1. 

a) Draw the angle $-\frac{11 \pi}{6}$ in standard position.

b) determine an angle that is co-terminal to $-\frac{11 \pi}{6}$

$$
\frac{\pi}{6}>-\frac{23 \pi}{6}
$$

c) Determine the reference angle of $-\frac{11 \pi}{6}$

$$
\frac{\pi}{6}
$$

2. Which pair of angles are co-terminal with $\frac{2 \pi}{3}$ ?
a. $-\frac{4 \pi}{3}, \frac{8 \pi}{3}$
b. $\frac{4 \pi}{3}, \frac{8 \pi}{3}$
c. $-\frac{4 \pi}{3},-\frac{8 \pi}{3}$
$a$
d. $-\frac{\pi}{3}, \frac{\pi}{3}$
3. During a routine, a figure skater completes $-13 \frac{11}{18}$ rotations. How many degrees has the figure skater turned?
a. $4900^{\circ}$
b. $-4900^{\circ}$
c. $6900^{\circ}$
d. $-6900^{\circ}$
4. If the angle $p$ is $-4000^{\circ}$ in standard position, it can be described as having made
a. $22 \frac{2}{9}$ rotations
b. $-22 \frac{2}{9}$ rotations
c. $11 \frac{1}{9}$ rotations
d. $-11 \frac{1}{9}$ rotations
5. $\frac{5 \pi}{3}$ radians is equal to how many degrees?
a. $300^{\circ}$
b. $600^{\circ}$
c. $60^{\circ}$
d. $120^{\circ}$
6. A grandfather clock shows a time of $8 o^{\prime}$ clock. What is the exact radian measure of the angle between the hour hand and the minute hand?
$2 \pi / 3$
7. $\frac{3 \pi}{4}$ radians is equal to how many degrees?
a. $135^{\circ}$
b. $315^{\circ}$
c. $45^{\circ}$

## $a$

d. $225^{\circ}$
8. Which of the following angles, in degrees, is co terminal with, but not equal to $\frac{3 \pi}{2}$ radians?
a. $270^{\circ}$
b. $450^{\circ}$
c. $630^{\circ}$

C
d. $540^{\circ}$
9. An angle of $240^{\circ}$ expressed in radians is:
a. $\frac{4 \pi}{3}$
b. $\frac{\pi}{3}$
c. $\frac{5 \pi}{4}$
d. $\frac{7 \pi}{6}$
10. Covert 3.8 radians to the nearest degree

$$
218^{\circ}
$$

11. Calculate the arc length that subtends an angle of 1.64 radians at the center of a circle with radius 20 m

## 32.4 m

12. Determine the arc length of a circle with radius 5.1 cm if it is subtended by a central angle of $\pi$ radians. Round your answer to one decimal place.
a. 14.2 cm
b. 3.1 cm
c. 30.2 cm
d. 16.02 cm
13. The radian measure of an angle is $\frac{2 \pi}{5}$. The arc that subtends the angle has a length of $2 \pi$. Determine the radius the of circle.

14. A bicycle has a front wheel that is 36 cm in diameter and a rear wheel that is 18 cm in diameter. If the front wheel rotates through an angle of $120^{\circ}$, through how many degrees does the rear wheel rotate?
a. $120^{\circ}$
b. $60^{\circ}$
c. $360^{\circ}$
d. $240^{\circ}$
15. A child swings on a playground swing set. If the length of the swing's chain is 4 m and the child swings through an angle of $\pi / 16$, what is the exact arc length through which the child travels?

16. Identify a measure for the central angle p in the interval $0 \leq \theta \leq 2 \pi$ such that the point $(0,-1)$ is on the terminal arm.
a. 0
b. $\frac{\pi}{2}$
C. $\pi$
d. $\frac{3 \pi}{2}$
17. The angle $\theta$ lies in Quadrant $I I$ with point $Q$ on the terminal arm and $\tan \theta=-\frac{2}{5}$

Determine the exact value of $\sin \theta$.

18. Angle $\theta$ lies in Quadrant $I I I$ with point $\mathrm{A}(-4,-6)$ on the terminal arm. Point p is the point of intersection of the terminal arm of $\theta$ and the unit circle centered at $(0,0)$. Determine the exact value of the $x$-coordinate of point $P$.

19. If $\cos \theta=\frac{5}{13}$ and angle $\theta$ is in Quadrant $I V$. Determine the possible coordinates for point P on the terminal arm of angle $\theta$.

$$
P(5,-12)
$$

20. Determine the exact value of the following.
a) $\sin \frac{-11 \pi}{6}$
e) $\cot \frac{\pi}{6}$

$$
\frac{1}{2}
$$

$$
\sqrt{3}
$$

b) $\cos \frac{-3 \pi}{4}$
f) $\sec \frac{\pi}{4}$

$$
-\frac{\sqrt{2}}{2}
$$

$$
\sqrt{2}
$$

c) $\tan \frac{2 \pi}{3}$
g) $\csc \pi$

$$
-\sqrt{3}
$$

undefined
d) $\cos \frac{7 \pi}{4}$
h) $\tan \left(-\frac{5 \pi}{3}\right)$

$$
\frac{\sqrt{2}}{2}
$$

$$
\sqrt{3}
$$

i)

$$
\left[\cos \left(\frac{5 \pi}{6}\right)\right]^{2}-\left[\sin \left(\frac{5 \pi}{6}\right)\right]^{2}=\frac{1}{2}
$$

21. The point $(-2,5)$ is located on the terminal arm of $\angle \mathrm{A}$ in standard position.
a. Determine the primary trigonometric ratios for $\angle \mathrm{A}$.

$$
\begin{array}{ll}
\sin A=\frac{5}{\sqrt{29}} & \operatorname{TAN} A=-\frac{5}{2} \\
\cos A=-\frac{2}{\sqrt{29}} &
\end{array}
$$

b. Determine the primary trigonometric ratios for $\angle \mathrm{B}$ with the same sine as $\angle \mathrm{A}$, but different signs for the other two primary trigonometric ratios.

$$
\begin{aligned}
& \sin B=\frac{5}{\sqrt{29}} \\
& \cos B=\frac{2}{\sqrt{29}} \\
& \operatorname{Tan} B=\frac{5}{2}
\end{aligned}
$$

c. Use a calculator to determine the measures of $\angle \mathrm{A}$ and $\angle \mathrm{B}$, to the nearest degree.

$$
\begin{aligned}
& \angle B=68^{\circ} \\
& \angle A=112^{\circ}
\end{aligned}
$$

22. Determine the period of each function
a) $y=\tan \frac{2}{3} x$

$$
\frac{3 \pi}{2}
$$

b) $y=\sin 4 x$

$$
\frac{\pi}{2}
$$

c) $y=5 \cos (t x)$

$$
\frac{2 \pi}{\pi}
$$

23. The number of solutions to the equation $\cos 5 x=-\frac{\sqrt{3}}{2}$, for $0 \leq x \leq 2 \pi$ is:

10
24. The number of solutions to the equation $\tan \frac{1}{2} x=\frac{\sqrt{3}}{1}$, for $0 \leq x \leq 2 \pi$ is:

1
25. The number of solutions to the equation $\sin 2 x=\frac{\sqrt{2}}{2}$, for $0 \leq x \leq 2 \pi$ is:

26. Determine the amplitude, period, phase shift and vertical displacement of the following functions:

$$
\begin{aligned}
& y=-5 \sin \left(4 x+\frac{5}{4}\right)+10 \\
& a_{n p}=5 \\
& \text { Period }=\frac{\pi}{2} \\
& \text { PiS }=\text { Left } \frac{5}{16} \\
& \text { VD. }=10 \mathrm{up} \\
& y=12 \cos (5 x+10)-2 \\
& a_{m p}=12 \\
& \text { Period }=\frac{2 \pi}{5} \\
& \text { PiS. }=\text { Left } 2 \\
& \text { VD. }=2 \text { down } \\
& y=100 \tan (30 x+1)+2 \\
& \text { Amp }=\text { none } \\
& \text { Period }=\frac{\pi}{30} \\
& \text { PiS. }=\text { Left } \frac{1}{30} \\
& V \cdot P=2 u p
\end{aligned}
$$

27. A cosine function has a maximum value of 10 and a minimum value of -2 , a period of 7 , and a phase shift of 12 . Write an equation representing this cosine function.

$$
y=6 \cos \left[\frac{2 \pi}{7}(x-12)\right]+4
$$

28. Given an equation for a transformed cosine function with an amplitude of $\frac{2}{7}$, a period of 3 , a phase shift of $\frac{3}{2}$ rad to the left, and a vertical translation of 3 units up.
a. $\frac{2}{7} \cos \left(3 x+\frac{3}{2}\right)+3$
b. $\frac{2}{7} \cos \left(3 x+\frac{9}{2}\right)+3$
c. $\frac{2}{7} \cos \left(\frac{2 \pi}{3} x+\frac{3}{2}\right)+3$
d. $\frac{2}{7} \cos \left(\frac{2 \pi}{3} x+\pi\right)+3$
29. The period of the graph $y=\cos x$ is
a. $360^{\circ}$
b. $180^{\circ}$
c. $720^{\circ}$
d. $0^{\circ}$
30. The period of the graph $y=\tan x$ is
a. $360^{\circ}$
b. $180^{\circ}$
c. $720^{\circ}$
d. $0^{\circ}$
31.What is the period of the sinusoidal function $y=-4 \sin (-2(x-\pi))+5$
a. $\frac{\pi}{2}$
b. $-\pi$
c. $2 \pi$
d. $\pi$
31. Determine the period of the sinusoidal graph below.

a. $2 \pi$
b. $\pi$

C. $\frac{1}{2}$
d. $\frac{3 \pi}{2}$
32. The graph of a periodic function is shown below. Determine the amplitude.

a. 4
b. 3
c. 2

d. 1
33. Which of the following is an equation for the graph shown?

a. $y=\sin \frac{1}{2} x$
b. $y=\sin \frac{1}{3} x$
c. $y=\sin \frac{2 \pi}{3} x$
d. $y=\sin 3 x$
34. Which of the following is an equation for cosine function graphed below?

a. $y=7 \cos \frac{3}{2}\left(x-\frac{\pi}{3}\right)-2$
b. $y=7 \cos \frac{3}{2}\left(x+\frac{\pi}{3}\right)-2$
c. $y=3 \cos \frac{4}{3}\left(x+\frac{\pi}{3}\right)-2$
d. $y=3 \cos \frac{4}{3}\left(x-\frac{\pi}{3}\right)-2$
35. For the graph below, write the equation in form $y=a \cos \frac{2 \pi}{12} x+b$


$$
y=25 \cos \left[\frac{2 \pi}{12}(x)\right]+20
$$

37. Which two functions below have the same period?
a. $y=2 \sin \left(3 x-\frac{\pi}{6}\right)$
b. $y=-5 \cos \left(3 x-\frac{\pi}{12}\right)$
$a$ and $b$
c. $y=3 \sin \frac{x}{6}$
d. $y=2 \cos \left(x-\frac{x}{12}\right)$
38. The range of the function defined by $y=3 \sin \left(x-\frac{\pi}{6}\right)-4$ is :
a. $1 \leq y \leq 7$
b. $-7 \leq y \leq-1$
c. $-3 \leq y \leq 3$
d. $-2 \pi \leq y \leq 2 \pi$
39. Determine the range of the function $y=\operatorname{acos}\left(x-\frac{\pi}{2}\right)+b$, given a is less than zero.
a. $-a-b \leq y \leq a-b$
b. $-a \leq y \leq a$
c. $-b a \leq y \leq b a$
d. $a+b \leq y \leq-a+b$
40. Graph two periods of $y=3 \sin \left(x-\frac{\pi}{2}\right)-2$

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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|  |  |  |  |  |  |  |  |  |  |  | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |
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|  |  |  |  |  |  |  |  | - | - |  |  |  |  | 7 | $\pi$ | $\pi$ | $\bigcirc$ |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | $\cdots$ |  |  | , |  |  |  | f |  |  |  |  |  |  |  |  |  |  |
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41. Graph two periods of $y=-3 \cos 2\left(x-\frac{\pi}{6}\right)+1$

42. Graph two periods of $y=\tan \frac{1}{2} x$

43. Sketch the graph of the following for two cycles, where angles are in radians

$$
y=\frac{1}{2} \sin [\pi(x-1)]+3
$$


44. The time of sunrise, $t$ hours on a given day of the year $d$ is given by:

$$
t=2.65 \cos 2 \pi \frac{(d+11)}{365}+6.15
$$

What time does the sunrise on May 23rd, the 143rd day of a non-leap year.
a. 3:49am
b. 5:41am
c. 7:32 am

## $a$

d. 9:13am
45. The voltage, $v$, in an electric circuit is measured in millivolts and is given by the formula $v=0.2 \sin 0.1 \pi(T-0.5)+0.3$, where $T$ is the time in seconds from the start of an experiment. Use the graph of the function to estimate how many seconds in the 40 second interval starting at $\mathrm{T}=0$ during which the voltage is below 0.21 mV
a. 7.03 s
b. 14.06 s

c. 12.97 s
d. 27.16 s
46. One of the largest Ferris wheels ever built is the British Airways London Eye which was completed in year 2000. The diameter is 135 m and passengers get on at the bottom 4 m above the ground. The wheel rotates once every 3 minutes.
a. Draw a graph which represents the height of a passenger in meters as a function of time in minutes.

b. Determine the equation that express your height $h$ as a function of elapsed time $t$.

$$
h(t)=-67.5 \cos \left[\frac{2 \pi}{3}(t)\right]+71.5
$$

c. How high is a passenger 5 minutes after the wheel starts rotating?

$$
105.25 \mathrm{~m}
$$

d. How many seconds after the wheel starts rotating is a passenger 90 m above the ground for the first time? Answer to the nearest tenth.

$$
53 \mathrm{sec}
$$

47. At a certain ocean bay, the maximum height of the water is 2 m above mean sea level at 7:00 a.m. The height is at a maximum again at 7:24 p.m. Assuming that the relationship between the height, $h$, in metres, and the time, $t$, in hours, is sinusoidal, determine the height of the water above mean sea level, to the nearest tenth of a metre, at 11:00 a.m.
$-0.9 m$
