Vectors – Equations of Lines (HL)

No calculator allowed on all exercises [**answers** on next page]

1. The Cartesian equations of a line are $\frac{x-1}{3}=\frac{y+2}{2}=\frac{z-1}{3}$. Find the vector equation of the line.

2. The two lines, whose vector equations are given below, intersect. Find the point of intersection.

$$\mathbf{r}=\begin{pmatrix}3\\4\\1\end{pmatrix}+s\begin{pmatrix}4\\1\\0\end{pmatrix} \qquad \mathbf{r}=\begin{pmatrix}-1\\7\\5\end{pmatrix}+t\begin{pmatrix}12\\6\\3\end{pmatrix}$$

3. The position vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are $2\mathbf{i}-\mathbf{j}+\mathbf{k}$ and $\mathbf{i}+\mathbf{j}-\mathbf{k}$ respectively.

(a) Show that a vector equation for line (AB) can be written as $\mathbf{i}(2+\lambda)+\mathbf{j}(-1-2\lambda)+\mathbf{k}(1+2\lambda)$.

(b) There exists a point P on line (AB) such that $\overrightarrow{OP}$ is perpendicular to (AB). Find the coordinates of P.

(c) Hence, or otherwise, find the perpendicular distance from the origin to the line (AB).

4. Consider the two lines L_1 and L_2 with the following parametric equations:

$$L_1:\ x=-1-2\mu,\ y=\mu,\ z=2+3\mu \qquad L_2:\ x=2+\lambda,\ y=-\lambda,\ z=2-\lambda$$

(a) Show that lines L_1 and L_2 are skew.

(b) A third line, L_3, has the direction vector $\begin{pmatrix}2\\1\\1\end{pmatrix}$. Verify that L_3 is perpendicular to L_1 and L_2.

(c) Find parametric equations for L_3 given that it passes through the point $A(1,1,3)$.

(d) Find the coordinates of the point B where L_2 and L_3 intersect.

5. Find the coordinates of the point on the line L (equation below) which is nearest to the origin.

$$L:\ x=1-\lambda,\ y=2+3\lambda,\ z=3+\lambda$$

6. The points A, B and C have position vectors $\mathbf{a}=\begin{pmatrix}3\\-1\\4\end{pmatrix}$, $\mathbf{b}=\begin{pmatrix}0\\1\\-4\end{pmatrix}$ and $\mathbf{c}=\begin{pmatrix}6\\4\\5\end{pmatrix}$ respectively.

(a) Find the position vector of the point P on line (BC) such that $\overrightarrow{AP}$ is perpendicular to $\overrightarrow{BC}$.

(b) Hence, or otherwise, find the shortest distance from A to the line (BC).

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ANSWERS

1. $\mathbf{r} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}$

2. $(-17, -1, 1)$

3. (b) $P\left(\frac{4}{3}, \frac{1}{3}, -\frac{1}{3}\right)$

(c) $\sqrt{2}$ units

4. (c) $x = 1 - 2t,\ y = 1 - t,\ z = 3 - t$

(d) $(-1, 0, 2)$

5. $(2, -1, 2)$

6. (a) $\overrightarrow{OP} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}$

(b) $\sqrt{21}$ units