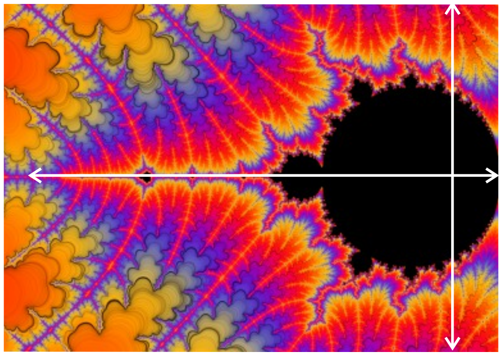
Complex Numbers & The Mandelbrot Set Acvtivity

NAME:________________________________ DATE:_________________

How can it be determined if a complex number belongs to the Mandelbrot set?

UNDERSTANDING



The ________________________ set is a set of points in a complex plain.

1. The image produces a ______________(a never-ending pattern), that results from separating the points into two categories.
    a. Points inside the Mandelbrot set are the _________________ points.

       The absolute value of the sequence of these points never approaches ___________.

    b. The points outside the Mandelbrot set are the non-black points.
        1) The points close to the Mandelbrot set "______________" approach infinity.
        2) The points far from the Mandelbrot set more quickly approach infinity.
        3) The _____________ of these points corresponds to the speed at which infinity is approached.

    c. A Mandelbrot fractal is infinitely complex, meaning you can ___________ in forever.

2. The function of a Mandelbrot set is the series: $\boxed{z_n = Z^2_{n-1} + \qquad .}$
    a. Note: z_0 = _____ .
    b. The output of the _______________ function is the input of the preceding function.

       Note: $z_1 = (f(z_0))^2 + c$, $z_2 = (f(z_1))^2 + c$, $z_3 = (f(z_2))^2 + c$, $z_4 = (f(z_3))^2 + c$ ...

    c. If $|z_0|, |z_1|, |z_2|, |z_3|, |z_4|$... does NOT approach infinity, then the point is inside the Mandelbrot set.
    d. If $|z_0|, |z_1|, |z_2|, |z_3|$,...approaches infinity, then the point is NOT inside the Mandelbrot set.
       Example: if $c = 1$, $z_n = z^2 + 1$ then $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 5$, $|z_4| = 26$,...

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EXPLORATION

1. Determine the values of z_1 - z_4 for each point and circle your answers (show your work).
2. Determine the values of $|z_1|$, $|z_2|$, $|z_3|$, $|z_4|$ for each point (show your work).
3. Conclude if each point is part of the Mandelbrot set or not.

1. $c = i$

$|z_0| = 0$

$|z_1|$

$|z_2|$

$|z_3|$

$|z_4|$

Yes-No

2. $c = 1 + i$

$|z_0| = 0$

$|z_1|$

$|z_2|$

$|z_3|$

$|z_4|$

Yes-No

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3. c = 1 - i

$|z_0| = 0$

$|z_1|$

$|z_2|$

$|z_3|$

$|z_4|$

Yes-No

4. c = -1 + .1i

$|z_0| = 0$

$|z_1|$

$|z_2|$

$|z_3|$

$|z_4|$

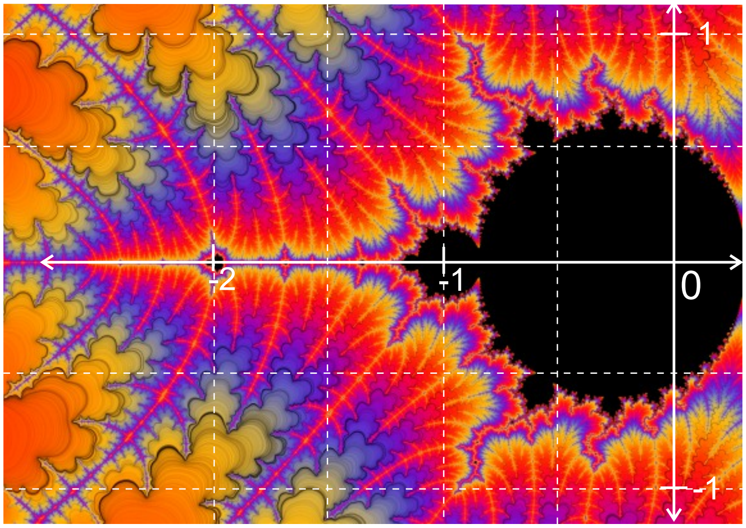
Yes-No

APPLICATION

1. Using the illustration to the left, assume if each point is part of the Mandelbrot set.

   a. c = -0.5 + i Yes-No

   b. c = -2 + 0i Yes-No



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2. What assumption did you have to make to answer “a” and “b” of question 1?

3. Can you think of any other sequences where a given term is a function of the previous term(s)? Explain.

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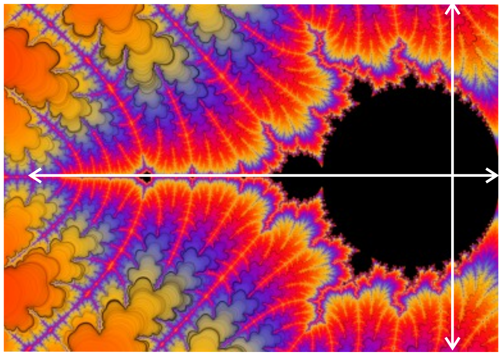
NAME: Answer Key DATE:____________

How can it be determined if a complex number belongs to the Mandelbrot set?



UNDERSTANDING

The **MANDELBROT** set is a set of points in a complex plain.

1. The image produces a **FRACTAL** (a never-ending pattern), that results from separating the points into two categories.
   a. Points inside the Mandelbrot set are the **BLACK** points.

      The absolute value of the sequence of these points never approaches **INFINITY**.

   b. The points outside the Mandelbrot set are the non-black points.
      1) The points close to the Mandelbrot set "**SLOWLY**" approach infinity.
      2) The points far from the Mandelbrot set more quickly approach infinity.
      3) The **COLOR** of these points corresponds to the speed at which infinity is approached.

   c. A Mandelbrot fractal is infinitely complex, meaning you can **ZOOM** in forever.

2. The function of a Mandelbrot set is the series: $\boxed{z_n = Z_{n-1}^2 + c\ .}$
   a. Note: z_0 = 0 .

   b. The output of the **PREVIOUS** function is the input of the preceding function.

      Note: $z_1 = (f(z_0))^2 + c$, $z_2 = (f(z_1))^2 + c$, $z_3 = (f(z_2))^2 + c$, $z_4 = (f(z_3))^2 + c$ ...

   c. If $|z_0|, |z_1|, |z_2|, |z_3|, |z_4|$... does NOT approach infinity, then the point is inside the Mandelbrot set.

   d. If $|z_0|, |z_1|, |z_2|, |z_3|$,...approaches infinity, then the point is NOT inside the Mandelbrot set.
      Example: if $c = 1$, $z_n = z^2 + 1$ then $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 5$, $|z_4| = 26$,...

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EXPLORATION

1. Determine the values of z_1 - z_4 for each point and circle your answers (show your work).
2. Determine the values of $|z_1|$, $|z_2|$, $|z_3|$, $|z_4|$ for each point (show your work).
3. Conclude if each point is part of the Mandelbrot set or not.

1. $c = i$

$z_1 = f(0) = Z^2 + i$

$z_1 = 0 + i = \boxed{i}$

$|z_1| = \sqrt{(0)^2 + 1^2} = 1$

$z_2 = f(i) = (i)^2 + i = \boxed{-1 + i}$

$|z_2| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

$z_3 = f(-1 + i) = (-1 + i)^2 + i$

$1 - 2i + i^2 + i \quad z_3 = \boxed{-i}$

$|z_3| = \sqrt{0^2 + (-1)^2} = 1$

$z_4 = f(-i) = (-i)^2 + i = \boxed{-1 + i}$

$|z_1| = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$

$|z_0| = 0$

$|z_1| = 1$

$|z_2| \approx 1.41$

$|z_3| = 1$

$|z_4| \approx 1.41$

[✓ Yes]-No

2. $c = 1 + i$

$z_1 = f(0) = Z^2 + 1 + i$

$z_1 = 0 + 1 + i = \boxed{1 + i}$

$|z_1| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

$z_2 = f(i) = (1 + i)^2 + 1 + i$

$1 + 2i + i^2 + 1 + i = \boxed{1 + 3i}$

$|z_2| = \sqrt{(1)^2 + 3^2} = \sqrt{10}$

$z_3 = f(1 + 3i) = (1 + 3i)^2 + 1 + i$

$1 + 6i + 9i^2 + 1 + i = = \boxed{-7 + 7i}$

$|z_3| = \sqrt{(-7)^2 + (7)^2} = \sqrt{98}$

$z_4 = f(-7 + 7i) = (-7 + 7i)^2 + 1 + i$

$49 - 98i + 49i^2 + 1 + i = = \boxed{1 - 97i}$

$|z_4| = \sqrt{(1)^2 + (-97)^2} = \sqrt{9,410}$

$|z_0| = 0$

$|z_1| \approx 1.41$

$|z_2| \approx 3.16$

$|z_3| \approx 9.90$

$|z_4| \approx 97.01$

Yes-[✓ No]

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3. c = 1 - i

$z_1 = f(0) = Z^2 + 1 - i$

$z_1 = 0 + 1 - i = \boxed{1 - i}$

$|z_1| = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$

$z_2 = f(1 - i) = (1 - i)^2 + 1 - i$

$1 - 2i + i^2 + 1 - i = \boxed{1 - 3i}$

$|z_2| = \sqrt{1^2 + (-3)^2} = \sqrt{10}$

$z_3 = f(1 - 3i) = (1 - 3i)^2 + 1 - i$

$1 - 6i + 9i^2 + 1 - i = \boxed{-7 - 7i}$

$|z_3| = \sqrt{(-7)^2 + (-7)^2} = \sqrt{98}$

$z_4 = f(-7 - 7i) = (-7 - 7i)^2 + 1 - i$

$49 + 98i + 49i^2 + 1 - i = \boxed{1 + 97i}$

$|z_4| = \sqrt{(1)^2 + (97)^2} = \sqrt{9410}$

$|z_0| = 0$

$|z_1| \approx 1.41$

$|z_2| \approx 3.16$

$|z_3| \approx 9.90$

$|z_4| \approx 97.01$

Yes-[✓ No]

4. c = -1 + .1i

$z_1 = f(0) = Z^2 + -1 + .1i$

$z_1 = 0 + -1 + .1i = \boxed{-1 + .1i}$

$|z_1| = \sqrt{(-1)^2 + (0.1)^2} \approx 1.005$

$z_2 = f(-1 + .1i) = (-1 + .1i)^2 + -1 + .1i$

$1 - .2i + .01i^2 - 1 + .1i = \boxed{-.01 - .1i}$

$|z_2| = \sqrt{(-.01)^2 + (-.1)^2} \approx .10$

$z_3 = f(-.01 - .1i) = (-.01 - .1i)^2 + -1 + .1i$

$.0001 + .002i + .01i^2 - 1 + .1i$

$\boxed{\approx -1.0099 - .102i}$

$|z_3| = \sqrt{(-1.0099)^2 + (-.102)^2} \approx 1.02$

$z_4 = (-1.0099 - .102i)^2 + -1 + .1i$

$1.02 + .206i + .01i^2 - 1 + .1i$

$\approx \boxed{0.01 - .306i}$

$|z_4| = \sqrt{(.01)^2 + (-.306)^2} \approx .306$

answers may vary slightly
due to rounding

$|z_0| = 0$

$|z_1| \approx 1.005$

$|z_2| \approx 0.10$

$|z_3| \approx 1.02$

$|z_4| \approx .306$

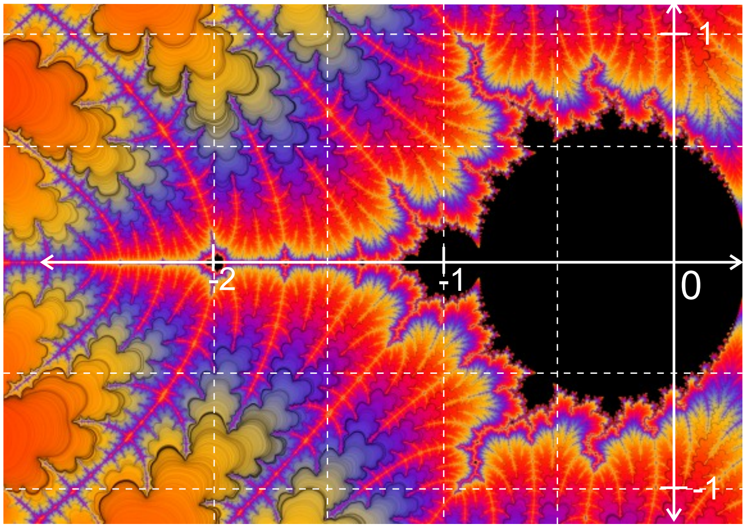
[✓ Yes]-No

APPLICATION

1. Using the illustration to the left, assume if each point is part of the Mandelbrot set.

   a. c = -0.5 + i Yes-[✓ No]

   b. c = -2 + 0i [✓ Yes]-No



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2. What assumption did you have to make to answer “a” and “b” of question 1?

It has to be assumed that when the illustration is zoomed-in, there will not be any change in the color of the graph at the point being considered. (answers may vary)

3. Can you think of any other sequences where a given term is a function of the previous term(s)? Explain. Fibonacci sequences (a term is the sum of the two previous terms).

Example: 1, 1, 2, 3, 5, 8, 13 ... The golden ratio is sum of the inverse of a Fibonacci sequence. Example: $1/f_0 + 1/f_1 + 1/f_2 + 1/f_3$ ... Arithmetic sequences are the result of adding a constant to the previous term. Example: 2, 7, 12, 17, 22... (5 is added) or 30, 22, 14, 6, -2 -10...(-8 is added). (answers will vary)

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