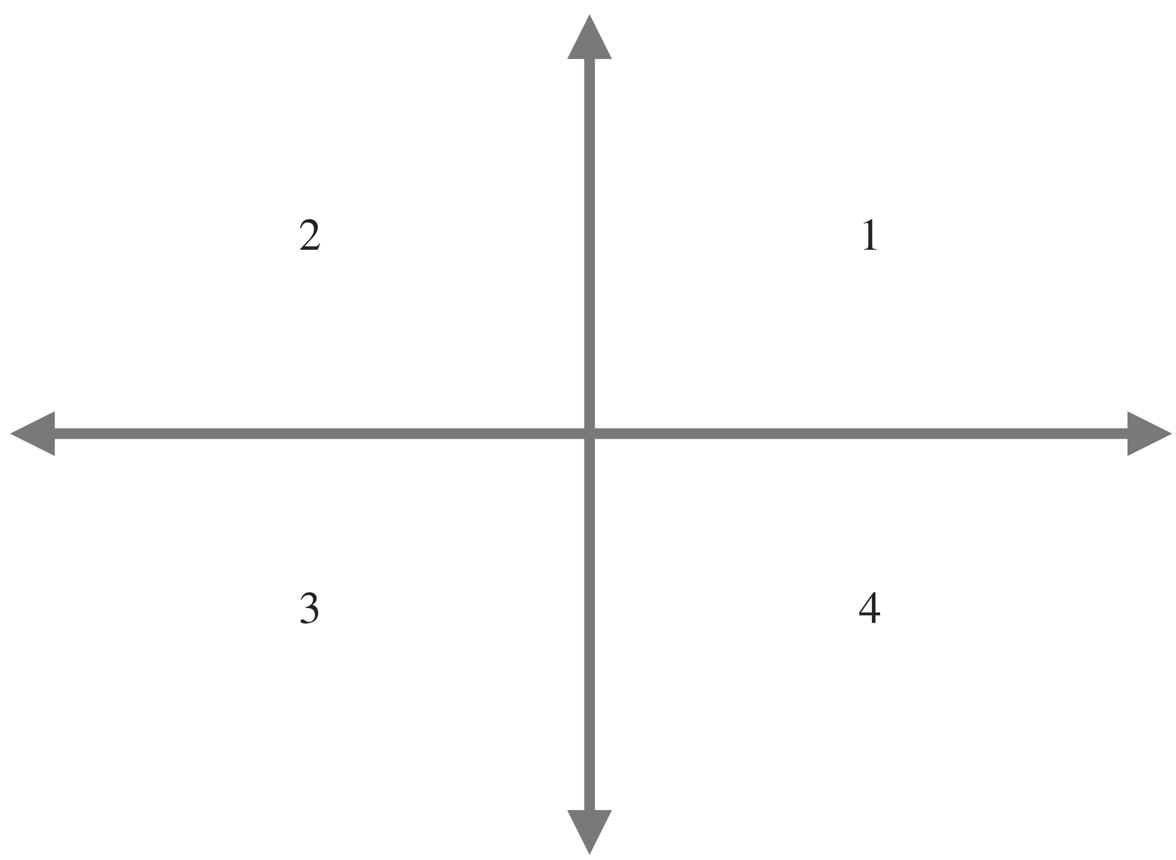
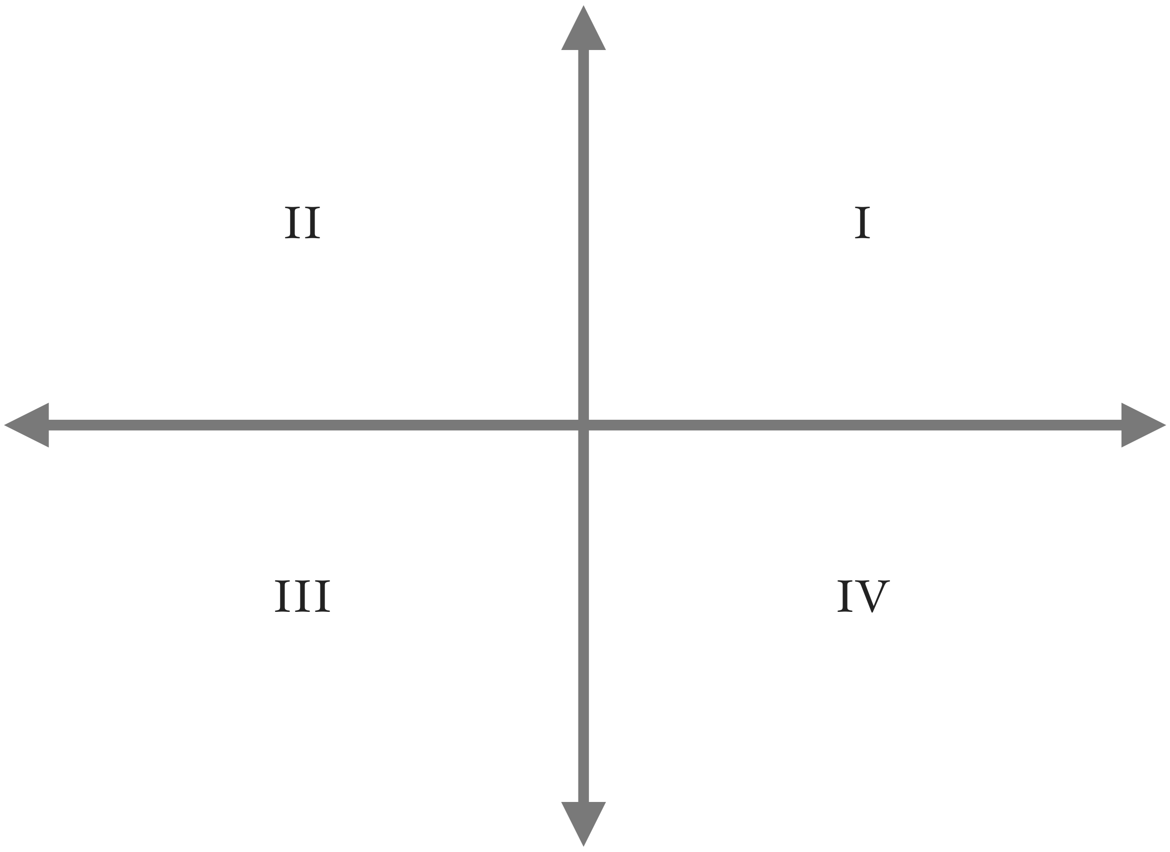
Quadrant of the angle

The horizontal and vertical axes in a two-dimensional coordinate system divide the rest of two-dimensional space (everything but the axes) into four parts. Each of these four parts is known as a quadrant.

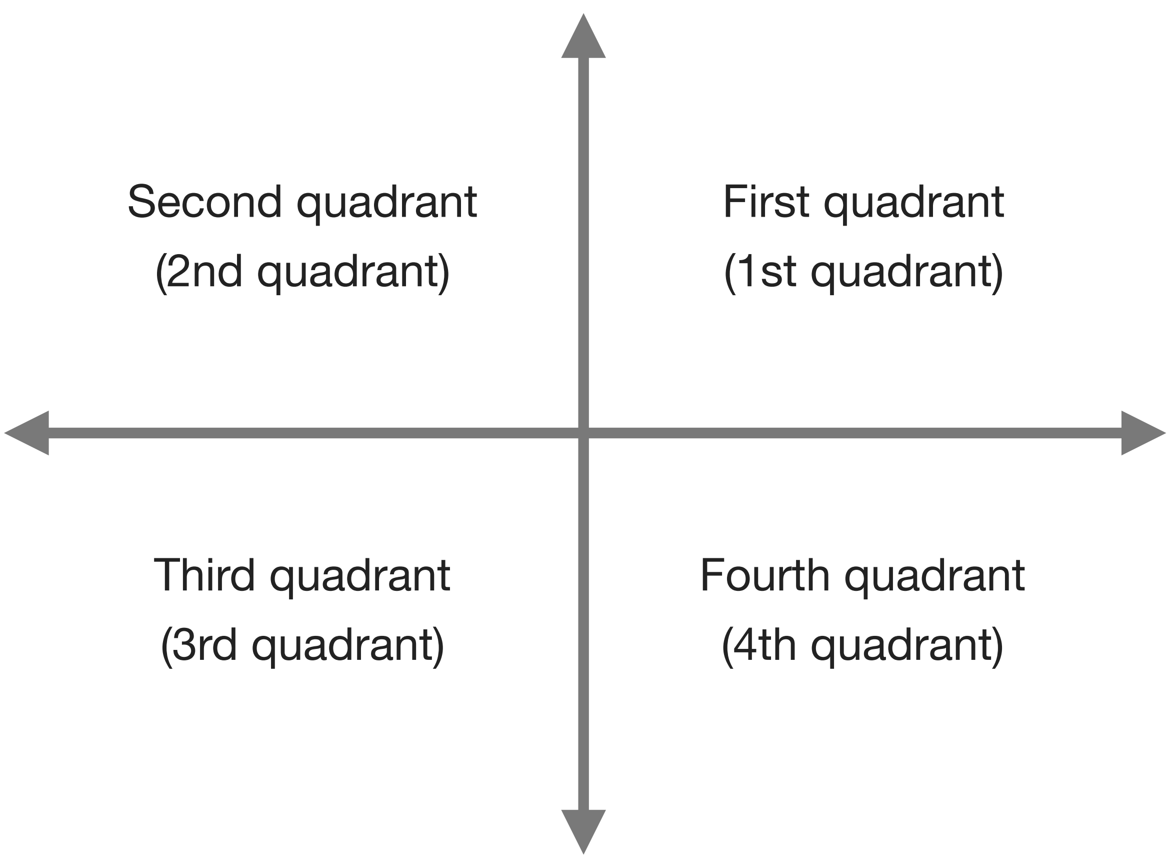
We label the quadrants with the numbers 1 through 4, beginning with the quadrant between the positive horizontal axis and the positive vertical axis (which is quadrant 1), and continuing in the positive (counterclockwise) direction to quadrants 2, 3, and 4.



Sometimes, the Roman numerals I through IV are used to label the quadrants.



We often refer to the quadrants as the first (or 1st) quadrant, the second (or 2nd) quadrant, the third (or 3rd) quadrant, and the fourth (or 4th) quadrant.



When we draw an angle in standard position, it is customary to state which quadrant the angle is in. By that we mean which quadrant the terminal side of the angle is in. There's no need to state which quadrant the initial side of any angle is in, because the initial side of any angle (in standard position) is on the positive horizontal axis.

The coordinate axes are the boundaries between quadrants, so they aren't part of any quadrant. Therefore, an angle whose terminal side is located on any of the coordinate axes isn't in any of the four quadrants. The four ``principal'' angles that are located on the coordinate axes are as follows:

Axis	**Principal angle (deg)**	**Principal angle (rad)**
Positive horizontal axis	0	0
Positive vertical axis	90	$\pi/2$
Negative horizontal axis	180	π
Negative vertical axis	270	$3\pi/2$

Using that table, we can find the range of angles θ with $0^\circ < \theta < 360^\circ$ (in radians: $0 < \theta < 2\pi$) that are located in each of the four quadrants.

Quadrant	**Range of angles (deg)**	**Range of angles (rad)**
First quadrant	$0 < \theta < 90$	$0 < \theta < \pi/2$
Second quadrant	$90 < \theta < 180$	$\pi/2 < \theta < \pi$
Third quadrant	$180 < \theta < 270$	$\pi < \theta < 3\pi/2$
Fourth quadrant	$270 < \theta < 360$	$3\pi/2 < \theta < 2\pi$

Of course, any angle that differs from one of the “principal” angles by a (positive or negative) integer multiple of 360° (in radians: 2π) is located on the same axis as that principal angle.

Positive horizontal axis

Angles (deg): $0,\ \pm 360,\ \pm 720,\ \pm 1{,}080...$

Angles (rad): $0,\ \pm 2\pi,\ \pm 4\pi,\ \pm 6\pi\ldots$

Positive vertical axis

Angles (deg): $90,\ 450,\ 810,\ 1{,}170...$ and $-270,\ -630,\ -990,\ -1{,}350...$

Angles (rad): $\frac{\pi}{2},\ \frac{5\pi}{2},\ \frac{9\pi}{2},\ \frac{13\pi}{2}\ldots$ and $-\frac{3\pi}{2},\ -\frac{7\pi}{2},\ -\frac{11\pi}{2},\ -\frac{15\pi}{2}\ldots$

Negative horizontal axis

Angles (deg): $\pm 180,\ \pm 540,\ \pm 900,\ \pm 1{,}260...$

Angles (rad): $\pm\pi,\ \pm 3\pi,\ \pm 5\pi,\ \pm 7\pi\ldots$

Negative vertical axis

Angles (deg): $270,\ 630,\ 990,\ 1{,}350...$ and $-90,\ -450,\ -810,\ -1{,}170...$

Angles (rad): $\frac{3\pi}{2},\ \frac{7\pi}{2},\ \frac{9\pi}{2},\ \frac{13\pi}{2}\ldots$ and $-\frac{\pi}{2},\ -\frac{5\pi}{2},\ -\frac{9\pi}{2},\ -\frac{13\pi}{2}\ldots$

If you have an angle θ with $0^\circ < \theta < 360^\circ$ (in radians: $0 < \theta < 2\pi$) and you want to determine the quadrant in which θ is located, all you need to do is figure out the two angles in the range 0 to 360 degrees (in radians: in the range 0 to 2π) which are located on axes and are closest in measure to θ. Then θ is between those two axes, which means that it's located in the quadrant bounded by them.

Example

Determine the quadrant in which an angle of 283° is located.

Note that

$$270^\circ < 283^\circ < 360^\circ$$

An angle of 270° is on the negative vertical axis, and an angle of 360° is on the positive horizontal axis. Thus an angle of 283° is between the negative vertical axis and the positive horizontal axis, which means that it's in the fourth quadrant.

If you have an angle θ that isn't in the range 0 to 360 degrees (in radians: in the range 0 to 2π), what you can do is add an appropriate integer multiple of 360° (in radians: an appropriate integer multiple of 2π) to θ, to get an angle α in the range 0° to 360° (in radians: 0 to 2π). If α is in one of the four quadrants, then θ is in the same quadrant as α; otherwise, α is on one of the axes, and θ is on the same axis as α.

Example

In which quadrant (or on which axis) is an angle of $-(33/5)\pi$ radians located?

The improper fraction $-(33/5)$ can be written as

$$-\frac{33}{5} = -\frac{30+3}{5} = -\left(\frac{30}{5} + \frac{3}{5}\right) = -6\frac{3}{5}$$

Therefore, an angle of $-(33/5)\pi$ radians isn't in the range 0 to 2π radians, but you can find an angle in the range 0 to 2π radians that differs from an angle of $-(33/5)\pi$ radians by some integer multiple of 2π radians. Since the given angle is negative, you will need to add positive integer multiples of 2π to $-(33/5)\pi$ to find such an angle.

$$-\left(6\frac{3}{5}\right)\pi + 2\pi = -\left(4\frac{3}{5}\right)\pi$$

$$-\left(6\frac{3}{5}\right)\pi + 2(2\pi) = -\left(4\frac{3}{5}\right)\pi + 2\pi = -\left(2\frac{3}{5}\right)\pi$$

$$-\left(6\frac{3}{5}\right)\pi + 3(2\pi) = -\left(2\frac{3}{5}\right)\pi + 2\pi = -\left(\frac{3}{5}\right)\pi$$

One more addition of 2π, and you'll get the angle you're looking for:

$$-\left(6\frac{3}{5}\right)\pi+4(2\pi)=-\left(\frac{3}{5}\right)\pi+2\pi=-\left(\frac{3}{5}\right)\pi+\left(\frac{10}{5}\right)\pi=\frac{7}{5}\pi$$

Note that

$$\pi<\frac{7}{5}\pi=1.4\pi<1.5\pi=\frac{3}{2}\pi$$

An angle of π radians is on the negative horizontal axis, and an angle of $(3/2)\pi$ radians is on the negative vertical axis. Thus an angle of $(7/5)\pi$ radians is between the negative horizontal axis and the negative vertical axis, which means that it's in the third quadrant. Since the original angle, $-(33/5)\pi$ radians, differs from $(7/5)\pi$ by an integer multiple of 2π radians, an angle of $-(33/5)\pi$ radians is also in the third quadrant.

Now let's deal with an angle in radians whose measure isn't given as a constant multiple of π.

Example

In which quadrant (or on which axis) is an angle of 21.9 radians located?

Division of 21.9 by π gives approximately 6.97, so $21.9\approx 6.97\pi$. Now this is outside the range 0 to 2π, and it's positive, so you will need to subtract positive integer multiples of 2π from 6.97π to get an angle that's in the range 0 to 2π.

$$6.97\pi-2\pi=4.97\pi$$

$$6.97\pi-2(2\pi)=4.97\pi-2\pi=2.97\pi$$

One more step will do the trick:

$$6.97\pi - 3(2\pi) = 2.97\pi - 2\pi = 0.97\pi$$

Note that

$$\frac{1}{2}\pi = 0.5\pi < 0.97\pi < \pi$$

An angle of $(1/2)\pi$ radians is on the positive vertical axis, and an angle of π radians is on the negative horizontal axis. Thus an angle of 21.9 (approximately 6.97π) radians is between the positive vertical axis and the negative horizontal axis, which means that it's in the second quadrant.