
Solvency II's Market Risk Standard Formula: How Credible Is the Proclaimed Ruin Probability?

Alexander Braun, Hato Schmeiser, and Florian Schreiber¹

Abstract: We assess the credibility of the ruin probability allegedly associated with the market risk standard formula of Solvency II, the new regulatory framework for the European insurance industry. For this purpose, we draw on the empirical risk-return profiles of six major asset classes and derive mean-variance efficient portfolio compositions, taking into account both short-sale constraints and the prevailing legal investment limits in Germany. In a next step, the capital requirements under the standard formula are calculated for each asset allocation. Employing the respective results, we then invert an internal model for market risk based on full statistical distributions instead of mere stress factors to estimate the actual ruin probabilities corresponding to the efficient portfolios. In most cases, the latter deviate substantially from the proclaimed target of the regulator. Since a large fraction of small to medium-sized companies is likely to resort to the standard formula, the introduction of Solvency II could lead to a lot more ambiguity about insolvency risk in the European insurance sector than currently expected. [Key words: Solvency II, standard formula, portfolio optimization, ruin probability.] JEL classifications: G11, G22, G24, G28, G32, G33.

INTRODUCTION

According to the latest announcements of the European Insurance and Occupational Pensions Authority (EIOPA), the new regulatory regime for insurance companies, Solvency II, will come into force at the beginning of 2016. It replaces its predecessor Solvency I and contains both quantitative and qualitative requirements, which are divided into three pillars. The main goal of the first pillar is to introduce capital requirements for several risk categories. In order to calculate these charges, the regulator

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provides a standard formula that is separated into distinct submodules, each of which has been calibrated to a target safety level of 99.5 percent per year, thus implying a default or ruin probability of 0.5 percent. Apart from the standard formula, the solvency capital requirements may be calculated based on internal models that have been preapproved by the regulator. However, in order to develop and maintain such a proprietary model, insurance companies need a critical degree of resources and risk management know-how. Thus, it is likely that many small to medium-sized insurers will resort to the standard formula.

There is little research on the accuracy of the standard formula. In an article by Sandstroem (2007), it is shown that the capital charges of the Solvency II submodules have to be corrected if the underlying probability distributions are skewed. Otherwise, the model is no longer consistent. A similar result is found by Pfeifer and Strassburger (2008), who analyze the stability of the standard formula in detail. Dhaene et al. (2008) consider the suitability of different risk measures for the calculation of the solvency capital requirements of financial institutions. They demonstrate that the subadditivity property which is often demanded can lead to undesirable outcomes in the context of mergers. Finally, Fuchs et al. (2012) mathematically derive a condition that has to be satisfied by the joint distribution of an insurer's risks such that the aggregation under the Solvency II standard formula is accurate.

Our analysis is an extension of the work by Braun et al. (2015). More specifically, we draw on the same time series data and set-up for the internal model. Based on the empirical risk-return profiles of the main asset classes held by European insurance companies, we derive mean-variance efficient portfolios, taking into account both short-sale constraints and the prevailing legal investment limits in Germany. Subsequently, the capital requirements under the Solvency II standard formula are calculated for each asset allocation. Employing the respective results, we then invert the internal model to estimate the actual ruin probabilities associated with the efficient portfolios when the insurer relies on the standard formula. The insights of this study should provide an additional impulse for the ongoing discussion of the new standards. Since the introduction of Solvency II is at hand, this research can be considered highly topical.

The remainder of the article is organized as follows. In the next section, a short description of the Solvency II standard formula for market risk is given and the partial internal model is introduced. Furthermore, in the third section we run the portfolio optimization, in the fourth section we calculate the solvency capital requirements for the efficient portfolios, and in the penultimate section we estimate the actual ruin probabilities that

arise for each asset allocation. Finally, in the last section we draw our conclusion.

MODEL FRAMEWORK

Solvency II Standard Formula—Market Risk Module

For the calculation of the solvency capital requirement (SCR) the regulator offers a standard formula, which is meant to be a reasonable alternative to an internal model for those insurance companies that lack the necessary risk management and modeling capacities. The standard formula consists of the six modules market risk, health risk, default risk, life risk, non-life risk, and intangible asset risk. Our analysis is centered on the market risk module, since it accounts for the largest fraction of the overall SCR for European insurance companies (see EC, 2010 and Fitch Ratings, 2011). The market risk module itself comprises seven submodules. We focus on those for interest rate risk, equity risk, property risk, and spread risk.² The basic own funds (BOF) or equity capital of an insurance company equal the difference between its assets and liabilities (see EIOPA, 2012b).³ For each type of market risk, stress factors determine a change in the basic own funds (ΔBOF) that needs to be covered by the firm's solvency capital. Once the charges originating from the individual submodules have been determined, they are combined into the insurer's overall solvency capital requirement for market risk (SCR_{Mkt}) according to a predetermined aggregation formula. A quantitative representation of this framework can be found in the Appendix.

Partial Internal Model for Market Risk

Insurers may also use an internal model for the calculation of SCR_{Mkt} . In this section, we introduce a parsimonious asset-liability approach based on structural credit modeling (see Merton, 1974) and portfolio theory (see Markowitz, 1952).⁴ Consistent with Solvency II, the capital requirements are calculated based on the value at risk measure (VaR) with a confidence level of 99.5 percent and a one-period time horizon. Insolvency occurs when the insurer's assets are lower than its liabilities. Under discrete

²These four submodules constitute roughly 80 percent of the overall market risk (see, e.g., Fitch Ratings, 2011).

³Note that in former quantitative impact studies (QIS), the BOF were termed net asset value (NAV) (see, e.g., EC, 2010).

⁴Please note that the internal model is adopted from Braun et al. (2015).

compounding, the assets at the end of the period ($t = 1$), \tilde{A}_1 , can be stated as:

$$\tilde{A}_1 = A_0(1 + \tilde{r}_A), \quad (1)$$

with A_0 denoting the deterministic market value of the assets at the beginning of the period and \tilde{r}_A the stochastic asset return over the period.

By drawing on the individual portfolio weights (w_i) for each asset class i , the asset portfolio return \tilde{r}_A can be calculated as weighted average of the individual asset returns $\tilde{r}_i \sim N(\mu_{\tilde{r}_i}, \sigma_{\tilde{r}_i})$ in the following manner:

$$\tilde{r}_A = \sum_{i=1}^n w_i \tilde{r}_i = \mathbf{w}' \mathbf{R}. \quad (2)$$

In Equation (2), n equals the number of asset classes in the portfolio, \mathbf{w} is a vector containing the portfolio weights, and \mathbf{R} a random vector of asset class returns. As a consequence, we have $\tilde{r}_A \sim N(\mu_A, \sigma_A)$.⁵

The corresponding mean and variance can be computed as:

$$\mu_A = E[\tilde{r}_A] = \mathbf{w}' \mathbf{M}, \quad (3)$$

and

$$\sigma_A^2 = \text{var}[\tilde{r}_A] = \mathbf{w}' \Sigma \mathbf{w}, \quad (4)$$

with the vector of mean returns \mathbf{M} , and the variance-covariance matrix of returns Σ .

The insurance liabilities represent the discounted expected future payments to the policyholders. Similar to the development of the insurer's assets, the liabilities at the beginning of the period (L_0) are assumed to grow by a rate $\tilde{g}_L \sim N(\mu_L, \sigma_L)$. Hence, the stochastic market value at time $t = 1$, \tilde{L}_1 , can be expressed as:

$$\tilde{L}_1 = L_0(1 + \tilde{g}_L). \quad (5)$$

⁵Despite its known shortcomings in describing empirically observed returns (see, e.g., Fama, 1965), the normality assumption is helpful to curtail deviations from the results for the market risk standard formula that occur purely due to differences in the model specification. It does not cause a loss of generality and could be relaxed at the expense of the closed-form solutions presented throughout this section.

In addition to the marginal distributions for the asset and liability values, we need to define a dependency structure. In this respect, we assume the asset return \tilde{r}_A and the liability growth rate \tilde{g}_L to adhere to a bivariate normal distribution:

$$(\tilde{r}_A, \tilde{g}_L)' \sim N_2(\mathbf{M}_{A,L}, \Sigma_{A,L}). \quad (6)$$

$\mathbf{M}_{A,L}$ in Equation (6) is a two-dimensional mean vector and $\Sigma_{A,L}$ the 2x2 variance-covariance matrix. Furthermore, suppose that the common variation of the insurer's assets and liabilities is caused by their sensitivity to interest rate movements. Hence, the associated correlation $\rho_{A,L}$ will be adapted to each portfolio composition by means of the following approximation (see Braun et al., 2014):

$$\rho_{A,L} \approx \begin{cases} D_A/D_L & \text{if } D_A \leq D_L \\ D_L/D_A & \text{otherwise,} \end{cases} \quad (7)$$

with D_A and D_L denoting the modified durations of the asset and the liability side, respectively. D_A depends on the fraction of bonds in the insurer's portfolio. Hence, for each asset allocation, a different modified duration is obtained:

$$D_A = \sum_{i=1}^6 D_i \cdot w_i, \quad (8)$$

where D_i is the duration of asset class i and w_i represents the respective portfolio weight.

Based on the above definitions, a distribution for the stochastic basic own funds of the insurer at time $t = 1$ ($B\tilde{O}F_1$) as well as their change over the considered period ($\Delta B\tilde{O}F$) can be derived:

$$B\tilde{O}F_0 = A_0 - L_0 \quad (9)$$

$$B\tilde{O}F_1 = \tilde{A}_1 - \tilde{L}_1 \quad (10)$$

$$\Delta B\tilde{O}F = B\tilde{O}F_1 - B\tilde{O}F_0. \quad (11)$$

Using Equations (1) and (5), we can derive the first two central moments of the $\Delta B\tilde{O}F$ -distribution:

$$\mu_{\Delta B\tilde{O}F} = A_0\mu_A - L_0\mu_L, \quad (12)$$

and

$$\sigma_{\Delta B\tilde{O}F}^2 = A_0^2\sigma_A^2 + L_0^2\sigma_L^2 - 2A_0L_0\sigma_A\sigma_L\rho_{A,L}. \quad (13)$$

The solvency capital requirements can now be computed by applying the VaR with a 99.5 percent confidence level to the $\Delta B\tilde{O}F$ -distribution. Let VaR_α denote the value at risk with a confidence level of $1 - \alpha$. It is defined as that loss (i.e., the negative change in the BOF) during the period, which, in absolute terms, is only exceeded with probability α . In our case, this means the z_α -quantile of the normal distribution. Hence, SCR_{Mkt} is given by:

$$SCR_{Mkt} = |(\mu_{\Delta B\tilde{O}F} + z_{0.5\%}\sigma_{\Delta B\tilde{O}F})|, \quad (14)$$

where $z_{0.5\%}$ is the 0.5-percent quantile of the standard normal distribution.

Calibration

Solvency II Standard Formula for Market Risk

We draw on the Solvency II directives of CEIOPS (see CEIOPS, 2010a; CEIOPS, 2010b; CEIOPS, 2010c) and the current proposal and errata document of EIOPA (see EIOPA, 2012a; EIOPA, 2012b). Regarding the interest rate risk submodule, CEIOPS derived its stress factors based on EUR- and GBP-denominated government bond yields as well as the respective LIBOR swap rates. In the following, we assume that the term structure is flat and that the insurance company only invests in EUR-denominated assets. Hence, foreign exchange (FX) risk can be neglected. Taking the mean of the AAA-rated Eurozone zero bond spot yield curve at the end of December 2012, we are left with an unstressed interest rate of 0.92 percent. Similarly, we compute a single upward stress factor of +45 percent and a single downward stress factor of -40 percent by averaging the parameter values provided by the regulator across all maturities. Due to the low level of our unstressed rate, we obtain absolute changes below one percentage point. Hence, the latter need to be manually adjusted to plus one percent and minus one percent for the upward and the downward state, respectively (see EIOPA, 2012b; EIOPA, 2012a). The insurer's assets and liabilities are assumed to react to these yield curve shifts as implied by their modified durations.

Based on an analysis of the MSCI World Developed Price Equity Index, the type 1 equity stress factor has been set to 39 percent (see CEIOPS, 2010c; EIOPA, 2012b). In contrast to that, the regulator has drawn on a broad range of indices for private equity, commodities, hedge funds, and emerging markets to derive the type 2 stress factor. Although the results for these benchmarks vary substantially, a single stress factor of 49 percent has been chosen (see EIOPA, 2012b).⁶ The correlation between both equity risk categories is set to 0.75 (see, e.g., EC, 2010; EIOPA, 2012b).

The stress factor for the property risk submodule has been calibrated based on the Investment Property Databank (IPD) in UK. The IPD indices represent several property market sectors such as retail, office, industrial and residential (see CEIOPS, 2010c). Nevertheless, CEIOPS refrained from a breakdown of the categories and defined a single property stress of 25 percent (see CEIOPS, 2010c; EIOPA, 2012b).

Finally, Merrill Lynch corporate bond indices with different maturity buckets and rating classes were used to calibrate the spread risk submodule (see CEIOPS, 2010c). In EIOPA (2012b), the corresponding stress factor depends on both a bond's duration and rating class. Assuming the insurer exclusively invests in investment-grade (IG) securities, we average the spread risk factors for corporate debt over all IG rating classes (AAA to BBB) for the duration category of between five and ten years, obtaining a single spread shock of 9.10 percent (see EIOPA, 2012b).⁷ An overview of the interest rate, equity, property, and spread risk parameter values for the Solvency II standard formula that enter our analysis can be found in Table 1.

Partial Internal Model for Market Risk

To calibrate our internal model, we follow Braun et al. (2015) and select the asset categories stocks, government bonds, corporate bonds, real estate, hedge funds, and money market instruments. Each subportfolio is assumed to behave like a representative index for which we obtained time series of monthly returns from January 1993 to December 2012. We decided in favor of a 20-year time horizon since the extended periods of financial market turmoil throughout the last decade lead to negative mean returns. Consequently, our calibration covers several business cycles as well as high

⁶In accordance with former research on the topic, such as the work by Braun et al. (2014, 2015), we refrain from incorporating the *symmetric adjustment mechanism* into the analysis (for further information, refer to, e.g., CEIOPS, 2010a; CEIOPS, 2010c; EIOPA, 2012b).

⁷This proceeding is motivated by the fact that the index that we have chosen to represent the insurer's corporate bond portfolio enters our analysis with a modified duration of 7.09 (see Table 2).

Table 1. Input Data for the Solvency II Standard Formula

Submodule	Shock %
Interest rate risk	−40.00 / +45.00
Type 1 equity	−39.00
Type 2 equity	−49.00
Property risk	−25.00
Spread risk	−9.10

This table contains the percentage shocks of the considered submodules of the Solvency II standard formula for market risk (see EIOPA, 2012b). For the interest rate risk submodule, the regulator provides an upward as well as a downward shock. In order to obtain a single interest rate shock for both scenarios, the CEIOPS values for all maturities have been averaged. The same procedure has been applied to the shock of the spread risk submodule by averaging the parameters for all investment-grade ratings and a maturity range between five and ten years. All input data can be found in the directives of CEIOPS (see CEIOPS, 2010a, CEIOPS, 2010b, and CEIOPS, 2010c).

and low interest rate environments. All time series have been obtained from Bloomberg or Datastream.

In order to model the insurer's equity subportfolio, the EURO STOXX 50 Index is used, which comprises 50 stocks of large companies located in twelve countries of the Eurozone. Owing to its diversity, we consider it to be a suitable benchmark for the European stock markets. For the aforementioned time period, we compute an expected return of 9.21 percent and a standard deviation of 19.26 percent. In addition, the German Stock Exchange REX Performance Index (REXP) is used as a proxy for the government bond subportfolio. This index covers 30 German Bunds with a series of different maturities and coupons.⁸ Over the considered 20-year period from 1993 until 2012, the REXP exhibited a mean return of 5.96 percent and a standard deviation of 3.34 percent. Its modified duration at the end of December 2012 was 4.92. The other interest rate-sensitive asset class in the portfolio of our exemplary insurance company are corporate bonds. Due to the lack of a suitable index representing corporate debt in Europe before 1998, we decided to draw on the Barclays U.S. Corporate

⁸Due to lack of data for the overall Eurozone, we concentrate on the German government bond market only. Considering its size and importance, we deem this approach to be appropriate.

Bond Index, which reflects the performance of IG fixed income instruments of U.S. corporations. It exhibited a mean return of 6.99 percent and a standard deviation of 5.55 percent over our calibration period. The corresponding modified duration for December 2012 equals 7.09. Moreover, the asset category real estate is reflected by the Grundbesitz Europa Fund, which conducts investments in residential and commercial real estate across Europe. We corrected the respective time series for annual dividend payouts to investors and calculated a mean return of 4.81 percent and a standard deviation of 1.76 percent between 1993 and 2012. The hedge fund portfolio of the insurer is assumed to behave like the HFRI Fund Weighted Composite Index (HFRI), for which we estimate a mean return of 9.65 percent and a standard deviation of 7.08 percent.⁹ Finally, the one-month Euro Interbank Offered Rate (EURIBOR) with a mean return of 3.14 percent and a standard deviation of 0.50 percent is employed for the insurer's money market portfolio.¹⁰ Table 2 provides some descriptive statistics for the six asset classes as well as the applicable legal investment limits for insurance companies in Germany, which will enter the optimization procedure in the next section. The corresponding variance-covariance matrix (Σ) of the returns is shown in Table 3.

For the liability side of the insurer's balance sheet, reliable data is not available. Therefore, we decide to apply suitable approximations (see Braun et al., 2014). First of all, we set μ_L to 0.0175.¹¹ Furthermore, we assume that, in line with the asset portfolio, the firm's liabilities are EUR-denominated, implying that they are exclusively affected by changes in the EUR term structure. Between January 1995 and December 2012, the standard deviation of the EUR interest rate amounted to 69 basis points.¹² Based on this figure, we estimate $\sigma_{\tilde{g}_L}$ as follows:

$$\sigma_{\tilde{g}_L} \approx \sigma_{i_{EUR}} \cdot D_L = 0.0069 \cdot 10.00 = 0.069. \quad (15)$$

Consistent with evidence from practitioner studies for the German life insurance market, we set D_L to 10 (see, e.g., Steinmann, 2006).

⁹ The HFRI comprises more than 2,000 single funds and therefore represents a very well diversified hedge fund portfolio, which may somewhat overestimates the performance of alternative investments conducted by a typical insurance company.

¹⁰ Before 1999, we rely on the one-month FIBOR.

¹¹ This figure equals the current technical interest rate in Germany (see BaFin, 2012b).

¹² Note that earlier data is unavailable.

Table 2. Descriptive Statistics and Investment Limits of Asset Classes (01/01/1993–12/31/2012)

Asset class	Index	μ_i	r_{med}	σ_i	Duration	I.L.
Stocks	EURO STOXX 50	9.21%	15.92%	19.26%	–	20.00%
Government bonds	REXP Index	5.96%	7.88%	3.34%	4.92	–
Corporate bonds	Barclays U.S. Corp. Index	6.99%	8.16%	5.55%	7.09	10.00%
Real estate	Grundbesitz Europa Fund	4.81%	3.99%	1.76%	–	25.00%
Hedge funds	HFRI Fund Weighted Index	9.65%	11.94%	7.08%	–	5.00%
Money market	1-month FIBOR / EURIBOR	3.14%	3.26%	0.50%	–	–

For each return time series, mean (μ), median (r_{med}), standard deviation (σ) on an annual basis are shown. In addition, the table contains the durations for government and corporate bonds as well as the investment limit (I.L.) for all asset classes as applicable in Germany.

Table 3. Annualized Variance-Covariance Matrix of Returns (01/01/1993–12/31/2012)

	(a)	(b)	(c)	(d)	(e)	(f)
(a) Stocks	0.0371	–0.0014	0.0016	–0.0001	0.0094	0.0000
(b) Government bonds		0.0011	0.0008	0.0001	–0.0005	0.0000
(c) Corporate bonds			0.0031	0.0000	0.0011	0.0000
(d) Real estate				0.0003	0.0000	0.0000
(e) Hedge funds					0.0050	0.0000
(f) Money market						0.0000

PORTFOLIO OPTIMIZATION

The insurance company is assumed to choose its asset allocation based on the six asset classes introduced above. Assume that, in doing so, it acts as a risk-averse investor wanting to minimize the standard deviation (or variance) for a fixed level of return. Therefore, it needs to solve the following quadratic optimization problem (see, e.g., Kroll et al., 1984):

$$\min_w w' \Sigma w \quad (16)$$

$$\text{subject to } w' M = \bar{\mu}_A, \quad (17)$$

$$w' \mathbf{1} = 1, \quad (18)$$

$$w_i \geq 0, \quad (19)$$

$$\text{and } w_i \leq u_i \quad i \in \{1, 2, \dots, 6\}. \quad (20)$$

Equation (17) sets the required expected return to $\bar{\mu}_A$. Equation (18) and Inequality (19) represent the insurer's budget and short-sale constraints, respectively. The former rules out borrowing and requires the insurer to invest 100 percent of the available capital. Based on these three constraints, we run a first optimization. Subsequently, a second optimization is conducted, additionally including the investment limits (20). For each asset class i , u_i reflects the upper bound on the portfolio weight for those assets of the insurer that back its technical reserves. Since the investment limits vary across EU member states, we restrict our analysis to Germany, where the portfolio choice is governed by the so-called "Regulation on the Investment of Restricted Assets of Insurance Undertakings" (see BMJ, 2011).¹³ In the latter, it is stated that the asset classes stocks, corporate bonds, and hedge funds are in sum limited to 35 percent of the insurer's restricted assets, whereby the latter may account for no more than 5 percent. The remaining 30 percent have been assigned to stocks with an upper bound of 20 percent and corporate bonds with an upper limit of 10 percent. Similarly, the real estate asset class must not exceed a share of

¹³Note that it is currently unclear whether these investment limits will remain in effect after the introduction of Solvency II. In the current situation, however, their inclusion enriches the analysis by an additional degree of realism.

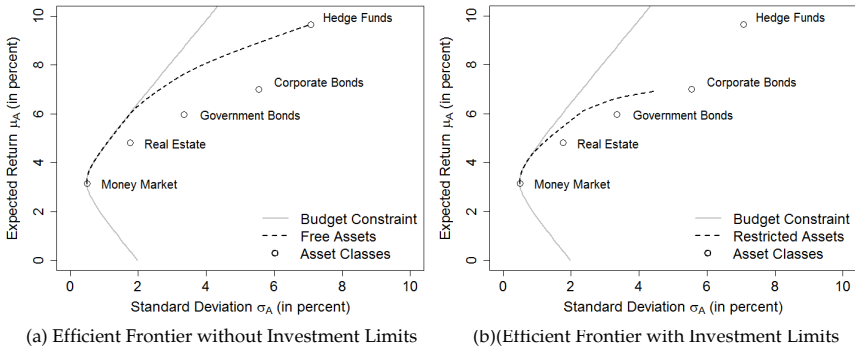


Fig. 1. Efficient frontiers for free and restricted assets. In this figure, the μ - σ spaces for our two optimizations are shown. Both subfigures contain a solid gray curve, representing the efficient frontier with budget constraint only. The dashed curves represent the efficient frontiers with (a) budget and short-sale constraints as well as (b) additional investment limits. The original asset classes are marked by small circles. In both subfigures, the minimum-variance portfolio is located slightly to the upper left of the money market asset class.

25 percent. Taking these considerations into account, the upper bounds u_i are defined.

Figure 1 shows the efficient portfolios obtained from the two optimizations together with the original asset classes in the μ - σ space.¹⁴ In the following, the efficient portfolios that resulted from the optimization with budget and short-sale constraints will be termed “free asset portfolios,” while those obtained from the optimization with the additional investment limit constraint will be termed “restricted asset portfolios.” Both subfigures contain a solid gray line that represents the efficient frontier with budget constraints only. When short-sale constraints are added, we receive the frontier marked by the dashed line in subfigure (a) (130,241 portfolios). The dashed curve in subfigure (b), on the other hand, indicates the frontier in case all previously discussed constraints are incorporated (75,080 portfolios). Due to the additional investment limits, it is located inside the space of possible portfolio choices shown in subfigure (a). Therefore, both the highest achievable expected return and the highest possible level of market risk are lower.

Furthermore, the corresponding portfolio compositions are depicted in Figure 2. To ensure consistency with Figure 1, the portfolios are sorted in ascending order of asset risk. Both subfigures exhibit at least two

¹⁴With a standard deviation of 19.26 percent (see Table 2), stocks are located outside the bounds of Figure 1.

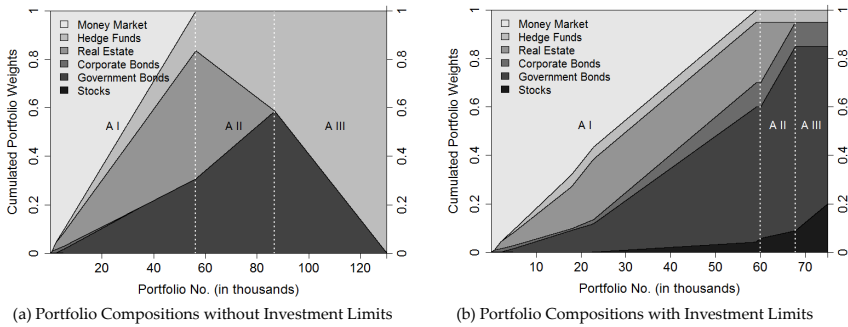


Fig. 2. Portfolio compositions for free and restricted assets. This figure shows the portfolio compositions that correspond to the efficient frontiers (a) without and (b) with investment limits. In both subfigures, the portfolios are arranged in ascending order of risk, beginning with the minimum-variance portfolio on the very left. Vertical dotted lines separate three areas, in which the portfolio compositions differ substantially.

significant kinks at which the compositions change substantially. These kinks are highlighted by dotted vertical lines and mark the borders of three distinct areas. At the left end of Figure 2(a), the portfolios are characterized by a high share of money market investments, which continuously declines until it vanishes completely at the end of area A I (portfolio 56,500). Area A II begins with portfolios that purely consist of government bonds, real estate, and hedge funds. Further to the right, however, the fraction of real estate declines and is increasingly substituted by government bonds and hedge funds. Finally, all portfolios in area A III consist of government bonds and hedge funds only, with the former being slowly crowded out by the latter. The portfolio with the highest risk is located at the right end of the efficient frontier (see Figure 1(a)) and consists of hedge funds only.

In Figure 2(b), we have plotted the compositions of the efficient portfolios in the presence of investment limits. Just as without this additional constraint, the portfolios located on the very left mainly consist of money market instruments. Throughout area A I, we then witness increases in the portfolio weights of all remaining asset classes, with the strongest being attributable to government bonds. To the right of portfolio 20,000, the hedge fund allocation is already at its maximum of five percent and at the end of area A I, the money market asset class is no longer included. Moreover, area A II is characterized by relatively well diversified portfolios. From its left to its right border, the real estate assets are gradually removed and substituted by government bonds as well as some stock investments. The latter are further expanded in area A III, where the investment limit of 10 percent for corporate bonds is reached as well. Consequently, the portfolio with the highest attainable standard deviation

contains a maximum fraction of stocks, corporate bonds, and hedge funds, whereas the remaining capital is invested in government bonds.

CAPITAL REQUIREMENTS

In this section, the market risk capital requirements for the efficient portfolios are calculated. As the standard formula does not rely on portfolio weights, however, we first need to determine the balance sheet of our exemplary insurance company in absolute terms. Based on average figures for life insurers in the German and Swiss markets, we assume a capital structure with 12 percent equity and 88 percent technical reserves and decide to fix the balance sheet size to EUR 10 bn.¹⁵ For a portfolio to be admissible under Solvency II, the following condition needs to hold:

$$SCR_{Mkt} \leq BOF_0. \quad (21)$$

Figure 3(a) shows the capital requirements for the free asset portfolios under both solvency models. The firm's equity is marked by the dashed horizontal line. For those portfolios with low return volatilities, the capital charges under the standard formula are below this line. Hence, the insurer may select them. In Figure 2(a), we see that the aforementioned portfolios exhibit a high share of money market instruments and relatively low shares of the risky asset classes stocks, corporate bonds, real estate, and hedge funds. However, due to increasing investments in the latter, the capital requirements surpass the insurer's equity after portfolio 16,000 and therefore the remaining asset allocations are inadmissible. Between portfolios 58,000 and 80,000, we detect a small reduction in the capital charges, which is caused by an increase in the firm's government bond holdings and the associated closing of the duration gap. Since, beyond portfolio 87,000, the money market, real estate, and stock subportfolios are substituted by hedge fund investments, the capital charges again increase sharply. The maximum capital requirements amount to approximately EUR 5.4 bn, which corresponds to 450 percent of the insurer's equity capital (BOF_0).

Turning to the results for the internal model in Figure 3(a), we notice that the capital charges for those portfolios with very low return standard deviations are above the insurer's basic own funds. Although this might appear counterintuitive at first glance, one should bear in mind that the

¹⁵See <http://www.bafin.de> for German life insurers, and <http://www.finma.ch> for Swiss life insurers.

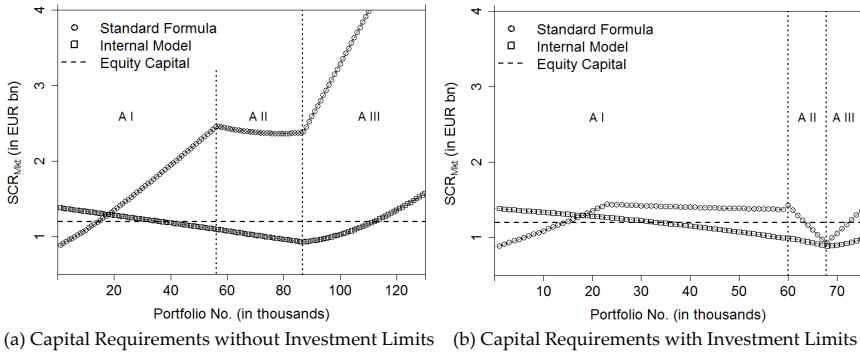


Fig. 3. Capital requirements for free and restricted assets. This figure shows the capital requirements for the optimization (a) without and (b) with investment limits. In both subfigures, the results for the standard formula are marked by circles, while those for the internal model are marked by squares. The insurer's equity capital is depicted as a dashed horizontal line located at EUR 1.2 bn.

internal model is based on a full-fledged asset-liability approach. Hence, the observed capital charges are attributable to a wide duration gap, which is caused by the low shares of government and corporate bonds in these portfolios. As the weights of the bond subportfolios increase, the duration gap narrows and the capital requirements decline. At portfolio 36,300, they fall below the available equity capital, thus rendering the asset allocations admissible. The capital charges are at their minimum for portfolio 87,000, which consists of approximately 58.5 percent government bonds. The subsequent substitution of government bonds with hedge fund investments widens the duration gap again and thus leads to rising capital charges. As a result, all portfolios beyond portfolio 112,000 are inadmissible.

In Figure 3(b), we see that the optimization with investment limits yields a completely different pattern for the capital requirements. Again, the large money market subportfolio causes the low asset-risk portfolios in area A I to be admissible under the standard formula. Beginning with portfolio 15,000, investments in corporate bonds, real estate, and hedge funds start to increase and the capital charges exceed the basic own funds until portfolio 63,000. However, due to fact that the growing weight of government bonds closes the duration gap, we also observe a flattening of the curve. Recall from Figure 2(b) that the government bond holdings are expanded all across area A II and reach their greatest size at the beginning of area A III. As a consequence, the asset allocations surrounding portfolio 67,840 are admissible again. The highest return volatilities are caused by maximum permissible investments in stocks, corporate bonds, and hedge funds. These push the capital charges above the available equity again.

Similar to our findings for the free assets, the capital requirements under the internal model shown in Figure 3(b) start at their peak and then gradually decline. Again, this is caused by the closing duration gap, which results in an increasing correlation between the market values of assets and liabilities. From portfolio 34,000 onwards, the curve lies below the insurer's basic own funds. Although the investments in the risky asset classes stocks, corporate bonds, and hedge funds increase for the portfolios with higher numbers, they always remain admissible. Owing to its 65 percent allocation to government bonds, even the portfolio with the highest return standard deviation may be chosen, since it provides a good asset-liability hedge.

RUIN PROBABILITIES

Actual Ruin Probabilities of the Standard Formula

According to QIS 5, the Solvency II standard formula has been calibrated to correspond to a *VaR*-approach with a confidence level of 99.5 percent and a time-horizon of one year (see EC, 2010; EIOPA, 2012b). That way, the regulator wants to ensure that the annual ruin probability equals 0.5 percent, implying on average one insolvent insurer in 200 years. Since the *SCR* from our internal model equals the 0.5-percent quantile of the $\Delta B\tilde{O}F$ -distribution, it exactly matches the targeted ruin probability (see Equation (14)). According to CEIOPS (2010c), the stress factors for the Solvency II standard formula have mainly been derived based on normal distributions as well. Hence, one would expect similar capital requirements to arise under both approaches. Due to the substantial deviations documented in the last section, however, it must be suspected that the actual ruin probabilities α under the standard formula deviate from the proclaimed target. In order to reveal this potential mismatch, we rearrange Equation (14) and employ the standard normal cumulative distribution function (cdf) Φ :

$$\alpha = \Phi(z_\alpha) = \Phi\left(-\frac{(SCR_{Mkt} + \mu_{\Delta B\tilde{O}F})}{\sigma_{\Delta B\tilde{O}F}}\right). \quad (22)$$

For each individual portfolio, we now insert the capital requirements that have been calculated with the standard formula. Figure 4(a) shows the α for the free assets. The dashed horizontal line marks the regulator's target level of 0.5 percent. Those portfolios, which are admissible in case the insurer uses the standard formula, are located inside area B I. At first glance it can be seen that, for the vast majority of the portfolios, the estimated ruin

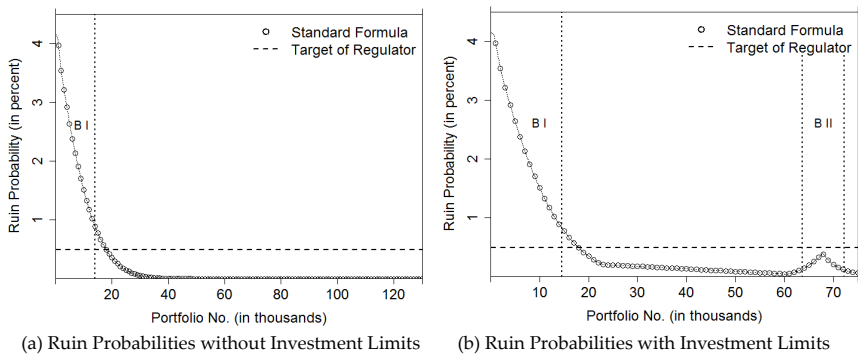


Fig. 4. Ruin probabilities for free and restricted assets. This figure shows the estimated ruin probabilities associated with the capital charges of the standard formula for the case (a) without and (b) with investment limits. The dashed horizontal line depicts the Solvency II target ruin probability of 0.5 percent. Areas B I and B II contain those portfolios that are admissible under the Solvency II market risk standard formula.

probabilities deviate substantially from the target. In fact, only portfolio 17,932 exhibits a ruin probability of exactly 0.5 percent. However, the more alarming finding is that all of the alleged low-risk portfolios in area B I are associated with considerably higher outcomes. The minimum-variance portfolio, for instance, exhibits a ruin probability of 4.16 percent, which equals 8.32 times the target of the regulator.

In Figure 4(b), we have plotted the α for the restricted assets. Again, the minimum-variance portfolio is associated with a ruin probability of approximately 4.16 percent and all admissible portfolios in area B I exhibit substantially higher ruin probabilities than targeted by the regulator. In contrast to that, the admissible portfolios in area B II lead to ruin probabilities below 0.5 percent, indicating that their capital requirements are excessive.

A more detailed presentation of selected efficient portfolios from the optimization with investment limits (restricted assets) can be found in Table 4. The first six rows contain the weights for each asset class. In addition to that, the expected returns and standard deviations, the correlation between the assets and liabilities, and the capital charges under the standard formula as well as the internal model are shown. Admissibility is indicated by a checkmark and inadmissibility by a small x. The respective quantiles of the standard normal distribution as well as the ruin probability and the safety level of the insurer under the market risk standard formula (SF) are presented in the last three rows.

As discussed in the previous section, the standard formula and the internal model produce a different pattern with regard to the admissibility

Table 4. Characteristics of Efficient Portfolios with Investment Limits (01/01/1993–12/31/2012)

Number of Portfolio	1	1,000	5,000	10,000	15,000	20,000	25,000	30,000	35,000
Subportfolio share									
Stocks	0.00%	0.08%	0.01%	0.00%	0.00%	0.00%	0.29%	0.86%	1.42%
Government bonds	0.00%	0.00%	1.48%	4.39%	7.30%	10.15%	14.38%	20.45%	26.53%
Corporate bonds	0.00%	0.91%	1.25%	1.05%	0.84%	1.15%	2.19%	3.34%	4.49%
Real estate	0.00%	0.62%	5.43%	10.04%	14.66%	20.60%	25.00%	25.00%	25.00%
Hedge funds	0.00%	0.00%	1.05%	2.59%	4.10%	5.00%	5.00%	5.00%	5.00%
Money market instruments	100.00%	98.39%	90.78%	81.93%	73.10%	63.10%	53.14%	45.35%	37.56%
Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
Expected return of portfolio	3.14%	3.19%	3.39%	3.64%	3.89%	4.14%	4.39%	4.64%	4.89%
Standard deviation of portfolio	0.50%	0.50%	0.50%	0.56%	0.64%	0.75%	0.89%	1.07%	1.26%
Correlation $\rho_{A,L}$	0.00	0.01	0.02	0.03	0.04	0.06	0.09	0.12	0.16
Capital charges standard formula (SF) (EUR mn)	880.000	887.150	975.929	1,088.514	1,214.506	1,353.854	1,433.974	1,421.802	1,410.812
Admissibility standard formula	✓	✓	✓	✓	x	x	x	x	x
Capital charges internal model (EUR mn)	1,386.428	1,380.493	1,359.454	1,333.616	1,308.069	1,282.024	1,253.385	1,221.373	1,187.487
Admissibility internal model	x	x	x	x	x	x	x	x	✓
Quantile of standard normal distribution (SF)	-1.732	-1.754	-1.936	-2.167	-2.420	-2.696	-2.878	-2.913	-2.954
Estimated ruin probability (SF)	4.16%	3.97%	2.64%	1.51%	0.78%	0.35%	0.20%	0.18%	0.16%
Safety level (SF)	95.84%	96.03%	97.36%	98.49%	99.22%	99.65%	99.80%	99.82%	99.84%

Number of Portfolio	40,000	45,000	50,000	55,000	60,000	65,000	68,000	70,000	75,000
Subportfolio share									
Stocks	1.99%	2.56%	3.13%	3.70%	5.60%	7.70%	9.09%	12.17%	19.88%
Government bonds	32.61%	38.69%	44.76%	50.84%	54.40%	68.15%	75.91%	72.83%	65.12%
Corporate bonds	5.65%	6.80%	7.95%	9.11%	10.00%	10.00%	10.00%	10.00%	10.00%
Real estate	25.00%	25.00%	25.00%	25.00%	25.00%	9.15%	0.00%	0.00%	0.00%
Hedge funds	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%
Money market instruments	29.75%	21.95%	14.16%	6.35%	0.00%	0.00%	0.00%	0.00%	0.00%
Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
Expected return of portfolio	5.14%	5.39%	5.64%	5.89%	6.14%	6.39%	6.54%	6.64%	6.89%
Standard deviation of portfolio	1.48%	1.70%	1.92%	2.16%	2.41%	2.87%	3.16%	3.42%	4.41%
Correlation $\rho_{A,L}$	0.20	0.24	0.28	0.31	0.34	0.42	0.44	0.43	0.39
Capital charges standard formula (SF) (EUR mn)	1,400.951	1,392.174	1,384.448	1,377.747	1,423.039	1,097.750	935.841	1,054.067	1,358.566
Admissibility standard formula	x	x	x	x	x	✓	✓	✓	x
Capital charges internal model (EUR mn)	1,151.483	1,113.217	1,072.585	1,029.487	993.411	927.723	886.709	899.281	979.732
Admissibility internal model	✓	✓	✓	✓	✓	✓	✓	✓	✓
Quantile of standard normal distribution (SF)	-3.001	-3.055	-3.118	-3.188	-3.337	-2.886	-2.667	-2.859	-3.220
Estimated ruin probability (SF)	0.13%	0.11%	0.09%	0.07%	0.04%	0.20%	0.38%	0.21%	0.06%
Safety level (SF)	99.87%	99.89%	99.91%	99.93%	99.96%	99.80%	99.62%	99.79%	99.94%

This table contains the characteristics of selected efficient portfolios with investment limits. The first six rows include the portfolio weights for the asset classes stocks, government bonds, corporate bonds, real estate, hedge funds, and money market instruments. In addition, expected returns, standard deviations, and correlations between assets and liabilities are shown. Admissibility is indicated by a checkmark and inadmissibility by a small x. The quantiles of the standard normal distribution, the estimated ruin probabilities, and the corresponding safety levels under the Solvency II standard formula for market risk are presented in the last three rows.

of portfolios. Consider the following specific example: The rather well-diversified portfolios no. 30,000, 35,000, 40,000, 45,000, 50,000, 55,000, and 60,000, that also offer a good asset-liability hedge, are disallowed under the standard formula. In contrast, the four least diversified asset allocations in Table 4, i.e., no. 1, 1,000, 5,000, and 10,000, with the lowest expected returns, may be chosen by the insurer. Unfortunately the estimated ruin probability estimates for the latter, amounting to 4.16 percent, 3.97 percent, 2.64 percent, and 1.51 percent, are alarmingly high. Furthermore, a comparison between portfolio 10,000 and portfolio 65,000 yields interesting insights. Although both are admissible and lead to almost the same capital charges under the standard formula, the former is associated with a 7.55 times higher ruin probability than the latter. The reason is the huge difference in the correlation between asset returns and liability growth rates (0.03 vs. 0.42). As the internal model reacts much more sensitively to such an asset-liability mismatch, it assigns substantially higher capital charges to portfolio 10,000. At the same time, the stress factors of the standard formula do not adequately account for the risk-return profiles of the portfolios and thus seem to largely overlay the effect of the duration gap. Clearly, such distortions have the potential to severely harm the stability of the financial system, since insurers are prompted to invest in portfolios whose ruin probabilities are in fact much higher than assumed.

The Effect of Increasing Equity Capital

In the following, we carry out a sensitivity analysis with respect to the insurer's basic own funds at time $t = 0$ for the efficient portfolios with investment limits. More specifically, we compare three scenarios with constant balance sheet totals of EUR 10 bn, but equity capital that is 5 percent, 10 percent, and 15 percent higher than in the base case. Table 5 summarizes the input parameter values and the resulting maximum, minimum, and average capital charges as well as ruin probabilities in case the insurer runs the standard formula. The first column contains the figures for the base case with an equity capital of EUR 1.2 bn, average capital charges of EUR 1.271 bn, and an average ruin probability of 0.58 percent. Scenarios (i) to (iii) illustrate how the maximum, minimum, and average capital charges as well as ruin probabilities decline with an increasing amount of basic own funds. Note that, on average, the ruin probability targeted by the regulator is exceeded in all of the scenarios.

Based on Table 5, it is tempting to conclude that higher equity buffers help to reduce the overall level of market risk in the insurance sector. In fact, however, the relationship is more complex. To illustrate this point, Figure 5 shows both the capital requirements and the ruin probabilities under the standard formula for scenarios (i) to (iii). The dashed horizontal

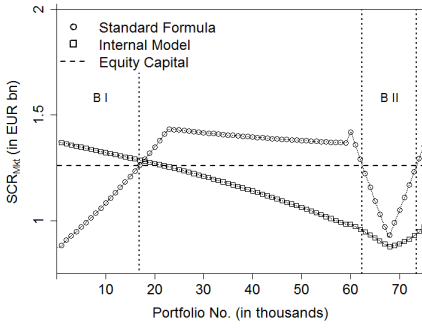
Table 5. Sensitivity Analysis: Change in Equity Capital

Scenario	Base case	Scenario (i)	Scenario (ii)	Scenario (iii)
Equity capital (EC) (EUR mn)	1,200	1,260	1,320	1,380
– <i>Level of EC</i> –	100.00%	105.00%	110.00%	115.00%
– <i>Equity ratio</i> –	12.00%	12.60%	13.20%	13.80%
Liabilities (EUR mn)	8,800	8,740	8,680	8,620
Balance sheet total (EUR mn)	10,000	10,000	10,000	10,000
Capital charges max. (EUR mn)	1,439.5	1,434.3	1,429.1	1,423.9
Capital charges min. (EUR mn)	879.3	873.3	867.3	861.3
Capital charges avg. (EUR mn)	1,271.2	1,266.0	1,260.9	1,255.7
Admissibility of average portfolio	✗	✗	✓	✓
Ruin probability max.	4.16%	4.13%	4.10%	4.07%
Ruin probability min.	0.04%	0.04%	0.04%	0.03%
Ruin probability avg.	0.58%	0.57%	0.56%	0.55%

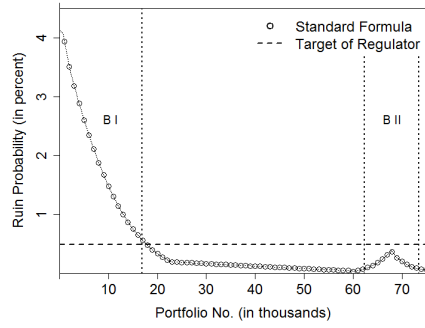
This table shows the maximum, minimum, and average market risk capital requirements as well as ruin probability estimates under the standard formula for a constant balance sheet size but different percentages of equity capital.

lines highlight either the insurer's available equity capital or the 0.5 percent target ruin probability of the regulator. Just as in Figure 4, dotted vertical lines border the areas with admissible portfolios under the standard formula. At first glance, there seem to be no major differences: Neither the shape of the capital requirements nor that of the estimated ruin probabilities changes significantly for the different levels of equity capital.

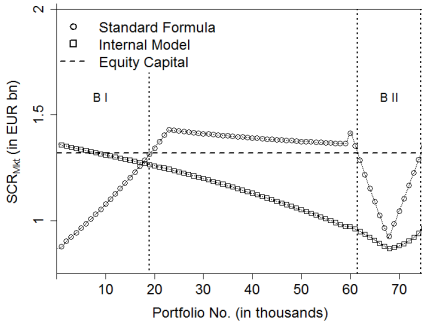
When looking at the capital charges in subfigures (a), (c), and (e) in closer detail, however, we notice that an increase in the insurer's equity leads to an expansion of the areas that contain admissible portfolios. In the base case, the first 14,445 portfolios are admissible, whereas the first 16,913, 19,221, and 21,393 portfolios may be chosen by the insurer in scenarios (i), (ii), and (iii). A similar effect can be observed for areas B II and B III. The flip side is that, compared to the base case, more portfolios with ruin probabilities in excess of the targeted 0.5 percent become admissible. This can be seen in subfigures (b), (d), and (f). Thus, the probability of the insurer selecting an asset allocation that is associated with an inadequately high level of default risk increases with its available equity capital.



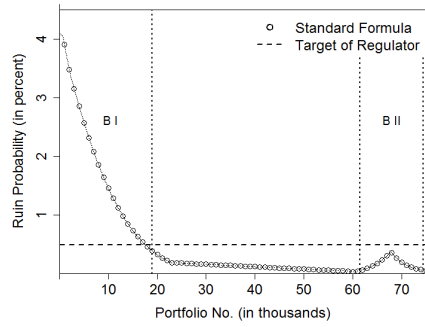
(a) Capital Requirements—Scenario (a)



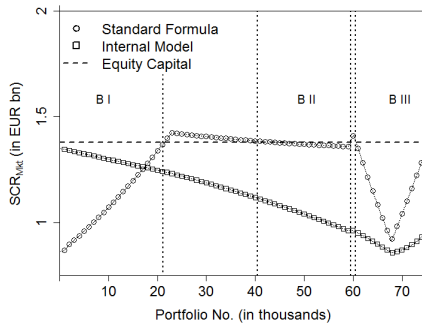
(b) Ruin Probabilities—Scenario (a)



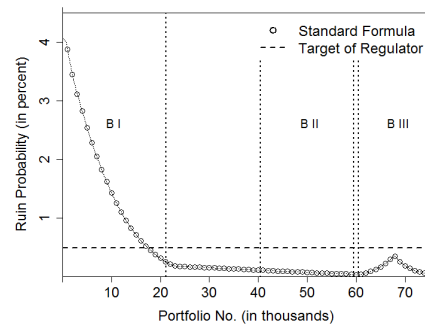
(c) Capital Requirements—Scenario (b)



(d) Ruin Probabilities—Scenario (b)



(e) Capital Requirements—Scenario (c)



(f) Ruin Probabilities—Scenario (c)

Fig. 5. Sensitivity Analysis: Change in Equity Capital. In this figure, the insurer's capital requirements as well as the ruin probabilities for each scenario introduced in Table 5 are depicted. The dashed horizontal line in subfigures (a), (c), and (e) represents the insurer's equity capital. In subfigures (b), (d), and (f), it marks the regulator's target ruin probability of 0.5 percent per year. Admissible portfolios under the market risk standard formula are located in areas B I, B II, and B III.

Ruin Probabilities of German Insurance Companies

We now assess the actual ruin probabilities that would be associated with the average portfolios of German property-liability insurers, life insurers, pension funds, and death benefit funds, if the market risk capital requirements were calculated by means of the standard formula.¹⁶ For this purpose, we draw on balance sheet figures for the fourth quarter of 2012 as published by the German regulatory authority BaFin (see BaFin, 2012a).¹⁷ In addition to that, we analyze the reference portfolio for the typical European insurance company as included in a report by Fitch Ratings (2011). Both input parameter values and results are shown in Table 6. Since the liabilities of a property-liability insurer differ substantially from those of a life insurer, we reduced the respective duration (D_L) from 10 to 5.¹⁸

Note that some portfolio weights, such as that for the corporate bond holdings of the average death benefit fund, seem to be above the respective investment limits. This can be explained by the fact that the company has chosen the maximum possible allocation within the restricted assets, while also investing a major part of the free assets in the asset class.¹⁹ Based on our assumed equity capital of EUR 1.2 bn, all portfolios but the one reported by Fitch Ratings (2011) are admissible. Furthermore, the estimated ruin probabilities for the average property-liability insurer and the Fitch Ratings portfolio are substantially lower than the targeted 0.5 percent. The average life insurer, pension fund, as well as death benefit fund portfolios, on the other hand, are associated with much higher default probabilities.

¹⁶We decided to exclude health insurance companies from our analysis, since the capital charges originating from market risk constitute a much smaller fraction of their overall solvency capital requirement.

¹⁷BaFin provides statistics on a total of 19 different asset classes. To ensure comparability with our previous results, we have consolidated those into the six subportfolios for stocks, government bonds, corporate bonds, real estate, hedge funds, and money market instruments.

¹⁸Please note that the design of our internal model is most suitable for life insurers. In the case of nonlife insurers, which exhibit heavy-tailed claims, a liability distribution allowing for skewness and kurtosis would be superior. Consequently, our model is likely to underestimate the actual ruin probabilities of the latter.

¹⁹In our previous analyses, free and restricted assets have been considered separately.

Table 6. Ruin Probabilities of German Insurance Companies
(Fourth Quarter 2012)

Type of insurer	P.-L. insurer	Life insurer	Pension fund	D.-B. fund	Fitch Ratings
Subportfolio share					
Stocks	15.00%	5.20%	5.20%	2.70%	10.00%
Government bonds	51.37%	57.80%	57.52%	39.65%	30.00%
Corporate bonds	7.76%	6.80%	5.32%	14.45%	35.00%
Real estate	7.67%	6.40%	7.86%	9.60%	15.00%
Hedge funds	2.80%	3.40%	2.60%	1.40%	5.00%
Money market instruments	15.40%	20.40%	21.50%	32.20%	5.00%
Balance sheet total	100.00%	100.00%	100.00%	100.00%	100.00%
Capital charges (SF) (EUR mn)	976.7	940.5	940.1	935.4	1,482.1
<i>Admissibility</i>	✓	✓	✓	✓	✗
Ruin probability (SF)	0.000%	0.827%	0.891%	1.122%	0.020%
No. of companies	212	92	148	38	n.a.

The first six rows show the average asset allocations of property-liability (P.-L.) insurers, life insurers, pension funds, and death benefit (D.-B.) funds in Germany at the end of the fourth quarter 2012 (see BaFin, 2012a). Furthermore, the typical portfolio composition for European insurance groups as published in Fitch Ratings (2011) is included (without unit-linked assets). The lower part of the table contains the capital charges resulting from the market risk standard formula and the corresponding ruin probability estimates.

ECONOMIC IMPLICATIONS AND CONCLUSION

We draw on portfolio theory and empirical time series data in order to derive efficient portfolios for insurance companies both without and with an investment limit constraint. Subsequently, the market risk capital charges under the Solvency II standard formula are calculated for each asset allocation. Based on the respective results and the inversion of a partial internal model for market risk, we are able to estimate the actual ruin probabilities corresponding to the efficient portfolios.

Our analyses reveal that the current set-up and calibration of the Solvency II standard formula for market risk are inadequate. The evaluation of asset portfolios based on stress factors only is not compatible with

the insurance business as it merely emphasizes the goal of avoiding risks instead of balancing risks and opportunities. Clearly, such an approach cannot properly distinguish the risk-return profiles of different investment types. Hence, more volatile asset classes such as stocks and hedge funds become much less attractive for asset management purposes, which greatly reduces the degrees of freedom with regard to portfolio choice. Furthermore, even quite well-diversified portfolios that include relatively small fractions of stocks, corporate bonds, real estate, and hedge funds are severely penalized under the standard formula, thus being unavailable unless the insurer holds a very large amount of equity capital. However, insurers are allowed to select asset allocations that exhibit low return volatilities, but cause relatively wide duration gaps and are therefore considerably more risky from a proper asset-liability stance.

Due to the aforementioned aspects, most admissible portfolios were found to be associated with ruin probabilities clearly above the regulator's target. In contrast to common intuition, these problems worsen for insurance companies with greater amounts of equity capital, since even more such portfolios become available. On the other hand, we identified feasible asset allocations with ruin probabilities far below 0.5 percent, implying excessive capital requirements under the standard formula. Despite the fact that it achieves a high safety level, such an outcome is also not desirable, as it threatens the profitability of the insurance industry. Consequently, the introduction of Solvency II could lead to a lot more ambiguity about the insolvency risk in the European insurance sector than currently expected. Against this background, we strongly encourage regulators and industry professionals to maintain an unprepossessed discussion with the goal of further improving the new regulatory standards. By highlighting substantial weaknesses of the market risk module, we hope that our results may provide an additional impulse in this regard and spawn follow-up analyses by other researchers.

APPENDIX: SOLVENCY II MARKET RISK STANDARD FORMULA

The interest rate risk submodule needs to be applied to all assets and liabilities of the insurance company that are affected by changes in the yield curve (see, e.g., EC, 2010; CEIOPS, 2010b; EIOPA, 2012b). Hence, we have capital requirements covering upward shocks (Mkt_{int}^{up}) as well as downward shocks (Mkt_{int}^{down}) to the term structure of interest rates (see, e.g., EIOPA, 2012b):

$$Mkt_{int}^{up} = \Delta BOF|_{up}, \quad (23)$$

$$Mkt_{int}^{down} = \Delta BOF|_{down}. \quad (24)$$

For both scenarios, the prevailing yield curve is modified by predefined stress factors in the following manner (see, e.g., EC, 2010; EIOPA, 2012b):

$$\begin{aligned} r_t \cdot (1 + s_t^{up}) \quad \forall t, \text{ for the upward shock,} \\ r_t \cdot (1 + s_t^{down}) \quad \forall t, \text{ for the downward shock,} \end{aligned} \quad (25)$$

with r_t being the interest rate for maturity t , and s_t^{up} and s_t^{down} denoting the upward and downward stress, respectively.

The equity risk charge (Mkt_{eq}) is based on the ΔBOF caused by declining equity prices. It consists of two categories: “type 1 equities” and “type 2 equities” (see EIOPA, 2012b).²⁰ Type 1 equities are those from developed markets in the *European Economic Area* (EEA) and the *Organization for Economic Cooperation and Development* (OECD). Type 2 equities comprise, amongst others, hedge funds, private equity, commodities, and emerging market stocks. Owing to this categorization, two steps are needed for the calculation of the equity risk capital requirements. In a first step, the latter are calculated for each category on a separate basis (see, e.g., EC, 2010; EIOPA, 2012b):

$$Mkt_{eq,i} = \max(\Delta BOF|_{equity \ shock_i}; 0), \quad (26)$$

²⁰In former QIS proposals, the equity categories were named as “global equities” and “other equities” (see CEIOPS, 2010a).

with $i = \{\text{type 1 equities; type 2 equities}\}$. In a second step, the individual capital requirements are aggregated through the following formula (see EIOPA, 2012b):

$$Mkt_{eq} = \sqrt{\sum_i \sum_j CorrIndex_{eq} \cdot Mkt_{eq,i} \cdot Mkt_{eq,j}}, \quad (27)$$

where $i, j = \{\text{type 1 equities; type 2 equities}\}$ and $CorrIndex_{eq}$ equals the correlation coefficient between the two categories.

Similar to equity risk, the capital requirement for property risk, Mkt_{prop} , is based on ΔBOF due to drops in real estate prices (see EIOPA, 2012b):

$$Mkt_{prop} = \max(\Delta BOF | \text{property shock}; 0). \quad (28)$$

Moreover, the change in basic own funds resulting from a widening of credit spreads is captured by the spread risk charge (Mkt_{sp}). Although a great variety of fixed income instruments is covered by Mkt_{sp} , we confine our analysis to corporate bonds. The corresponding spread shock is given by (see, e.g., EC, 2010; EIOPA, 2012b):

$$\text{spread shock on bonds} = \sum_{i=1}^n MV_i \cdot D_i \cdot F^{up}(\text{rating}_i), \quad (29)$$

with MV_i denoting the insurer's exposure to bond $i = \{1, \dots, n\}$, D_i representing its modified duration, and $F^{up}(\text{rating}_i)$ being a function of the bond's external rating (see EIOPA, 2012b). Once the shock has been determined, the capital requirements for spread risk can be derived as follows (see, e.g., EC, 2010; EIOPA, 2012b):

$$Mkt_{sp}^{bonds} = \max(\Delta BOF | \text{spread shock on bonds}; 0). \quad (30)$$

Finally, to calculate the overall capital requirement for market risk SCR_{Mkt} , the aforementioned components are aggregated in the following way (see, e.g., EC, 2010; EIOPA, 2012b):

$$SCR_{Mkt} = \max \left\{ \sqrt{\sum_i \sum_j CorrMkt_{i,j}^{up} \cdot Mkt_i^{up} \cdot Mkt_j^{up}}; \sqrt{\sum_i \sum_j CorrMkt_{i,j}^{down} \cdot Mkt_i^{down} \cdot Mkt_j^{down}} \right\}, \quad (31)$$

where $i, j \in \{int; eq; prop; sp\}$. The upward and downward scenarios of the interest rate risk submodule are expressed by the superscripts, whereas $CorrMkt^{up}$ and $CorrMkt^{down}$ represent the applicable correlation coefficients (see Table 7).

Table 7. Correlation Matrices of the Solvency II Market Risk Module

(a) Upward stress scenario				
$CorrMkt^{up}$	Equity	Interest	Property	Spread
Equity	1.00	0.00	0.75	0.75
Interest		1.00	0.00	0.00
Property			1.00	0.50
Spread				1.00
(b) Downward stress scenario				
$CorrMkt^{down}$	Equity	Interest	Property	Spread
Equity	1.00	0.50	0.75	0.75
Interest		1.00	0.50	0.50
Property			1.00	0.50
Spread				1.00

This table shows the correlations between the four main market risk categories of the Solvency II standard formula for both the upward and the downward scenario. The matrices can be found in EIOPA (2012b).

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