

$$\begin{aligned}
13) \quad & h(n) = 2n - 1 \\
& g(n) = 3n - 5 \\
& h(0) \div g(0) \\
& h(0) = 2(0) - 1 \\
& h(0) = 0 - 1 \\
& h(0) = -1 \\
& g(0) = 3(0) - 5 \\
& g(0) = 0 - 5 \\
& g(0) = -5 \\
& h(0) \div g(0) = -1 \div -5 = -\frac{1}{5}
\end{aligned}$$

$$\begin{aligned}
15) \quad & f(a) = -2a - 4 \\
& g(a) = a^2 + 3 \\
& \left(\frac{f}{g}\right)(7) = \frac{f(7)}{g(7)} \\
& f(7) = -2(7) - 4 \\
& f(7) = -14 - 4 \\
& f(7) = -18 \\
& g(7) = 7^2 + 3 \\
& g(7) = 49 + 3 \\
& g(7) = 52 \\
& \frac{f(7)}{g(7)} = \frac{-18}{52} = -\frac{9}{26}
\end{aligned}$$

$$\begin{aligned}
17) \quad & g(x) = -x^3 - 2 \\
& h(x) = 4x \\
& (g - h)(x) = g(x) - h(x) \\
& g(x) - h(x) = (-x^3 - 2) - (4x) \\
& g(x) - h(x) = -x^3 - 2 - 4x
\end{aligned}$$

$$\begin{aligned}
19) \quad & f(x) = -3x + 2 \\
& g(x) = x^2 + 5x \\
& (f - g)(x) = f(x) - g(x) \\
& f(x) - g(x) = (-3x + 2) - (x^2 + 5x) \\
& f(x) - g(x) = -3x + 2 - x^2 - 5x \\
& f(x) - g(x) = -x^2 - 8x + 2
\end{aligned}$$

$$\begin{aligned}
21) \quad & g(x) = 4x + 5 \\
& h(x) = x^2 + 5x \\
& g(x) \cdot h(x) \\
& g(x) \cdot h(x) = (4x + 5)(x^2 + 5) \\
& g(x) \cdot h(x) = 4x^3 + 20x^2 + 5x^2 + 25x \\
& g(x) \cdot h(x) = 4x^3 + 25x^2 + 25
\end{aligned}$$

$$\begin{aligned}
23) \quad & f(x) = x^2 - 5x \\
& g(x) = x + 5 \\
& (f + g)(x) = f(x) + g(x) \\
& f(x) + g(x) = (x^2 - 5x) + (x + 5) \\
& f(x) + g(x) = x^2 - 4x + 5
\end{aligned}$$

$$\begin{aligned}
25) \quad & g(n) = n^2 + 5 \\
& f(n) = 3n + 5 \\
& g(n) \div f(n) = \frac{g(n)}{f(n)} \\
& \frac{g(n)}{f(n)} = \frac{n^2 + 5}{3n + 5}
\end{aligned}$$

$$\begin{aligned}
27) \quad & g(a) = -2a + 5 \\
& f(a) = 3a + 5 \\
& \left(\frac{g}{f}\right)(a) = \frac{g(a)}{f(a)} \\
& \frac{g(a)}{f(a)} = \frac{-2a + 5}{3a + 5}
\end{aligned}$$

$$\begin{aligned}
29) \quad & h(n) = n^3 + 4n \\
& g(n) = 4n + 5 \\
& h(n) + g(n) \\
& h(n) + g(n) = (n^3 + 4n) + (4n + 5) \\
& h(n) + g(n) = n^3 + 8n + 5
\end{aligned}$$

$$\begin{aligned}
31) \quad & g(n) = n^2 - 4n \\
& h(n) = n - 5 \\
& g(n^2) \cdot h(n^2) \\
& g(n^2) \cdot h(n^2) = [(n^2)^2 - 4(n^2)][(n^2) - 5] \\
& g(n^2) \cdot h(n^2) = (n^4 - 4n^2)(n^2 - 5) \\
& g(n^2) \cdot h(n^2) = n^6 - 5n^4 - 4n^4 + 20n^2 \\
& g(n^2) \cdot h(n^2) = n^6 - 9n^4 + 20n^2
\end{aligned}$$

$$\begin{aligned}
33) \quad & f(x) = 2x \\
& g(x) = -3x - 1 \\
& (f + g)(-4 - x) = f(-4 - x) + g(-4 - x) \\
& f(-4 - x) + g(-4 - x) = [2(-4 - x)] + [-3(-4 - x) - 1] \\
& f(-4 - x) + g(-4 - x) = (-8 - 2x) + (12 + 3x - 1) \\
& f(-4 - x) + g(-4 - x) = x + 3
\end{aligned}$$

$$\begin{aligned}
35) \quad & f(t) = t^2 + 4t \\
& g(t) = 4t + 2 \\
& f(t^2) + g(t^2) \\
& f(t^2) + g(t^2) = [(t^4) + 4(t^2)] + [4(t^2) + 2] \\
& f(t^2) + g(t^2) = t^4 + 8t^2 + 2
\end{aligned}$$

$$\begin{aligned}
37) \quad & g(a) = a^3 + 2a \\
& h(a) = 3a + 4 \\
& \left(\frac{g}{h}\right)(-x) = \frac{g(-x)}{h(-x)} \\
& \frac{g(-x)}{h(-x)} = \frac{(-x)^3 + 2(x)}{3(x) + 4} = \frac{-x^3 - 2x}{-3x + 4}
\end{aligned}$$

$$\begin{aligned}
39) \quad & f(n) = -3n^2 + 1 \\
& g(n) = 2n + 1 \\
& (f - g)\left(\frac{n}{3}\right) = f\left(\frac{n}{3}\right) - g\left(\frac{n}{3}\right) \\
& f\left(\frac{n}{3}\right) - g\left(\frac{n}{3}\right) = \left[-3\left(\frac{n}{3}\right)^2 + 1\right] - \left[2\left(\frac{n}{3}\right) + 1\right] \\
& f\left(\frac{n}{3}\right) - g\left(\frac{n}{3}\right) = \left[-3\left(\frac{n^2}{9}\right) + 1\right] - \left[\frac{2n}{3} + 1\right] \\
& f\left(\frac{n}{3}\right) - g\left(\frac{n}{3}\right) = \left(-\frac{n^2}{3} + 1\right) - \frac{2n}{3} - 1 \\
& f\left(\frac{n}{3}\right) - g\left(\frac{n}{3}\right) = \frac{-n^2 - 2n}{3}
\end{aligned}$$

$$\begin{aligned}
43) \quad & f(x) = -4x + 1 \\
& g(x) = 4x + 3 \\
& (f \circ g)(x) = f(g(9)) \\
& g(9) = 4(9) + 3 \\
& g(9) = 36 + 3 \\
& g(9) = 39 \\
& f(39) = -4(39) + 1 \\
& f(39) = -156 + 1 \\
& f(39) = -155
\end{aligned}$$

$$\begin{aligned}
45) \quad & h(a) = 3a + 3 \\
& g(a) = a + 1 \\
& (h \circ g)(5) = h(g(5)) \\
& g(5) = 5 + 1 \\
& g(5) = 6 \\
& h(6) = 3(6) + 3 \\
& h(6) = 18 + 3 \\
& h(6) = 21
\end{aligned}$$

$$\begin{aligned}
45) \quad & g(x) = x + 4 \\
& h(x) = x^2 - 1 \\
& (g \circ h)(10) = g(h(10)) \\
& h(10) = 10^2 - 1 \\
& h(10) = 100 - 1 \\
& h(10) = 99 \\
& g(99) = 99 + 4 \\
& g(99) = 103
\end{aligned}$$

$$\begin{aligned}
47) \quad & f(n) = -4n + 2 \\
& g(n) = n + 4 \\
& (f \circ g)(9) = f(g(9)) \\
& g(9) = 9 + 4 \\
& g(9) = 13 \\
& f(13) = -4(13) + 2 \\
& f(13) = -52 + 2 \\
& f(13) = -50
\end{aligned}$$

$$\begin{aligned}
49) \quad & g(x) = 2x - 4 \\
& h(x) = 2x^3 + 4x^2 \\
& (g \circ h)(3) = g(h(3)) \\
& h(3) = 2(3^3) + 4(3^2) \\
& h(3) = 2(27) + 4(9) \\
& h(3) = 54 + 36 \\
& h(3) = 90 \\
& g(90) = 2(90) - 4 \\
& g(90) = 180 - 4 \\
& g(90) = 176
\end{aligned}$$

$$\begin{aligned}
51) \quad & g(x) = x^2 - 5x \\
& h(x) = 4x + 4 \\
& (g \circ h)(x) = g(h(x)) \\
& g(4x + 4) = (4x + 4)^2 - 5(4x + 4) \\
& g(4x + 4) = 16x^2 + 32x + 16 - 20x - 20 \\
& g(4x + 4) = 16x^2 + 12x - 4
\end{aligned}$$

$$\begin{aligned}
53) \quad & f(a) = -2a + 2 \\
& g(a) = 4a \\
& (f \circ g)(a) = f(g(a)) \\
& f(4a) = -2(4a) + 2 \\
& f(4a) = -8a + 2
\end{aligned}$$

$$\begin{aligned}
55) \quad & g(x) = 4x + 4 \\
& f(x) = x^3 - 1 \\
& (g \circ f)(x) = g(f(x)) \\
& g(x^3 - 1) = 4(x^3 - 1) + 4 \\
& g(x^3 - 1) = 4x^3 - 4 + 4 \\
& g(x^3 - 1) = 4x^3
\end{aligned}$$

$$\begin{aligned}
57) \quad & g(x) = -x + 5 \\
& f(x) = 2x - 3 \\
& (g \circ f)(x) = g(f(x)) \\
& g(2x - 3) = -(2x - 3) + 5 \\
& g(2x - 3) = -2x + 3 + 5 \\
& g(2x - 3) = -2x + 8
\end{aligned}$$

$$\begin{aligned}
59) \quad f(t) &= 4t + 3 \\
g(t) &= -4t - 2 \\
(f \circ g)(t) &= f(g(t)) \\
f(-4t - 2) &= 4(-4t - 2) + 3 \\
f(-4t - 2) &= -16t - 8 + 3 \\
f(-4t - 2) &= -16t - 5
\end{aligned}$$

10.3

$$\begin{aligned}
1) \quad g(x) &= -x^5 - 3 \\
f(x) &= \sqrt[5]{-x - 3} \\
(g \circ f)(x) &= g(f(x)) \\
g(\sqrt[5]{-x - 3}) &= -(\sqrt[5]{-x - 3})^5 - 3 \\
&= -(-x - 3) - 3 \\
&= x + 3 - 3 \\
&= x \\
&\text{yes}
\end{aligned}$$

$$\begin{aligned}
3) \quad f(x) &= \frac{-x-1}{x-2} \\
g(x) &= \frac{-2x+1}{-x-1} \\
(f \circ g)(x) &= f(g(x)) \\
f\left(\frac{-2x+1}{-x-1}\right) &= \frac{-\left(\frac{-2x+1}{-x-1}\right)-1}{\frac{-2x+1}{-x-1}-2} \\
&= \frac{\left(\frac{-x-1}{-x-1}\right)\frac{2x-1}{-x-1}-1(-x-1)}{\left(\frac{-x-1}{-x-1}\right)\frac{-2x+1}{-x-1}-2(-x-1)} \\
&= \frac{2x-1+x+1}{-2x+1+2x+2} \\
&= \frac{3x}{3} = x \\
&\text{yes}
\end{aligned}$$

$$\begin{aligned}
5) \quad g(x) &= -10x + 5 \\
f(x) &= \frac{x-5}{10} \\
(g \circ f)(x) &= g(f(x)) \\
g\left(\frac{x-5}{10}\right) &= -10\left(\frac{x-5}{10}\right) + 5 \\
&= -x + 5 + 5 \\
&= -x + 10 \\
&\text{No}
\end{aligned}$$

$$\begin{aligned}
7) \quad f(x) &= -\frac{2}{x+3} \\
g(x) &= \frac{3x+2}{x+2} \\
(f \circ g)(x) &= f(g(x)) \\
f\left(\frac{3x+2}{x+2}\right) &= \frac{-2(x+2)}{(x+2)\frac{3x+2}{x+2}+3(x+2)} \\
&= \frac{-2x-4}{3x+2+3x+6} \\
&= \frac{-2x-4}{6x+12} = \frac{-2(x+2)}{6(x+2)} = -\frac{2}{6} = -\frac{1}{3} \\
&\text{No}
\end{aligned}$$

$$\begin{aligned}
9) \quad g(x) &= \sqrt[5]{\frac{x-1}{2}} \\
f(x) &= 2x^5 + 1 \\
(g \circ f)(x) &= g(f(x)) \\
g(2x^5 + 1) &= \sqrt[5]{\frac{2x^5+1-1}{2}} \\
&= \sqrt[5]{\frac{2x^5}{2}} \\
&= \sqrt[5]{x^5} \\
&= x \\
&\text{yes}
\end{aligned}$$

$$\begin{aligned}
11) \quad f(x) &= (x-2)^5 + 3 \\
y &= (x-2)^5 + 3 \\
x &= (y-2)^5 + 3 \\
-3 &\quad -3 \\
\sqrt[5]{x-3} &= \sqrt[5]{y-2} \\
\sqrt[5]{x-3} &= y-2 \\
\frac{+2}{+2} &\quad \frac{+2}{+2} \\
\sqrt[5]{x-3} + 2 &= y \\
f^{-1}(x) &= \sqrt[5]{x-3} + 2
\end{aligned}$$

$$\begin{aligned}
 13) \quad g(x) &= \frac{4}{x+2} \\
 y &= \frac{4}{x+2} \\
 x &= \frac{4}{y+2} \\
 (y+2)x &= \frac{4}{y+2}(y+2) \\
 \frac{x(y+2)}{x} &= \frac{4}{x} \\
 y+2 &= \frac{4}{x} \\
 \frac{-2}{-2} \quad \frac{-2}{-2} \\
 y &= 4x - 2 \\
 g^{-1}(x) &= \frac{4}{x} - 2
 \end{aligned}$$

$$\begin{aligned}
 15) \quad f(x) &= \frac{-2x-2}{x+2} \\
 y &= \frac{-2x-2}{x+2} \\
 x &= \frac{-2y-2}{y+2} \\
 (y+2)x &= \frac{-2y-2}{y+2}(y+2) \\
 xy + 2x &= -2y - 2 \\
 \frac{+2y - 2x + 2y - 2x}{xy + 2y} &= \frac{-2 - 2x}{-2 - 2x} \\
 \frac{y(x+2)}{x+2} &= \frac{-2-2x}{x+2} \\
 y &= \frac{-2-2x}{x+2} \\
 f^{-1}(x) &= \frac{-2-2x}{x+2}
 \end{aligned}$$

$$\begin{aligned}
 17) \quad f(x) &= \frac{10-x}{5} \\
 y &= \frac{10-x}{5} \\
 5(x) &= \frac{10-y}{5}(5) \\
 5x &= 10 - y \\
 \frac{-10}{-1} \quad \frac{-10}{-1} \\
 \frac{5x}{-1} - \frac{10}{-1} &= \frac{-y}{-1} \\
 -5x + 10 &= y \\
 f^{-1}(x) &= -5x + 10
 \end{aligned}$$

$$\begin{aligned}
 19) \quad g(x) &= -(x-1)^3 \\
 y &= -(x-1)^3 \\
 \frac{x}{-1} &= \frac{-(y-1)^3}{-1} \\
 \sqrt[3]{-x} &= \sqrt[3]{(y-1)^3} \\
 \sqrt[3]{-x} &= y-1 \\
 \frac{+1}{\sqrt[3]{-x} + 1} \quad \frac{+1}{+1} \\
 \sqrt[3]{-x} + 1 &= y \\
 g^{-1}(x) &= \sqrt[3]{-x} + 1
 \end{aligned}$$

$$\begin{aligned}
 21) \quad f(x) &= (x-3)^3 \\
 y &= (x-3)^3 \\
 \sqrt[3]{x} &= \sqrt[3]{(y-3)^3} \\
 \sqrt[3]{x} &= y-3 \\
 \frac{+3}{\sqrt[3]{x} + 3} \quad \frac{+3}{+3} \\
 \sqrt[3]{x} + 3 &= y \\
 f^{-1}(x) &= \sqrt[3]{x} + 3
 \end{aligned}$$

$$\begin{aligned}
 23) \quad g(x) &= \frac{x}{x-1} \\
 y &= \frac{x}{x-1} \\
 (y-1)x &= \frac{y}{y-1}(y-1) \\
 xy - x &= y \\
 \frac{-xy}{-x} \quad \frac{-xy}{-x} \\
 -x &= y - xy \\
 -\frac{x}{1-x} &= \frac{y(1-x)}{1-x} \\
 -\frac{x}{1-x} &= y \\
 g^{-1}(x) &= -\frac{x}{1-x}
 \end{aligned}$$

$$\begin{aligned}
 25) \quad f(x) &= \frac{x-1}{x+1} \\
 y &= \frac{x-1}{x+1} \\
 (y+1)x &= \frac{y-1}{y+1}(y+1) \\
 xy + x &= y-1 \\
 \frac{-y-x}{xy-y} \quad \frac{-y-x}{-y-x} \\
 xy - y &= -1 - x \\
 \frac{y(x-1)}{x-1} &= \frac{-1-x}{x-1} \\
 y &= \frac{-1-x}{x-1} \\
 f^{-1}(x) &= \frac{-1-x}{x-1}
 \end{aligned}$$

$$27) g(x) = \frac{8-5x}{4}$$

$$y = \frac{8-5x}{4}$$

$$4(x) = \frac{8-5y}{4} \quad (4)$$

$$4x = 8 - 5y$$

$$\frac{-8 - 8}{4x-8} = -\frac{5y}{-5}$$

$$\frac{4x-8}{-5} = -\frac{5y}{-5}$$

$$\frac{4x-8}{-5} = y$$

$$g^{-1}(x) = \frac{4x-8}{-5}$$

$$29) g(x) = -5x + 1$$

$$y = -5x + 1$$

$$x = -5y + 1$$

$$\frac{-1 - 1}{x-1} = -\frac{5y}{-5}$$

$$\frac{x-1}{-5} = -\frac{5y}{-5}$$

$$\frac{x-1}{-5} = y$$

$$g^{-1}(x) = \frac{x-1}{-5}$$

$$31) g(x) = -1 + x^3$$

$$y = -1 + x^3$$

$$x = -1 + y^3$$

$$\frac{+1 + 1}{\sqrt[3]{x+1}} = \sqrt[3]{y^3}$$

$$\sqrt[3]{x+1} = \sqrt[3]{y^3}$$

$$y = \sqrt[3]{x+1}$$

$$g^{-1}(x) = \sqrt[3]{x+1}$$

$$33) h(x) = \frac{4-\sqrt[3]{4x}}{2}$$

$$y = \frac{4-\sqrt[3]{4x}}{2}$$

$$(2)x = \frac{4-\sqrt[3]{4y}}{2} \quad (2)$$

$$2x = 4 - \sqrt[3]{4y}$$

$$\frac{-4 - 4}{2x-4} = \frac{-\sqrt[3]{4y}}{-1}$$

$$\frac{2x-4}{-1} = \frac{-\sqrt[3]{4y}}{-1}$$

$$(-2x+4)^3 = (\sqrt[3]{4y})^3$$

$$\frac{(-2x+4)^3}{4} = \frac{4y}{4}$$

$$\frac{(-2x+4)^3}{4} = y$$

$$h^{-1}(x) = \frac{(-2x+4)^3}{4}$$

$$35) f(x) = \frac{x+1}{x+2}$$

$$y = \frac{x+1}{x+2}$$

$$(y+2)x = \frac{y+1}{y+2}(y+2)$$

$$xy + 2x = y + 1$$

$$\frac{-y - 2x}{xy - y} = \frac{-y - 2x}{1 - 2x}$$

$$xy - y = 1 - 2x$$

$$\frac{y(x-1)}{x-1} = \frac{1-2x}{x-1}$$

$$y = \frac{1-2x}{x-1}$$

$$f^{-1}(x) = \frac{1-2x}{x-1}$$

$$37) f(x) = \frac{7-3x}{x-2}$$

$$y = \frac{7-3x}{x-2}$$

$$(y-2)x = \frac{7-3y}{y-2}(y-2)$$

$$xy - 2x = 7 - 3y$$

$$\frac{+3y + 2x}{xy + 3y} = \frac{7 + 2x}{7 + 2x}$$

$$xy + 3y = 7 + 2x$$

$$\frac{y(x+3)}{x+3} = \frac{y+2x}{x+3}$$

$$y = \frac{y+2x}{x+3}$$

$$f^{-1}(x) = \frac{y+2x}{x+3}$$

$$39) g(x) = -x$$

$$y = -x$$

$$\frac{x}{-1} = (-y)/-1$$

$$-x = y$$

$$g^{-1}(x) = -x$$

10.4

$$\begin{aligned}
 1) \quad 3^{1-2n} &= 3^{1-3n} \\
 1 - 2n &= 1 - 3n \\
 \quad \quad \quad +3n &\quad +3n \\
 \hline
 1 + n &= 1 \\
 -1 &\quad -1 \\
 \hline
 n &= 0
 \end{aligned}$$

$$\begin{aligned}
 3) \quad 4^{2a} &= 1 \\
 4^{2a} &= 1^0 \\
 \frac{2a}{a} &= \frac{0}{2} \\
 a &= 0
 \end{aligned}$$

$$\begin{aligned}
 5) \quad \left(\frac{1}{25}\right)^{-k} &= 125^{-2k-2} \\
 (5^{-2})^{-k} &= (5^3)^{-2k-2} \\
 5^{2k} &= 5^{-6k-6} \\
 2k &= -6k - 6 \\
 \quad \quad \quad +6k &\quad +6k \\
 \hline
 \frac{8k}{8} &= -\frac{6}{8} \\
 k &= -\frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 7) \quad 6^{2m+1} &= \frac{1}{36} \\
 6^{2m+1} &= 6^{-2} \\
 2m + 1 &= -2 \\
 \quad \quad \quad -1 &\quad -1 \\
 \hline
 \frac{2m}{2} &= \frac{-3}{2} \\
 m &= -\frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 9) \quad 6^{-3x} &= 36 \\
 6^{-3x} &= 6^2 \\
 -\frac{3x}{-3} &= \frac{2}{-3} \\
 x &= -\frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 11) \quad 64^b &= 2^5 \\
 (2^6)^b &= 2^5 \\
 2^{6b} &= 2^5 \\
 \frac{6b}{6} &= \frac{5}{6} \\
 b &= \frac{5}{6}
 \end{aligned}$$

$$\begin{aligned}
 13) \quad \left(\frac{1}{4}\right)^x &= 16 \\
 (4^{-1})^x &= 4^2 \\
 4^{-x} &= 4^2 \\
 -\frac{x}{-1} &= \frac{2}{-1} \\
 x &= -2
 \end{aligned}$$

$$\begin{aligned}
 15) \quad 4^{3a} &= 4^3 \\
 \frac{3a}{3} &= \frac{3}{3} \\
 a &= 1
 \end{aligned}$$

$$\begin{aligned}
 17) \quad 36^{3x} &= 216^{2x+1} \\
 (6^2)^{3x} &= (6^3)^{2x+1} \\
 6^{6x} &= 6^{6x+1} \\
 6x &= 6x + 1 \\
 \quad \quad \quad -6x &\quad -6x \\
 \hline
 0 &= 1 \\
 &false \\
 &No\ Solution
 \end{aligned}$$

$$\begin{aligned}
 19) \quad 9^{2n+3} &= 243 \\
 (3^2)^{2n+3} &= 3^5 \\
 3^{4n+6} &= 3^5 \\
 4n + 6 &= 5 \\
 \quad \quad \quad -6 &\quad -6 \\
 \hline
 \frac{4n}{4} &= \frac{-1}{4} \\
 n &= -\frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 21) \quad 3^{3x-2} &= 3^{3x+1} \\
 3x - 2 &= 3x + 1 \\
 \quad \quad \quad -3x &\quad -3x \\
 \hline
 -2 &= 1 \\
 &false \\
 &No\ Solution
 \end{aligned}$$

$$\begin{aligned}
 23) \quad 3^{-2x} &= 3^3 \\
 \frac{-2x}{-2} &= \frac{3}{-2} \\
 x &= -\frac{3}{2}
 \end{aligned}$$

$$25) 5^{m+2} + 5^{-m}$$

$$m + 2 = -m$$

$$\frac{-m}{-2} = \frac{-m}{-2}$$

$$\frac{2}{-2} = \frac{-2m}{-2}$$

$$-1 = m$$

$$27) \left(\frac{1}{36}\right)^{b-1} = 216$$

$$(6^{-2})^{b-1} = 6^3$$

$$6^{-2b+2} = 6^3$$

$$-2b + 2 = 3$$

$$\frac{-2}{-2} = \frac{-2}{-2}$$

$$\frac{-2b}{-2} = \frac{1}{-2}$$

$$b = -\frac{1}{2}$$

$$29) r6^{2-2x} = 6^2$$

$$2 - 2x = 2$$

$$\frac{-2}{-2} = \frac{-2}{-2}$$

$$\frac{-2x}{-2} = \frac{0}{-2}$$

$$x = 0$$

$$31) 4 \cdot 2^{(-3n-1)} = \frac{1}{4}$$

$$2^2 \cdot 2^{-3n-1} = 2^{-2}$$

$$2^{-3n+1} = 2^{-2}$$

$$-3n + 1 = -2$$

$$\frac{-1}{-3} = \frac{-1}{-3}$$

$$\frac{-3n}{-3} = \frac{-3}{-3}$$

$$n = 1$$

$$33) 4^{3k-3} \cdot 4^{2-2k} = 16^{-k}$$

$$4^{k-1} = (4^2)^{-k}$$

$$4^{k-1} = 4^{-2k}$$

$$k - 1 = -2k$$

$$\frac{-k}{-3} = \frac{-k}{-3}$$

$$\frac{-1}{-3} = \frac{-3k}{-3}$$

$$\frac{1}{3} = k$$

$$35) 9^{-2x} \left(\frac{1}{243}\right)^{3x} = 243^{-x}$$

$$(3^2)^{-2x} (3^{-5})^{3x} = (3^5)^{-x}$$

$$3^{-4x} \cdot 3^{-15x} = 3^{-5x}$$

$$3^{-19x} = 3^{-5x}$$

$$-19x = -5x$$

$$\frac{+19x}{14} = \frac{+19x}{14}$$

$$\frac{0}{14} = \frac{14x}{14}$$

$$0 = x$$

$$37) 64^{n-2} \cdot 16^{n+2} = \left(\frac{1}{4}\right)^{3n-1}$$

$$(4^3)^{n-2} \cdot (4^2)^{n+2} = (4^{-1})^{3n-1}$$

$$4^{3n-6} \cdot 4^{2n+4} = 4^{-3n+1}$$

$$4^{5n-2} = 4^{-3n+1}$$

$$5n - 2 = -3n + 1$$

$$\frac{+3n}{8n-2} = \frac{+3n}{8n-2}$$

$$8n - 2 = 1$$

$$\frac{+2}{8n} = \frac{+2}{8n}$$

$$\frac{8n}{8} = \frac{3}{8}$$

$$n = \frac{3}{8}$$

$$39) 5^{-3n-3} \cdot 5^{2n} = 1$$

$$5^{-n-3} = 5^0$$

$$-n - 3 = 0$$

$$\frac{+3}{-1} = \frac{+3}{-1}$$

$$\frac{-n}{-1} = \frac{3}{-1}$$

$$n = -3$$

10.5

$$1) \log_9 81 = 2$$

$$9^2 = 81$$

$$3) \log_7 \frac{1}{49} = -2$$

$$7^{-2} = \frac{1}{49}$$

$$5) \log_{13} 169 = 2$$

$$13^2 = 169$$

$$7) 8^0 = 1$$

$$\log_8 1 = 0$$

$$9) 15^2 = 225$$

$$\log_{15} 225 = 2$$

$$11) 64^{\frac{1}{6}} = 2$$

$$\log_{64} 2 = \frac{1}{6}$$

$$13) \log_{125} 5 = x$$

$$125^x = 5$$

$$(5^3)^x = 5^1$$

$$5^{3x} = 5^1$$

$$\frac{3x}{3} = \frac{1}{3}$$

$$x = \frac{1}{3}$$

$$15) \log_{343} \frac{1}{7} = x$$

$$343^x = \frac{1}{7}$$

$$(7^3)^x = 7^{-1}$$

$$7^{3x} = 7^{-1}$$

$$\frac{3x}{3} = \frac{-1}{3}$$

$$x = -\frac{1}{3}$$

$$17) \log_4 16 = x$$

$$4^x = 16$$

$$4^x = 4^2$$

$$x = 2$$

$$19) \log_6 36 = x$$

$$6^x = 36$$

$$6^x = 6^2$$

$$x = 2$$

$$21) \log_2 64 = x$$

$$2^x = 64$$

$$2^x = 2^6$$

$$x = 6$$

$$23) \log_5 x = 1$$

$$5^1 = x$$

$$5 = x$$

$$25) \log_2 x = -2$$

$$2^{-2} = x$$

$$\frac{1}{2^2} = x$$

$$\frac{1}{4} = x$$

$$27) \log_{11} k = 2$$

$$11^2 = k$$

$$121 = k$$

$$29) \log_9(n+9) = 4$$

$$9^4 = n+9$$

$$6561 = n+9$$

$$\frac{-9}{-9} \quad \frac{-9}{-9}$$

$$6552 = n$$

$$31) \log_5(-3m) = 3$$

$$5^3 = -3m$$

$$\frac{125}{-3} = \frac{-3m}{-3}$$

$$-\frac{125}{3} = m$$

$$33) \log_{11}(x+5) = -1$$

$$11^{-1} = x+5$$

$$\frac{1}{11} = x+5$$

$$\frac{-5}{-5} \quad \frac{-5}{-5}$$

$$-\frac{54}{11} = x$$

$$35) \log_4(6b+4) = 0$$

$$4^0 = 6b+4$$

$$1 = 6b+4$$

$$\frac{-4}{-4} \quad \frac{-4}{-4}$$

$$\frac{-3}{6} = \frac{6b}{6}$$

$$-\frac{1}{2} = b$$

$$37) \log_5(-10x+4) = 4$$

$$5^4 = -10x+4$$

$$625 = -10x+4$$

$$\frac{-4}{-4} \quad \frac{-4}{-4}$$

$$\frac{621}{-10} = \frac{-10x}{-10}$$

$$-\frac{621}{10} = x$$

$$39) \log_2(10-5a) = 3$$

$$2^3 = 10-5a$$

$$8 = 10-5a$$

$$\frac{-10}{-10} \quad \frac{-10}{-10}$$

$$\frac{-2}{-5} = \frac{-5a}{-5}$$

$$\frac{2}{5} = a$$

10.6

1) Find each of the following:

a. \$500 invested at 4% compounded annually for 10 years.

$$A = 500 \left(1 + \frac{.04}{1}\right)^{1 \cdot 10}$$

$$A = 500(1.04)^{10}$$

$$A = 500(1.48)$$

$$A = \$740.12$$

b. \$600 invested at 6% compounded annually for 6 years.

$$A = 600 \left(1 + \frac{.06}{1}\right)^{1 \cdot 6}$$

$$A = 600(1.06)^6$$

$$A = 600(1.42)$$

$$A = \$851.11$$

c. \$750 invested at 3% compounded annually for 8 years.

$$A = 750 \left(1 + \frac{.03}{1}\right)^{1 \cdot 8}$$

$$A = 750(1.03)^8$$

$$A = 750(1.27)$$

$$A = \$950.08$$

d. \$1500 invested at 4% compounded semiannually for 7 years.

$$A = 1500 \left(1 + \frac{.04}{2}\right)^{2 \cdot 7}$$

$$A = 1500(1.02)^{14}$$

$$A = 1500(1.32)$$

$$A = \$1979.22$$

e. \$900 invested at 6% compounded semiannually for 5 years.

$$A = 900 \left(1 + \frac{.06}{2}\right)^{2 \cdot 5}$$

$$A = 900(1.03)^{10}$$

$$A = 900(1.34)$$

$$A = \$1209.52$$

f. \$950 invested at 4% compounded semiannually for 12 years.

$$A = 950 \left(1 + \frac{.04}{2}\right)^{(2 \cdot 12)}$$

$$A = 950(1.02)^{24}$$

$$A = 950(1.61)$$

$$A = \$1528.02$$

- g. \$2000 invested at 5% compounded quarterly for 6 years.

$$A = 200 \left(1 + \frac{.05}{2}\right)^{4 \cdot 6}$$

$$A = 200(1.0125)^{24}$$

$$A = 200(1.35)$$

$$A = \$2694.70$$

- h. \$2250 invested at 4% compounded quarterly for 9 years.

$$A = 2250 \left(1 + \frac{.04}{4}\right)^{4 \cdot 9}$$

$$A = 2250(1.01)^{36}$$

$$A = 2250(1.43)$$

$$A = \$3219.23$$

- i. \$3500 invested at 6% compounded quarterly for 12 years.

$$A = 3500 \left(1 + \frac{.06}{4}\right)^{4 \cdot 12}$$

$$A = 3500(1.015)^{48}$$

$$A = 3500(2.04)$$

$$A = \$7,152.17$$

- j. All of the above compounded continuously.

$$A = 500e^{.04 \cdot 10}$$

$$A = 500e^{.4}$$

$$A = 500(1.49)$$

$$A = \$745.91$$

$$A = 600e^{.06 \cdot 6}$$

$$A = 600e^{.36}$$

$$A = 600(1.43)$$

$$A = \$860.00$$

$$A = 750e^{.03 \cdot 8}$$

$$A = 750e^{.24}$$

$$A = 750(1.27)$$

$$A = \$953.44$$

$$A = 1500e^{.04 \cdot 7}$$

$$A = 1500e^{.28}$$

$$A = 1500(1.32)$$

$$A = \$1984.69$$

$$A = 900e^{.06 \cdot 5}$$

$$A = 900e^{.30}$$

$$A = 900(1.35)$$

$$A = \$1214.87$$

$$A = 950e^{.04 \cdot 12}$$

$$A = 950e^{.48}$$

$$A = 950(1.62)$$

$$A = \$1535.27$$

$$A = 2000e^{.05 \cdot 6}$$

$$A = 2000e^{.3}$$

$$A = 2000(1.35)$$

$$A = \$2699.72$$

$$A = 2250e^{.04 \cdot 9}$$

$$A = 2250e^{.36}$$

$$A = 2250(1.43)$$

$$A = \$3224.99$$

$$A = 3500e^{.06 \cdot 12}$$

$$A = 3500e^{.72}$$

$$A = 3500(2.05)$$

$$A = \$7190.52$$

- 3) What principal will amount to S3500 if invested at 4% interest compounded quarterly for 5 years?

$$3500 = P \left(1 + \frac{.04}{4} \right)^{4 \cdot 5}$$

$$3500 = P(1.01)^{20}$$

$$\frac{3500}{1.22} = \frac{P(1.22)}{1.22}$$

$$\$2868.41 = P$$

- 5) What principal will amount to S2500 if invested at 5% interest compounded semiannually for 7.5 years?

$$2500 = P \left(1 + \frac{.05}{2} \right)^{2 \cdot 7.5}$$

$$2500 = P(1.025)^{15}$$

$$\frac{2500}{1.45} = \frac{P(1.45)}{1.45}$$

$$\$1726.16 = P$$

- 7) A thousand dollars is left in a bank savings account drawing 7% interest, compounded quarterly for 10 years. What is the balance at the end of that time?

$$A = 1000 \left(1 + \frac{.07}{4}\right)^{4 \cdot 10}$$

$$A = 100(1.0175)^{40}$$

$$A = 1000(2.00)$$

$$A = \$2001.60$$

- 9) \$1750 is invested in an account earning 13.5% interest compounded monthly for a 2 year period. What is the balance at the end of 9 years?

$$A = 1750 \left(1 + \frac{.135}{12}\right)^{12 \cdot 2}$$

$$A = 1750(1.01125)^{24}$$

$$A = 1750(1.31)$$

$$A = \$2288.98$$

- 11) A \$10,000 Treasury Bill earned 16% compounded monthly. If the bill matured in 2 years, what was it worth at maturity?

$$A = 10,000 \left(1 + \frac{.16}{12}\right)^{12 \cdot 2}$$

$$A = 10,000(1.01)^{24}$$

$$A = 10,000(1.37)$$

$$A = \$13,742.19$$

- 13) A savings institution advertises 7% annual interest, compounded daily, How much more interest would you earn over the bank savings account or credit union in problems 7 and 8?

$$A = 1000 \left(1 + \frac{.07}{365}\right)^{365 \cdot 10}$$

$$A = 1000(1.00019)^{3650}$$

$$A = 1000(2.01)$$

$$A = \$2013.62$$

$$\#7: 2001.60 - 2013.62 = \$12.02$$

$$\#8: 2009.66 - 2013.62 = \$ - 3,96$$

- 15) You lend \$100 at 10% continuous interest. If you are repaid 2 months later, what is owed?

$$A = 100e^{1\left(\frac{1}{6}\right)}$$

$$A = 100e^{.0167}$$

$$A = 100(1.02)$$

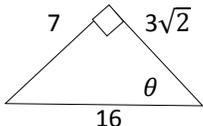
$$A = \$101.68$$

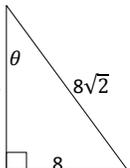
10. 7

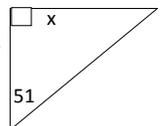
1) $\cos 71 = 0.3256$

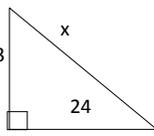
3) $\sin 75 = 0.9659$

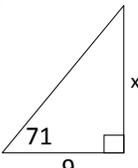
5)  $\sin \theta = \frac{7}{25}$

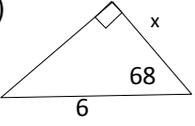
7)  $\sin \theta = \frac{7}{16}$

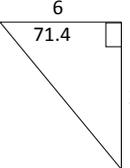
9)  $\sin \theta = \frac{8}{8\sqrt{2}} \left(\frac{\sqrt{2}}{\sqrt{2}} \right)$
 $\sin \theta = \left(\frac{8\sqrt{2}}{8 \cdot 2} \right)$
 $\sin \theta = \frac{\sqrt{2}}{2}$

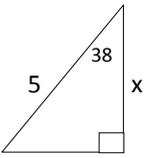
11)  (13) $\tan 51 = \frac{x}{13}$ (13)
 $13 \tan 51 = x$
 $13(1.235) = x$
 $16.05 = x$

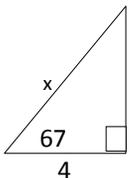
13)  (x) $\sin 24 = \frac{13}{x}$ (x)
 $\frac{x \sin 24}{\sin 24} = \frac{13}{\sin 24}$
 $x = \frac{13}{\sin 24}$
 $x = \frac{13}{0.407}$
 $x = 31.96$

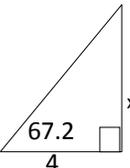
15)  (9) $\tan 71 = \frac{x}{9}$ (9)
 $9 \tan 71 = x$
 $9(2.904) = x$
 $26.14 = x$

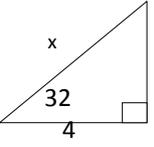
17)  (6) $\cos 68 = \frac{x}{68}$ (6)
 $6 \cos 68 = x$
 $6(.3746) = x$
 $2.25 = x$

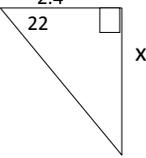
19)  (6) $\tan 71.4 = \frac{x}{6}$ (6)
 $6 \tan 71.4 = x$
 $6(2.9714) = x$
 $17.83 = x$

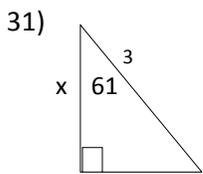
21)  (5) $\cos 38 = \frac{x}{38}$ (5)
 $5 \cos 38 = x$
 $5(.7880) = x$
 $3.94 = x$

23)  (x) $\cos 67 = \frac{4}{x}$ (x)
 $\frac{x \cos 67}{x \cos 67} = \frac{4}{\cos 67}$
 $x = \frac{4}{\cos 67}$
 $x = \frac{4}{.3907}$
 $x = 10.24$

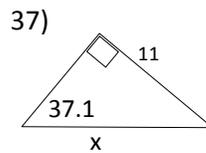
25)  (4) $\tan 67.2 = \frac{x}{4}$ (4)
 $4 \tan 67.2 = x$
 $4(2.3789) = x$
 $9.52 = x$

27)  (x) $\cos 32 = \frac{4}{x}$ (x)
 $\frac{x \cos 32}{\cos 32} = \frac{4}{\cos 32}$
 $x = \frac{4}{\cos 32}$
 $x = \frac{4}{0.8480}$
 $x = 4.72$

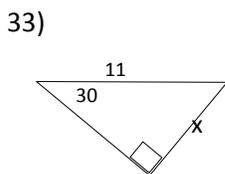
29)  (2.4) $\tan 22 = \frac{x}{2.4}$ (2.4)
 $2.4 \tan 22 = x$
 $2.4(0.4040) = x$
 $0.97 = x$



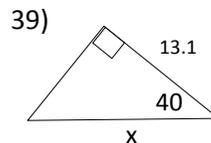
(3) $\cos 61 = \frac{x}{3}$ (3)
 $3 \cos 61 = x$
 $3(.4848) = x$
 $1.45 = x$



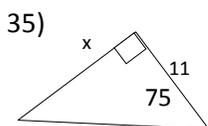
(x) $\sin 37.1 = \frac{11}{x}$ (x)
 $\frac{x \sin 37.1}{\sin 37.1} = \frac{11}{\sin 37.1}$
 $x = \frac{11}{\sin 37.1}$
 $x = \frac{11}{.6032}$
 $x = 18.24$



(11) $\sin 30 = \frac{x}{11}$ (11)
 $11 \sin 30 = x$
 $11(0.5) = x$
 $5.5 = x$



(x) $\cos 40 = \frac{13.1}{x}$ (x)
 $\frac{x \cos 40}{\cos 40} = \frac{13.1}{\cos 40}$
 $x = \frac{13.1}{\cos 40}$
 $x = \frac{13.1}{.7660}$
 $x = 17.1$

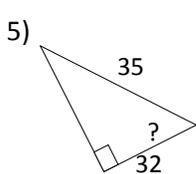


(11) $\tan 95 = \frac{x}{11}$ (11)
 $11 \tan 95 = x$
 $11(3.7321) = x$
 $41.05 = x$

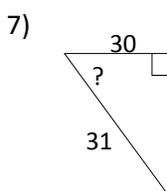
10.8

1) $\sin z = 0.4848$
 $z = 29^\circ$

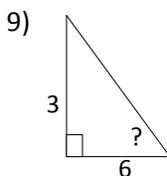
3) $\sin y = 0.6561$
 $y = 41^\circ$



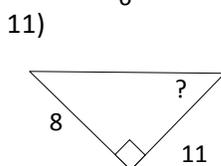
$\cos x = \frac{32}{35}$
 $\cos x = .9143$
 $x = 24^\circ$



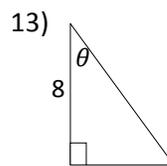
$\cos x = \frac{30}{31}$
 $\cos x = .9677$
 $x = 15^\circ$



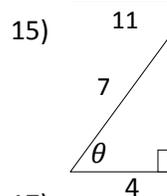
$\tan x = \frac{3}{6}$
 $\tan x = .5$
 $x = 27^\circ$



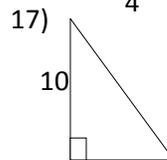
$\tan \theta = \frac{11}{8}$
 $\tan \theta = .7273$
 $\theta = 36^\circ$



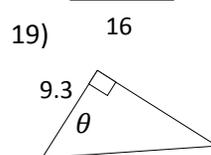
$\cos \theta = \frac{4}{7}$
 $\cos \theta = 0.5714$
 $\theta = 55.2^\circ$



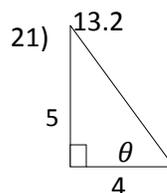
$\cos \theta = \frac{4}{7}$
 $\cos \theta = 0.5714$
 $\cos \theta = 55.2^\circ$



$\tan \theta = \frac{16}{10}$
 $\tan \theta = 1.6$
 $\tan \theta = 58^\circ$

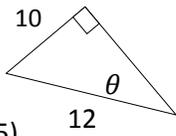


$\cos \theta = \frac{9.3}{13.2}$
 $\cos \theta = 0.7045$
 $\cos \theta = 45.2^\circ$



$\tan \theta = \frac{5}{4}$
 $\tan \theta = 1.25$
 $\tan \theta = 51.3^\circ$

23)

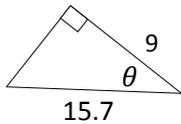


$$\sin \theta = \frac{10}{12}$$

$$\sin \theta = 0.8333$$

$$\sin \theta = 56.4^\circ$$

25)

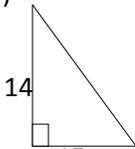


$$\cos \theta = \frac{9}{15.7}$$

$$\cos \theta = 0.5732$$

$$\cos \theta = 55^\circ$$

27)

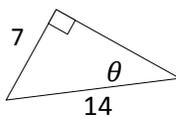


$$\tan \theta = \frac{15}{14}$$

$$\tan \theta = 1.0714$$

$$\tan \theta = 47^\circ$$

29)

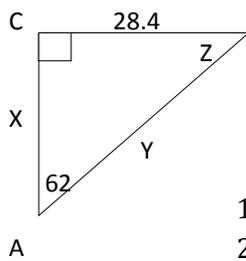


$$\sin \theta = \frac{7}{14}$$

$$\sin \theta = 0.5$$

$$\sin \theta = 30^\circ$$

31)



$$(x) \tan 62 = \frac{28.4}{x} (x)$$

$$\frac{x \tan 62}{\tan 62} = \frac{28.4}{\tan 62}$$

$$x = \frac{28.4}{\tan 62}$$

$$x = \frac{28.4}{1.8507}$$

$$x = 15.1$$

$$15.1^2 + 28.4^2 = y^2$$

$$228.01 + 806.56 = y^2$$

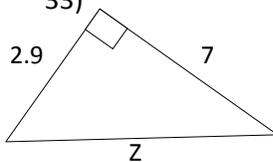
$$\sqrt{1034.57} = \sqrt{y^2}$$

$$32.2 = y$$

$$z = 90 - 62$$

$$z = 28^\circ$$

33)



$$\tan x = \frac{2.9}{7}$$

$$\tan x = .4143$$

$$x = 22.5^\circ$$

$$y = 90 - 22.5$$

$$y = 67.5^\circ$$

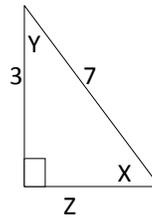
$$2.9^2 + 7^2 = z^2$$

$$8.41 + 49 = z^2$$

$$\sqrt{57.41} = \sqrt{z^2}$$

$$7.6 = z$$

35)



$$\sin x = \frac{3}{7}$$

$$\sin x = .4286$$

$$x = 25.4^\circ$$

$$y = 90 - 25.4$$

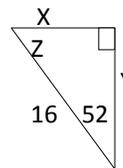
$$y = 64.6^\circ$$

$$3^2 + z^2 = 7^2$$

$$9 + z^2 = 49$$

$$\begin{array}{r} -9 \quad -9 \\ \hline \sqrt{z^2} = \sqrt{40} \\ z = 6.3 \end{array}$$

37)



$$(16) \sin 52 = \frac{x}{16} (16)$$

$$16 \sin 52 = x$$

$$16(.7880) = x$$

$$12.6 = x$$

$$12.6^2 + y^2 = 16^2$$

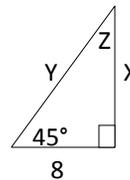
$$158.76 + y^2 = 256$$

$$\begin{array}{r} -158.76 \quad -158.76 \\ \hline \sqrt{y^2} = \sqrt{97.24} \\ y = 9.9 \end{array}$$

$$z = 90 - 52$$

$$z = 38^\circ$$

39)



$$(8) \tan 45 = \frac{x}{8} (8)$$

$$8 \tan 45 = x$$

$$8(1) = x$$

$$8 = x$$

$$8^2 + 8^2 = y^2$$

$$64 + 64 = y^2$$

$$\sqrt{128} = \sqrt{y^2}$$

$$11.3 = y$$

$$z = 90 - 45$$

$$z = 45^\circ$$

