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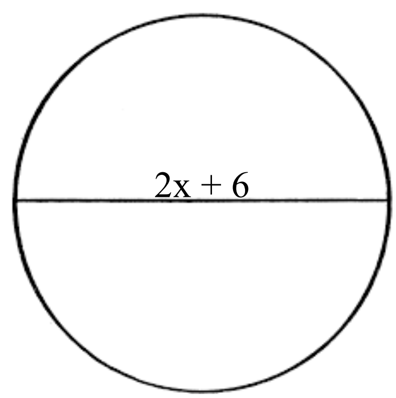
Algebra – Expressions and Formulas

The video covers the following exercises. Please print this sheet and work along!

$x = 4$ $y = -3$ $z = 2.5$	$x + y - 2z$	$2(x + y)$	$\frac{x^2 - 4y}{2 - 4z}$

Please write the appropriate expression for the area of the given circle:

$$A = \pi r^2$$



Algebra 2
Chapter 1 Notes

Name ______________________

Date ______________________

EQUATIONS AND INEQUALITIES

1.1: Expressions and Formulas

Order of Operations – PEMDAS

Parenthesis **E**xponents **M**ultiplication/**D**ivision **A**ddition **S**ubtraction

KeyConcept Order of Operations

Step 1 Evaluate the expressions inside grouping symbols.

Step 2 Evaluate all powers.

Step 3 Multiply and/or divide from left to right.

Step 4 Add and/or subtract from left to right.

Ex#1: Evaluate the following expressions if $m = 12$ and $q = -1$

a) $m + (3 - q)^2$

b) $m + 2q + 4$

Ex#2: Evaluate the following expressions if $a = 5$ and $b = -3.2$

c) $a + b^2(b - a)$

Ex#3: Evaluate the following expression if $h = 4$, $j = -1$, and $k = 0.5$

$$\frac{j^2 - 3h^2k}{j^3 + 2}$$

Formula – a mathematical "sentence" that creates relationships between certain values

The formula $F = \frac{9}{5}C + 32$ represents the conversion of temperature from Celsius to Fahrenheit.

Ex#4: What is the Fahrenheit equivalent of $40^\circ C$?

Ex#5: What is the Celsius equivalent of $41^\circ F$?

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Algebra – Properties of Real Numbers

The video covers the following information. Please print this sheet and work along!

List of number categories:

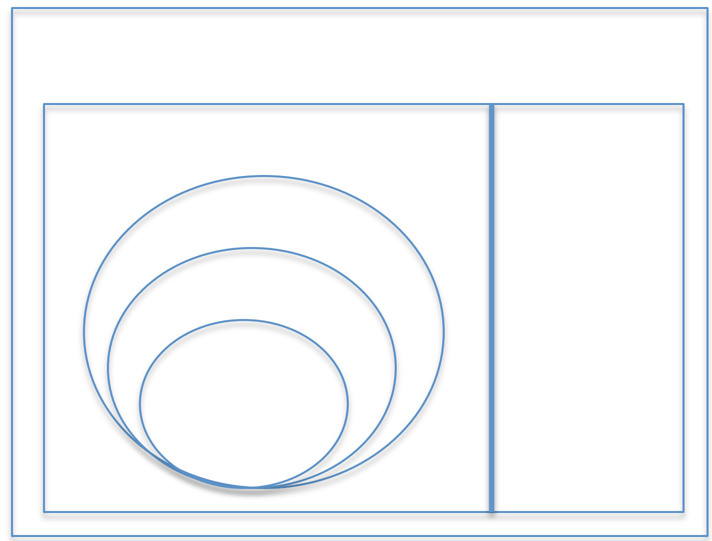
Natural #s –

Whole #s –

Integer #s –

Rational #s –

Irrational #s –



Please list which number categories each of the following are:

5

1.2

$1.\overline{22}$

–3

$\sqrt{36}$

$\sqrt{37}$

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Algebra – Properties of Numbers

The video covers the following exercises. Please print this sheet and work along!

Reflexive Property –

Symmetric Property –

Transitive Property –

Substitution Property –

a + _____ = a is the ________________________________

a · _____ = a is the ________________________________

a + _____ = 0 is the ________________________________, example: -3 + _____ = 0

a · _____ = 1 is the ________________________________, example: 4 · _____ = 1

↖ This number is often called the ___________________.

_______ · _______ · _______ · _______ · _______ · _______ · _______ = _______

This is called The ___________________ Property of _______.

Commutative Property –

Associative Property –

Please perform the math operations, and state the property you used in each step.

$$6(3-2)+4\cdot\frac{1}{4}+12(7-7)$$

$$6(1)\ +4\cdot\frac{1}{4}+12(7-7)$$ ____________________

$$6(1)\ +4\cdot\frac{1}{4}\ +\ 12(0)$$ ____________________

$$6\ +\ 4\cdot\frac{1}{4}\ +\ 12(0)$$ ____________________

$$6\ +\ 1\ +\ 12(0)$$ ____________________

$$6\ +\ 1\ +\ 0$$ ____________________

$$7\ +\ 0$$ ____________________

$$7$$ ____________________

Example	x = ?	Property
7 · x = 0		
x + 3 = 10 + 3		
4 + x = 4		
2 + 10 = 10 + x		
4 + x = 0		
x · 1 = 5 OR 1 · x = 5		
5 · x = 1		

1.2: Properties of Real Numbers

Real numbers are classified in a variety of ways.

Natural numbers: 1, 2, 3, …

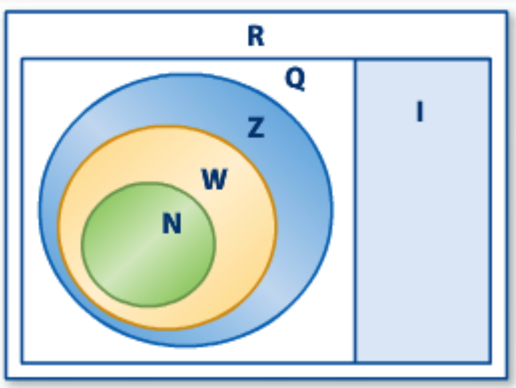
Whole numbers: all Natural numbers, and 0. So, 0, 1, 2, 3, …

Integers: all Whole numbers, and the negative countable numbers: … , -3, -2, -1, 0, 1, 2, 3, …

Rational numbers: all Integers, and *ratios* of integers, so fractions, ending decimals, and repeating decimals

Irrational numbers: cannot be represented by a ratio of integers. They're decimals that continue on without a pattern. Common examples include √ and π.

KeyConcept Real Numbers (R)



Letter	Set	Examples
Q	rationals	$0.125, -\frac{7}{8}, \frac{2}{3} = 0.66\ldots$
I	irrationals	$\pi = 3.14159\ldots$ $\sqrt{3} = 1.73205\ldots$
Z	integers	$-5, 17, -23, 8$
W	wholes	$2, 96, 0, \sqrt{36}$
N	naturals	3, 17, 6, 86

Ex#1: Name all of the sets of numbers to which each number belongs.

a) -185

b) $\sqrt{49}$

c) $\sqrt{95}$

d) $-\frac{7}{8}$

e) 0

f) $0.5\overline{8}$

Real Number Properties (and Examples)

For any real numbers, a, b, and c		
Property	Addition	Multiplication
Commutative	a + b = b + a	a • b = b • a
Associative	(a + b) + c = a + (b + c)	(a • b) • c = a • (b • c)
Identity	a + 0 = a	a • 1 = a
Inverse	a + (-a) = 0	$a \bullet \frac{1}{a}$
Distributive	a(b + c) = ab + ac	

Ex:#2: Please name the property illustrated by each of the following.

a) $(6 \cdot 8) \cdot 5 = 6 \cdot (8 \cdot 5)$

b) $84 + 16 = 16 + 84$

c) $(12 + 5)6 = 12 \cdot 6 + 5 \cdot 6$

Ex#3: Please find the additive and multiplicative inverses of each of the following numbers.

a) -7

b) 0.8 *(hint: turn into a fraction)*

Ex#4: Please simplify the following expressions.

a) $-2a + 4a(8 - 3a)$

b) $3(4x - 2y) - 2(3x + y)$

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Algebra – Solving Equations

The video covers the following exercises. Please print this sheet and work along!

Math Property:

Reflexive –

Symmetry –

Transitive –

Substitution –

Addition –

Subtraction –

Multiplication –

Division –

$2x-1=13$

$\frac{2}{3}x=30$

if $3x - 3 = 1/4$, then what is $3x + 7$?

$$V = \frac{1}{3}\pi r^2 h$$

Please solve for h.

1.3a: Solving Equations

Translating Verbal Expressions and Algebraic Expressions

Ex#1:

a) Please translate the verbal expressions into an algebraic expressions.

three times the difference of a number and eight

the cube of a number increased by 4 times the same number

b) Please translate the algebraic expression into a verbal expression.

$p^3 + 4p$

Ex#2: Please write a verbal sentence to represent the equation.

$2c = c^2 - 4$

Properties of Equality – common math operations, used to solve equations

For any real numbers, a, b, and c		
Property	Using only symbols	Additional examples
Reflexive	a = a	b + 8 = b + 8
Symmetric	If a = b, then b = a	If 2b + c = 20, Then 20 = 2b + c
Transitive	If a = b, and b = c, then a = c	If 2a + 12 = 30, and 30 = 5c – 8, then 2a + 12 = 5c – 8
Substitution	If a = b, then a can be replaced by b b can be replaced by a	If **(5 + 2)**x = 21, Then **7**x = 21

Ex#3: Please name the property illustrated by the following statement.

If $-11a+2=-3a$, then $-3a=-11a+2$

Additional **Properties of Equality**

"Whatever operation you do to one side of the equation, you must do to the other."

For any real number 'a'	
Property	Example
Addition	if a = a then a **+ 8** = a **+ 8**
Subtraction	if a = a then a **– 4** = a **– 4**
Multiplication	if a = a then a **• 3** = a **• 3**
Division	if a = a then a **÷ 7** = a **÷ 7**

Ex#4: Please solve the following equations, noting which property of equality is being utilized.

a) $x-14.29=25$

b) $\frac{2}{3}y=-18$

c) $-10x+3(4x-2)=6$

Ex#5: Please solve for *h* in the following formula for area of a trapezoid. $A=\frac{1}{2}h(b_1+b_2)$

Please note the property used for each step.

Algebra – Solving Equations (word problem)

If Suzy sells a total of 50 fruits in a day, and sells 8 more apples than plums…

(what will be
the question?)

1.3b: Solving Equations (word problems)

Ex. #1: Suppose that in my coffee shop, one day I sell 12 *more* regular coffees than decaffeinated. The total cups I sold that day were 60. How many of each kind of coffee did I sell?

(Hint: you can either play around with numbers to guess and check, or assign variables, such as D for the number of decaf cups sold.)

Ex. #2: Supplementary angles are defined as 2 angles that sum to 180º. Suppose that one angle is 3 times larger than its supplement. What are the measures of the 2 angles?

(Same hint as above. Maybe start with 100º and 80º. They're supplementary, but 100 is not 3 times as large as 80. So tinker with the numbers until one angle is 3 times larger than the other. Or, you can set variables to represent each of the 2 angles.)

Algebra
Absolute Value Equations
YAY MATH!

The following problems are solved in the video:

$|x+6|=18$

$\left|\frac{1}{2}x-1\right|=2$

$3|x+6|=36$

$|3x-1|=-450$

$|3t-5|=2t$

$|x-3|+7=2$

1.4: Solving Absolute Value Equations

The ***absolute value*** of a number is its *distance from zero* on a number line. Since distance is always non-negative, absolute values are always non-negative.

Symbol: $|x|$

Another way of understanding it is that the absolute value bars are like a "positivity machine." Any number that enters the positivity machine will come out *positive*. Zero will come out as zero.

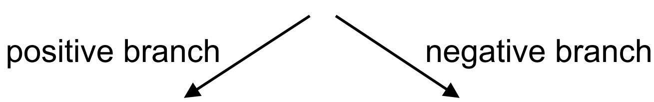
Ex #1: Please evaluate the following if $x = -2$.

a. $|4x+3|-3\frac{1}{2}$

b. $-2|3-x|+8$

Solving Absolute Value Equations – "BIFURCATE" – meaning, dividing into two branches

$$|x-5|=3$$

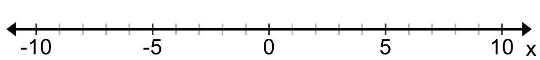


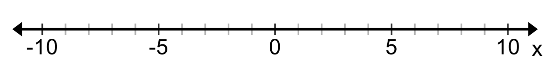
(then solve both branches)

Ex #2: Please solve each equation. Then graph your solution(s) on a number line.

a) $|x+3|=6$

b) $|x-7|=4$





No solution?

We know that an absolute value is always equal to a positive number.

Thus, whenever an absolute value equation equals a *negative number*, there is ***no solution***.

Here are some examples of an equation having "no solution" for the variable, 'a'.

$|a| = -8$ (there is no number that a can be that would make the equation true)

$-2|3a| = 8$ (divide both sides by –2, to see that abs. value = neg.)

Ex #3: **Extraneous Solutions** – When an absolute value expression is set equal to an expression containing a variable, ***extraneous solutions*** may be encountered.

(Hint: first combine like terms. Then isolate the absolute value. Then bifurcate, and solve each.)

$$2|x+1| - x = 3x - 4$$

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Algebra – Solving Inequalities

The video covers the following exercises. Please print this sheet and work along!

Add 3 to both sides

$2 < 6$

Divide both sides by 2

$2 < 6$

Multiply both sides by –1

$2 < 6$

$3x + 1 > 22$

$-3x + 1 > 22$

$10 > -2x$

$$x \leq \frac{3-x}{2}$$

$$\frac{2x-6}{4} > \frac{x-3}{2}$$

(please circle one)

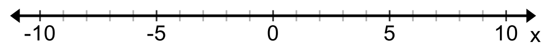
"at least" means:	$<$	$\leq$	$\geq$	$>$
"at most" means:	$<$	$\leq$	$\geq$	$>$
"no more than" means:	$<$	$\leq$	$\geq$	$>$
"no less than" means:	$<$	$\leq$	$\geq$	$>$

1.5: Solving Inequalities

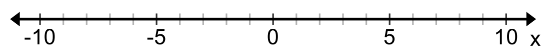
(circle one)
Adding and subtracting the same amount to each side of an inequality **DOES / DOES NOT** reverse the direction of the inequality sign.

Ex#1: Please solve the inequalities. Then graph the solution set.

a) $5x - 3 > 4x + 2$



b) $4x - 15 \leq 21$

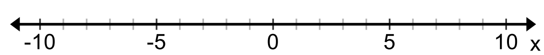


(circle one)
Multiplying or dividing by a ***positive number*** DOES / DOES NOT reverse the inequality sign.

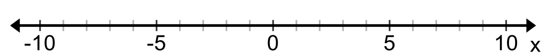
Multiplying or dividing by a ***negative number*** DOES / DOES NOT reverse the inequality sign.

Ex#2: Please solve and graph on the number line.

a) $-4.2x \leq 29.4$



b) $-3x \leq \frac{-4x + 22}{5}$



Algebra
Absolute Value Inequalities
YAY MATH!

The following problems are solved in the video:

$3x + 1 < 7$ OR $7 < 2x - 9$

$|x+2| > 3$

$|2x-9| \leq 27$

$|5x| + 10 < 3$

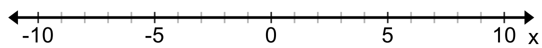
$|5x| > -7$

1.6: Solving Compound and Absolute Value Inequalities

A ***compound inequality*** consists of two inequalities joined by the word *"and"* or *"or."*

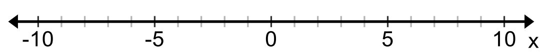
$$x \geq -4 \text{ and } x < 3$$

The compound inequality above involves "and". This means that BOTH statements need to be true. How would you graph all the numbers that are BOTH ≥ –4 *and* < 3?



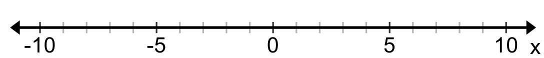
***"And"* inequalities** may also be rewritten in the following ways:

$4x+8 \geq -12 \quad and \quad 4x+8 \leq 32$ can be condensed to: $-12 \leq 4x+8 \leq 32$



Ex#1: Please solve and graph.

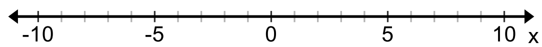
$$-5 \geq 3x-2 > -14$$



"Or" Inequalities is the ***union*** of the solution sets.

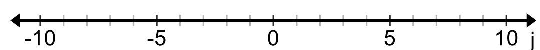
$$x \geq 5 \text{ or } x < -3$$

The compound inequality above involves "or". This means that ONE or BOTH of the statements need to be true. How would you graph all the numbers that are EITHER ≥ 5 *or* < –3?



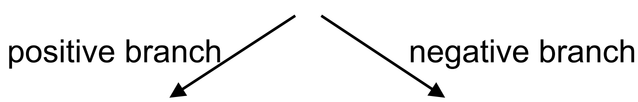
Ex#2: Please solve and graph the inequality.

$$5j \geq 15 \quad \text{or} \quad -3j \geq 21$$



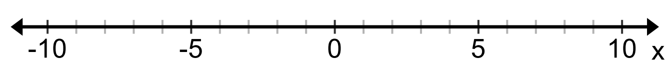
Absolute Value Inequalities – time to BIFURCATE into 2 separate statements

$$|x| < 3$$

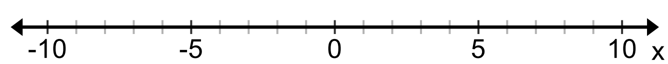


Ex#3: Please solve and graph.

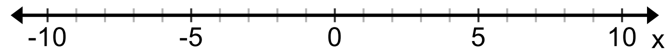
a) $|x| < 6$



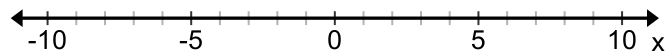
b) $|x| \geq 6$



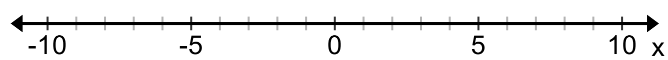
c) $|x-4| \le 6$



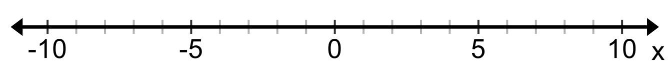
d) $|x+7| > 2$



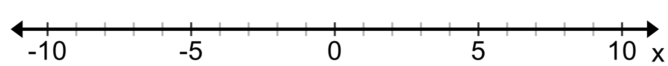
e) $|8x+3| \le 4$



f) $|x+3| \le -6$ *(Hint: can an absolute value expression ever be less than –6?)*



g) $|x+3| > -6$ *(Hint: how often is an absolute value expression greater than –6?)*



Remember to look for open circles or closed circles to decide which inequality to use, < vs ≤
> vs ≥

Let's check for understanding:

(circle one)

When using graphs, *open circles* over the numbers DO / DON'T include "or equal to" (as in, ≤)

When using graphs, *closed circles* over the numbers DO / DON'T include "or equal to"

To create an absolute value inequality, use this guide for "AND" problems:

$$\left|x - \text{middle \#}\right| \le \text{distance from middle to each value}$$

And use this for "OR" problems:

$$\left|x - \text{middle \#}\right| > \text{distance from middle to each value}$$

Remember this fun guide:

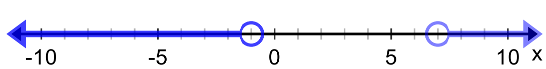
<, ≤ less than "less th<u>AND</u>"

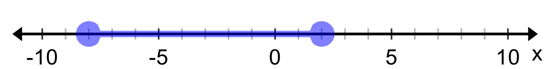
(so these abs. val. inequalities involve AND)

>, ≥ greater than "great<u>OR</u>"

(these abs. val. inequalities involve OR)

Ex#4: What is the absolute value inequality represented in each graph below?





a) ___________________

b) ___________________

Algebra 2 Name ___________________________
Chapter 1 Practice Test Date ___________________________

Please evaluate each expression.

1) $\frac{16-3\cdot 2}{1+4}$ 1) _____________

2) $21+[6-12\div 3]$ 2) _____________

3) $\frac{3}{4}(11-7)^2$ 3) _____________

Please evaluate each expression if $a = 3$, $b = -4$, and $c = \frac{1}{4}$.

4) $a^2(b-a)$ 4) _____________

5) $\frac{8c+ab}{c}$ 5) _____________

Please complete the table below by placing a check mark or X to indicate all sets of numbers that apply to the value of each expression.

		R real	**I** irrational	**Q** rational	**Z** integer	**W** whole	**N** natural
6)	0.4						
7)	$\sqrt{\frac{1}{4}}$						
8)	$-\sqrt{7}$						
9)	-15						

10) What are the additive and multiplicative inverses of $1\frac{2}{3}$? 10) Additive: _________

Multiplicative: _____

Please name the property illustrated by each equation or statement.

11) If $x - 2 = 5$, then $x = 7$. 11) ____________

12) $(3 \cdot 4) \cdot 9 = 3 \cdot (4 \cdot 9)$ 12) ____________

13) If $a = b$ and $b = -2$, then $a = -2$. 13) ____________

Please solve each equation or formula for the specified variable.

14) $y(x+z) - v = 3d$ for y 14) ____________

15) $\frac{10z + x}{y} = 4$ for x 15) ____________

Please solve each equation.

16) $6m - 4 = -46$ 16) ____________

17) $\frac{d}{2} + \frac{d}{4} = 3$ 17) ____________

18) $5-(2w-8)=6w-9$

18) ______________

19) $|x-3|=1$

19) ______________

20) $2|3e-2|=14$

20) ______________

21) $|3x-8|=-15$

21) ______________

Please solve each inequality. Then graph the solution set on a number line.

22) $-3y - 4 \geq -7$

22) ____________

23) $|2x + 3| \geq 11$

23) ____________

24) $|3x - 4| < -7$

24) ____________

25) $2a + 12 \leq 6$ or $3a - 1 > -13$

25) ____________