THE SCIENCE CUBE

CUBE NOTES

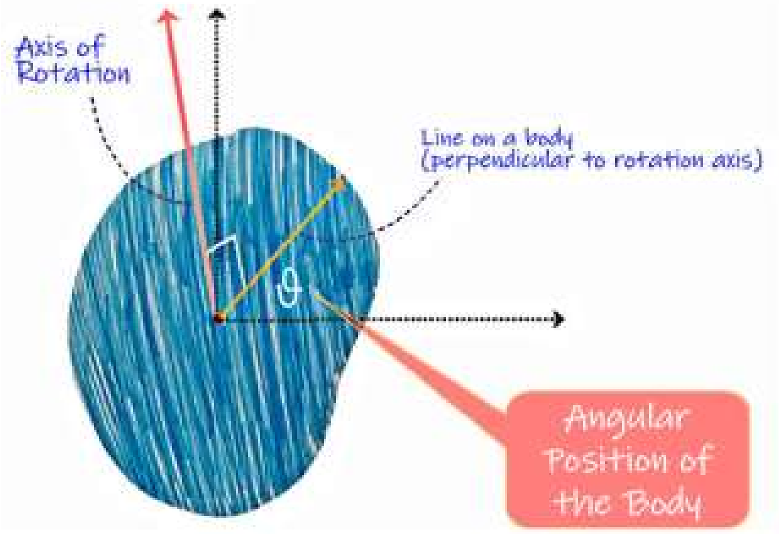
ROTATION: FULL LESSON SUMMARY

AP PHYSICS . JEE . NEET. GRADE 11/12

Rotation: A Comprehensive Overview

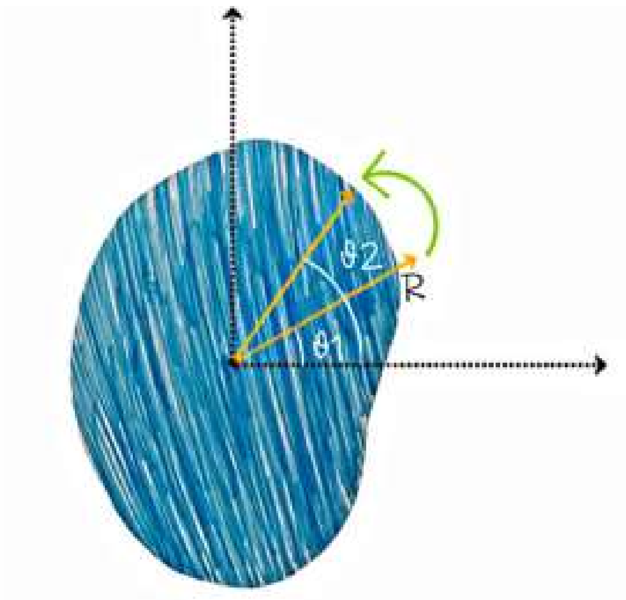
1. Angular Position (θ)

- Angular position describes the orientation of a body in rotational motion relative to a reference axis, often fixed. It is measured in radians.
- 1 revolution (rev) = 360° = 2π radians.
- The formula for angular position is:
  $\theta = s / r$
  where:
  s = arc length,
  r = radius of the circular path.



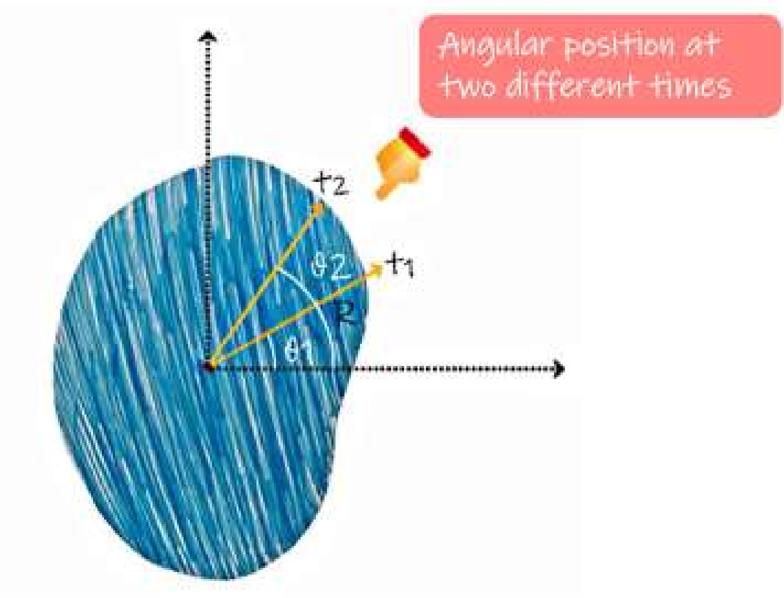
2. Angular Displacement (Δθ)

- Angular displacement is the change in the angular position of a rotating body between two points in time.
- It is positive for counterclockwise motion and negative for clockwise motion.
- Formula:
  $\Delta\theta = \theta_2 - \theta_1$
  where θ_1 and θ_2 are the initial and final angular positions.



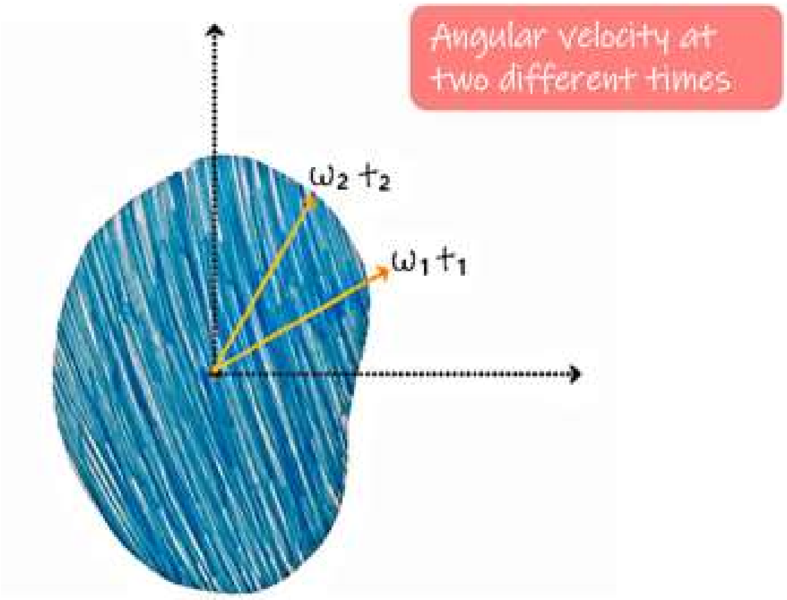
3. Angular Velocity (ω)

- **Average angular velocity** is the rate of change of angular displacement over time.
  $\omega_{avg} = \Delta\theta / \Delta t$
  where $\Delta\theta$ is angular displacement and Δt is the time interval.
- **Instantaneous angular velocity** is the rate of change of angular position at a specific moment in time.
  $\omega = d\theta / dt$
  (where $d\theta$ is the small change in angular position and dt is the small change in time).
- Both average and instantaneous angular velocities are vector quantities with direction given by the right-hand rule. If the curl of the fingers of your right hand matches the rotation direction, your thumb points along the axis of rotation to indicate the direction of ω.



4. Angular Acceleration (α)

- **Average angular acceleration** is the rate of change of angular velocity over time.
  $\alpha_{avg} = \Delta\omega / \Delta t$
  where $\Delta\omega$ is the change in angular velocity and Δt is the time interval.
- **Instantaneous angular acceleration** is the rate of change of angular velocity at a particular moment.
  $\alpha = d\omega / dt$
  (where $d\omega$ is the small change in angular velocity and dt is the small time interval).
- Like angular velocity, angular acceleration is a vector and follows the right-hand rule.



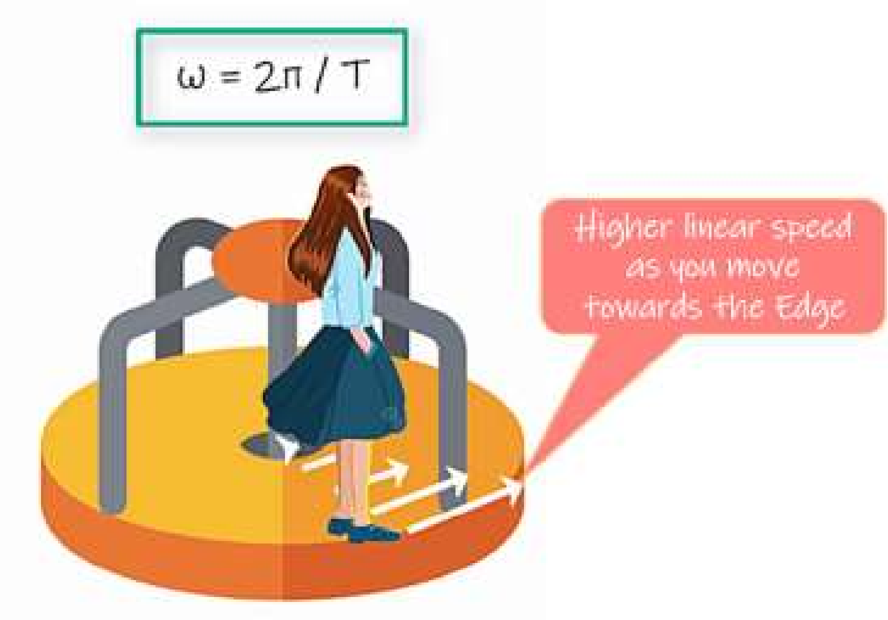
5. Kinematic Equations for Constant Angular Acceleration

These equations describe rotational motion when the angular acceleration is constant, analogous to linear motion equations:

1. $\omega = \omega_0 + \alpha t$
2. $\theta = \theta_0 + \omega_0 t + 0.5\alpha t^2$
3. $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
4. $\theta - \theta_0 = 0.5(\omega + \omega_0)t$

Here, ω_0 is the initial angular velocity, and θ_0 is the initial angular position.

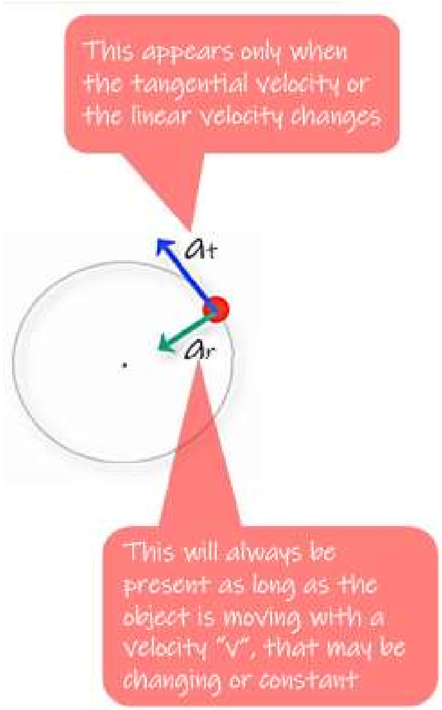
In rotation ω is always constant for any point on the body. However, the linear speed v varies

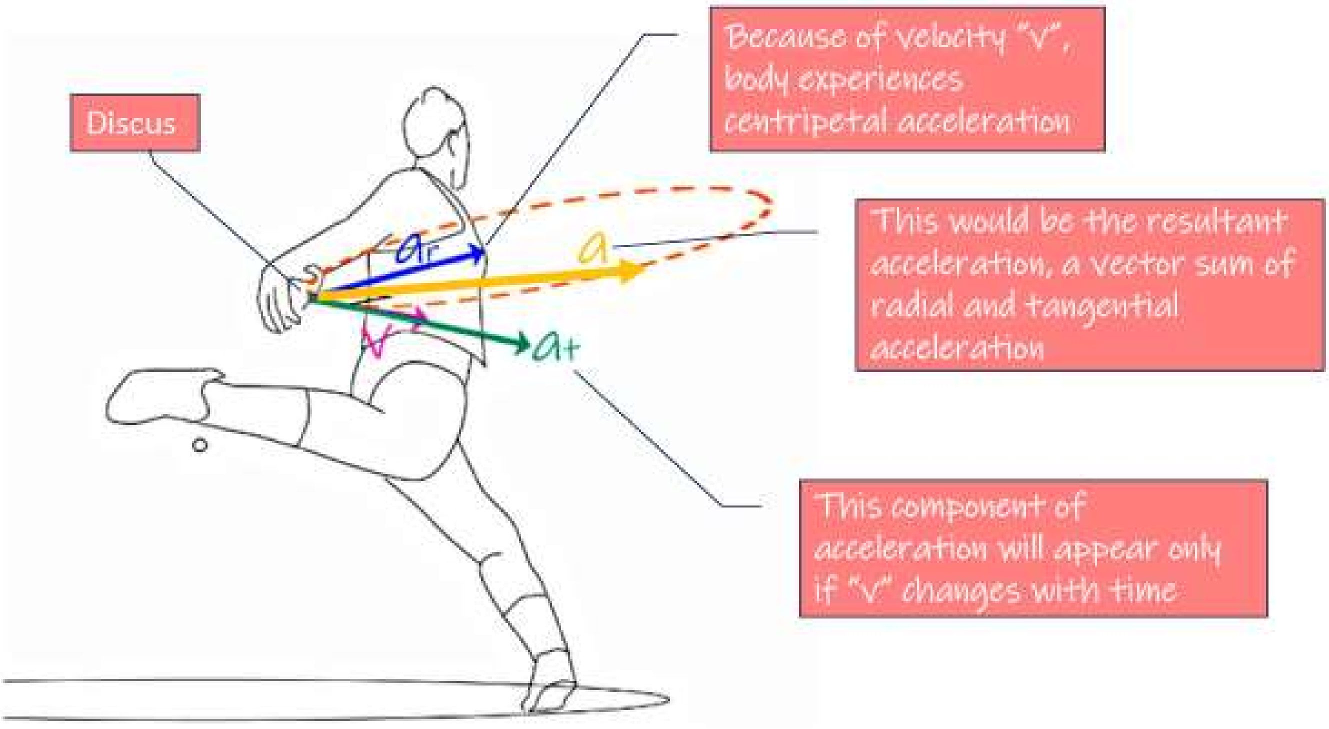


6. Relating Linear and Angular Kinematics

- The linear speed v of a point on a rotating object is related to its angular speed ω and its distance from the axis of rotation, r: $v = r\omega$
- The tangential acceleration (a_t) is related to the angular acceleration (α): $a_t = r\alpha$
- Radial (centripetal) acceleration (a_r) is related to the angular velocity (ω): $a_r = r\omega^2$

These relationships show how angular quantities translate to linear motion at any point on a rotating object.



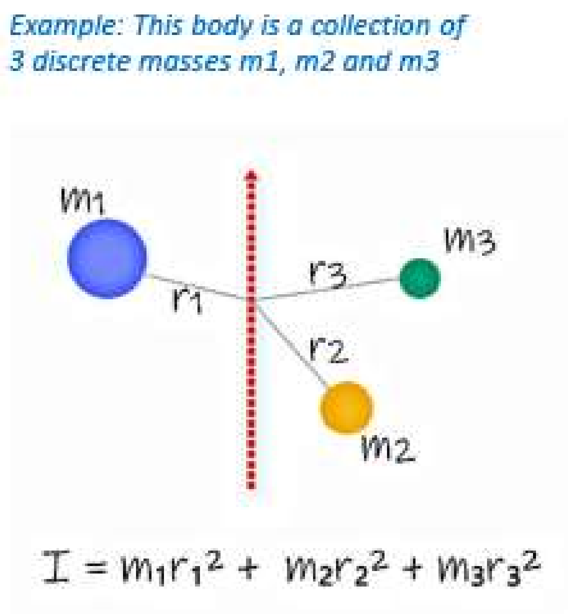


7. Rotational Kinetic Energy

- The rotational kinetic energy (K) of an object rotating about an axis is given by:
  $K = (1/2)\, I\, \omega^2$
  where:
  - I = moment of inertia of the object (depends on mass distribution relative to the axis).
  - ω = angular velocity.
- The moment of inertia (I) for a collection of particles is:
  $I = \Sigma\, m_i r_i^2$
  where:
  - m_i is the mass of particle i.
  - r_i is the perpendicular distance of particle i from the axis of rotation.

8. Moment of Inertia

- Moment of inertia (I) is the rotational equivalent of mass in linear motion. It represents the resistance to changes in rotational motion.
- For continuous bodies, I is calculated by integrating the mass elements over the body: $I = \int r^2 \, dm$
  (where dm is a mass element at a distance r from the axis).
- The moment of inertia depends on the axis of rotation. The farther the mass is from the axis, the larger the moment of inertia.

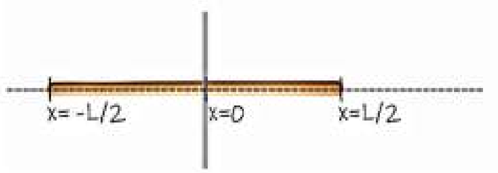


9. Parallel Axis Theorem

- The parallel axis theorem allows calculation of the moment of inertia about any axis parallel to one that passes through the center of mass (I_cm): $I = I_{cm} + Mh^2$
  where:
  - M = total mass of the object.
  - h = perpendicular distance between the center of mass and the new axis.

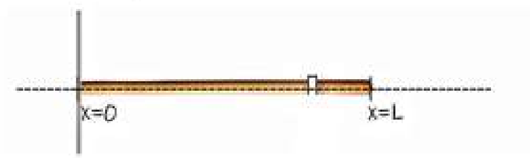
Example: Rod Rotated About 2 Different Axes

Central Axis:



$I_0 = (1/12)\, ML^2$

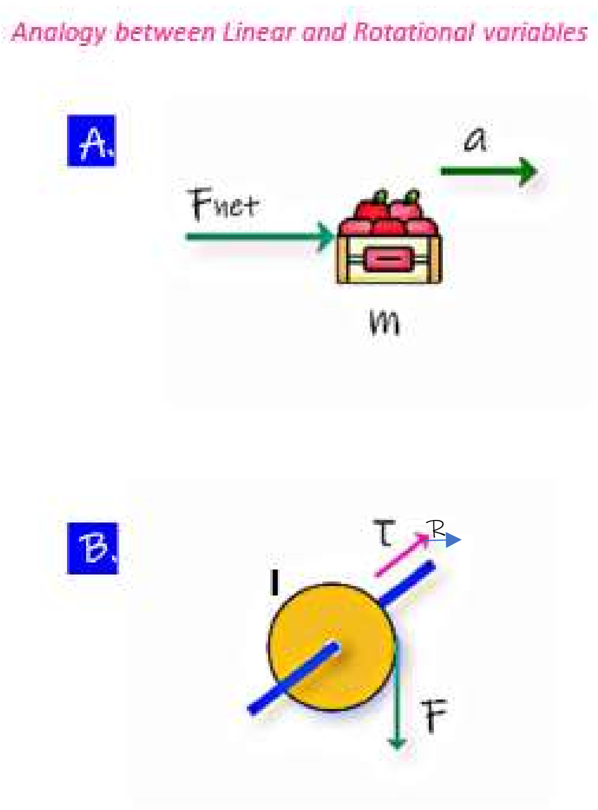
End Axis (Using Parallel Axis Theorem)



$I = (1/12)\, ML^2 + M(L/2)^2$
$I = (1/3)\, ML^2$

10. Torque (τ)

Torque is the measure of the force causing an object to rotate about an axis. It is the rotational equivalent of force.



- Torque (τ) is given by:
  $\tau = rF\sin\theta$ or the cross product $r \times F$

  where:

  - r = distance from the axis of rotation to the point where the force is applied.
  - F = applied force.
  - θ = angle between the force and the lever arm.
- Torque can be thought of as a force applied over a distance (lever arm) to create rotation.

11. Newton's Second Law for Rotation

- The rotational equivalent of Newton's second law is:
  $\Sigma\tau = I\alpha$
  where $\Sigma\tau$ is the net torque on the object, I is its moment of inertia, and α is its angular acceleration.

General approach to solving problems where both rotation and linear motion happen