STUDY GUIDE

BUOYANT FORCE

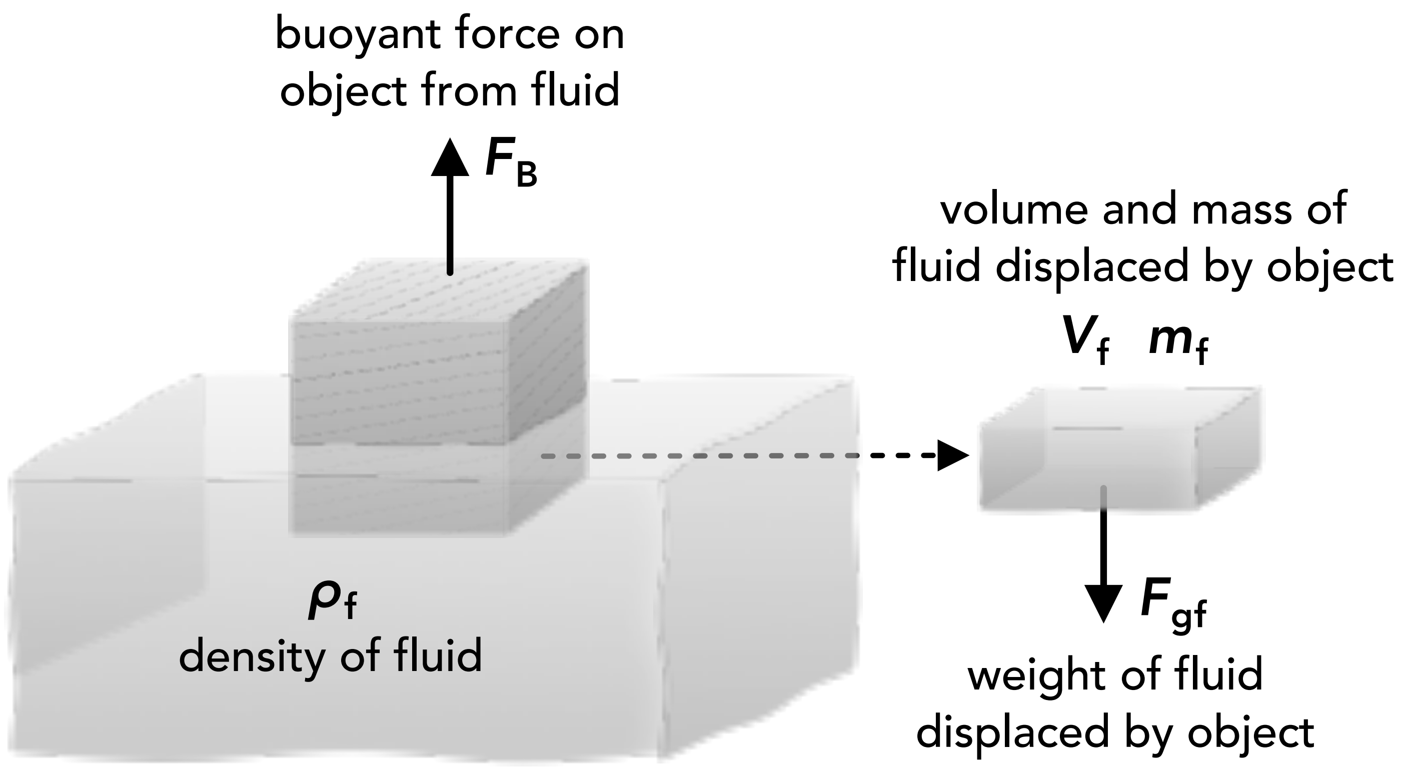
Buoyant Force

Values		Unit	Name
ρ_{water}	1,000	$\frac{kg}{m^3}$	density of water (4°C)
ρ_{ice}	916	$\frac{kg}{m^3}$	density of ice (0°C)
g	9.8	$\frac{m}{s^2}$	gravitational acceleration

Variables		SI Unit
F_B	buoyant force	N
	weight force	N
m	mass	kg
ρ	density	$\frac{kg}{m^3}$
V	volume	m³
A	area	m²

- A **buoyant force** is an upwards force exerted on an object by a fluid due to the fluid pressures around the object.
- This is what causes objects to float in a liquid like a boat in water, or to float in a gas like a hot air balloon in the atmosphere.
- There is always an upwards buoyant force acting on an object if it is in contact with a fluid (liquid or gas), even if the object sinks to the bottom of the fluid.
- Archimedes' principle says that the **upwards buoyant force** acting on an object by a fluid is equal to the **weight of the fluid that the object is displacing**. The mass of the fluid displaced is equal to the volume of the fluid displaced multiplied by the density of the fluid, giving us the equation below for buoyant force.
- The volume of fluid displaced by an object is the amount of fluid that has to move out of the way when the object is placed in the fluid. If an object is completely submerged in a fluid then the volume of the fluid displaced is equal to the volume of the object. If an object is only partially submerged in a fluid (part of the object is above the fluid) then the volume of the fluid displaced is equal to the submerged volume of the object (the amount of the object that is "underwater" in the fluid).

Archimedes' principle: the upwards buoyant force on an object
is equal to the weight of the fluid displaced by the object



Buoyant force on object from fluid

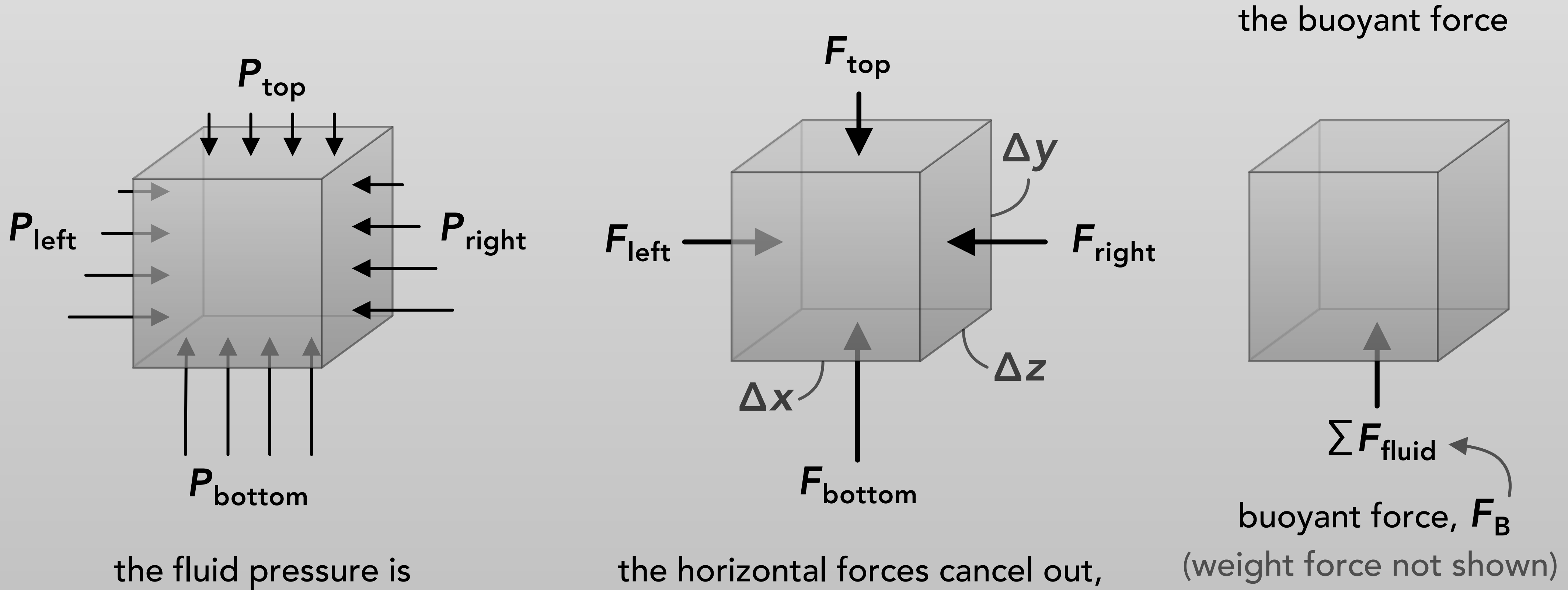
$$\boxed{F_B = \rho_f V_f g}$$

$$F_B = F_{gf} = (m_f g) = (\rho_f V_f) g$$

$$\rho_f = \frac{m_f}{V_f} \rightarrow m_f = \rho_f V_f$$

- A buoyant force is the **net force** caused by all of the **fluid pressure forces** acting on an object in all directions.
- As we learned in a previous section, the pressure in a fluid increases with depth due to the weight of the fluid above that point. Because of this, the **upwards force** exerted by the fluid on the bottom of the object is **greater** than the **downwards force** exerted by the fluid on the top of the object. All of the horizontal forces exerted on the object **cancel out** because they have equal magnitudes and act in opposite directions.
- The resulting net pressure force (the buoyant force) is **upwards**. This is true even if the density of the object is greater than the density of the fluid, or if the object sinks instead of floats. There is always an upwards force exerted by the fluid on the object because of the difference in pressure between the top and bottom of the object.
- As we see in the buoyant force equation, the force depends on the density of the **fluid** and not on the density of the object. The buoyant force is caused by the fluid pressure which depends on the fluid density. The buoyant force is completely independent of the mass of the object (the mass of the object will determine its weight force when we study the overall free body diagram for the object).

Fluid pressure forces acting on a submerged object, the front and back forces are not shown
(only the fluid forces are shown, not the weight of the object)



fluid pressure increases with depth

the fluid exerts a force on each surface of the object due to the fluid pressure

the net force on the object from the fluid pressure forces is upwards, this is the buoyant force

the fluid pressure is greater at the bottom of the object than the top

the horizontal forces cancel out, they are equal in magnitude and opposite in direction

Deriving the equation for buoyant force using the example above:

$$F_{left} = F_{right} \rightarrow \sum F_x = F_{left} - F_{right} = 0$$

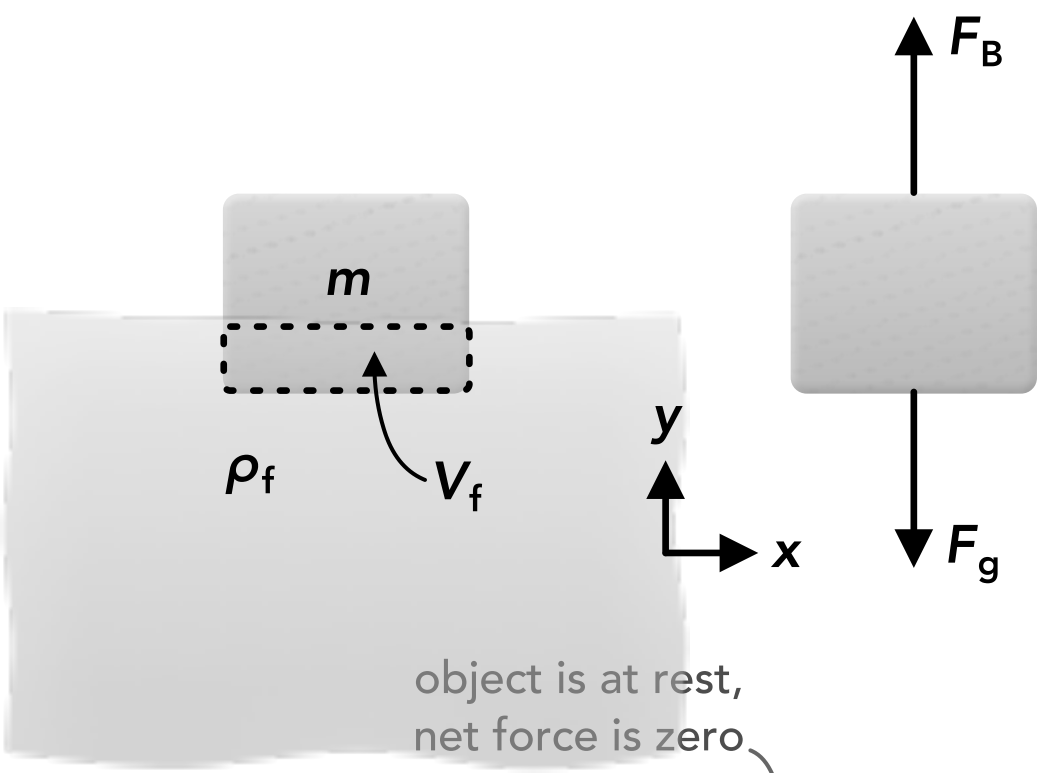
$$F_{front} = F_{back} \rightarrow \sum F_z = F_{front} - F_{back} = 0$$

$$\begin{aligned}
F_{bot} > F_{top} \rightarrow \sum F_y &= F_{bot} - F_{top} \\
&= (P_{bot}A_{bot}) - (P_{top}A_{top}) \longleftarrow PA = F \quad P = \frac{F}{A} \\
&= (\rho_f g h_{bot})A_{bot} - (\rho_f g h_{top})A_{top} \longleftarrow P = \rho g h \\
&= \rho_f g(h_{bot} - h_{top})A \longleftarrow A = A_{bot} = A_{top} \\
&= \rho_f g(\Delta y)(\Delta x \Delta z) \longleftarrow \Delta y = h_{bot} - h_{top} \quad A = \Delta x \Delta z
\end{aligned}$$

$$F_B = \sum F_y = \rho_f g V \longleftarrow V = \Delta x \Delta y \Delta z$$

- When we draw a free body diagram of the object we treat the buoyant force as a **single force** caused by the fluid, we don't need to draw the separate fluid pressure forces acting in each direction.
- We always need to include the **weight force** (gravitational force) acting on the object in the free body diagram.
- Typically those are the only two forces to include if the object is floating in the fluid. However, some scenarios involve an object being suspended by a string or a spring while in a fluid, or an object resting on surface while in a fluid. In those cases we need to include a tension force, spring force or normal force.
- It's important to remember that the buoyant force is the force exerted by the fluid on the object (similar to how the ground exerts a normal force on an object). **The buoyant force is not the overall net force on the object**, we also need to consider the weight force (and maybe other forces) to determine the net force on the object.

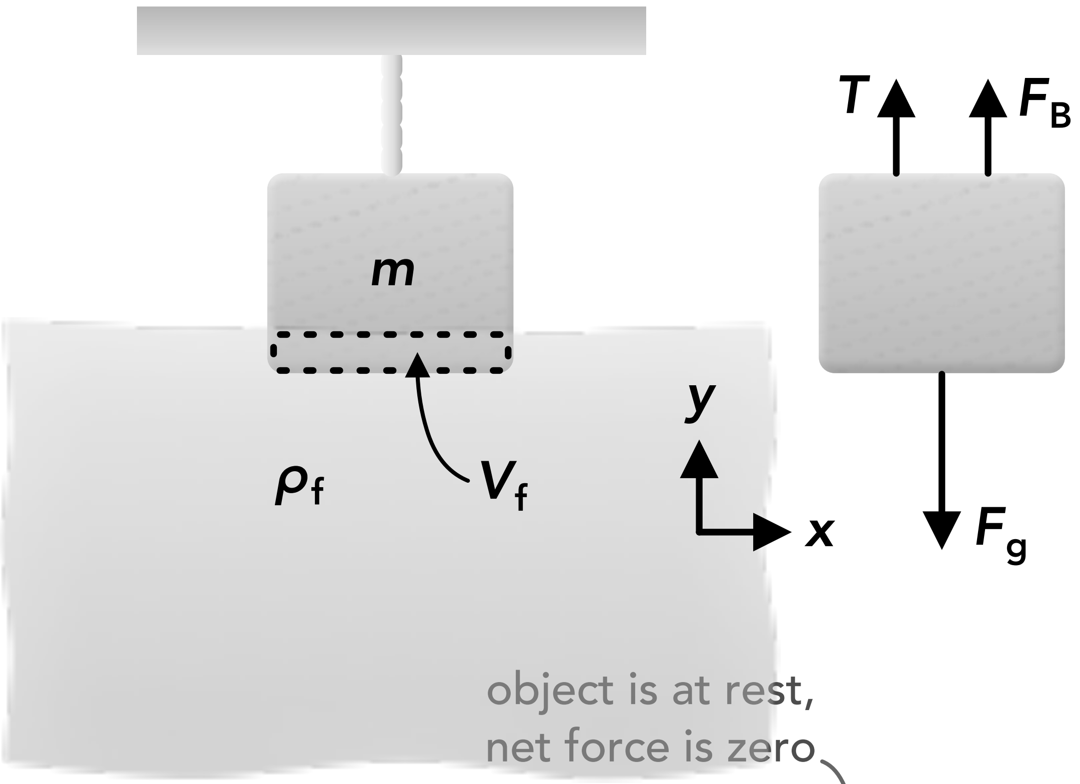
Free body diagram and Newton's 2nd law for an object floating in a fluid



object is at rest, net force is zero

$$\sum F_y = F_B - F_g = 0$$
$$F_B = F_g$$
$$\rho_f V_f g = mg$$

Free body diagram and Newton's 2nd law for an object floating in a fluid and suspended by a string

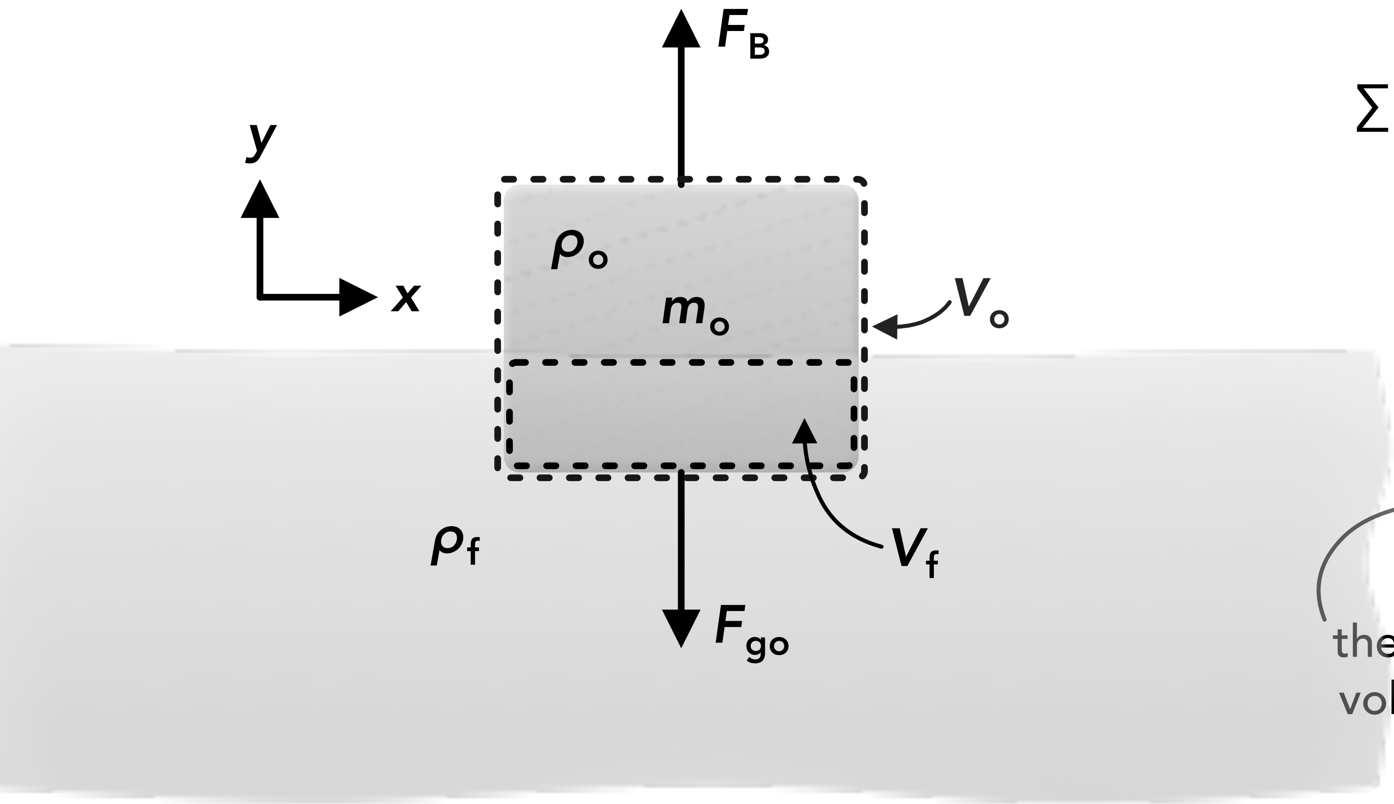


object is at rest, net force is zero

$$\sum F_y = F_B + T - F_g = 0$$
$$F_B + T = F_g$$
$$\rho_f V_f g + T = mg$$

- A common buoyant force question is to find the fraction of an object's volume that is submerged in the fluid if the object is floating at rest.
- We may also be asked to find the height of the object above or below the fluid surface. If the shape of the object is the same above and below the surface, like a cylinder or a rectangular prism, then the object's height is proportional to its volume, so the fraction of the object's volume submerged and height submerged are equal.
- We can draw a free body diagram, apply Newton's 2nd law ($\sum \boldsymbol{F} = m\boldsymbol{a}$) and set the net force to 0 because the object is floating at rest. The upwards buoyant force will be equal in magnitude to the downwards weight force. We may be given the density of the object instead of the mass, so we'll need to remember that the object's mass is equal to its density multiplied by the total volume of the object.

What fraction of the object's volume is submerged in the fluid?



$$\sum F_y = F_B - F_{go} = 0 \longleftarrow$$

object is at rest, net force is zero

$$F_B = F_{go}$$

$$\rho_f V_f g = m_o g$$

$$\rho_f V_f g = (\rho_o V_o) g \longleftarrow$$

mass of the object: $m_o = \rho_o V_o$

$$\frac{V_f}{V_o} = \frac{\rho_o}{\rho_f}$$

the fraction of the object's volume that is submerged