SIMPLE HARMONIC MOTION

Mass-Spring Simple Harmonic Motion

1. For any mass-spring system, the equilibrium position is at which point? (Select all that apply)
   - A The point of maximum acceleration
   - B The point where the net force on the mass is zero
   - C The point where the acceleration is zero
   - D The point where the spring force on the mass is zero

2. A block is attached to the bottom of a vertical spring hanging from the ceiling. The block is at rest and is then lifted up 10 cm and released, and it begins to oscillate up and down. What is the amplitude of the oscillation?
   - A 5 cm
   - B 10 cm
   - C 20 cm
   - D -10 cm

3. Two blocks are attached to two separate but idential springs to form two horizontal mass-spring systems. Block A has a mass of m_A and block B has a mass of $3m_A$. Block A oscillates back and forth with an amplitude of A_A and block B oscillates with an amplitude of $2A_A$. If block A oscillates with a period of T_A, what is the period of the oscillation of block B in terms of T_A?
   - A $\sqrt{3}\,T_A$
   - B $\sqrt{6}\,T_A$
   - C $3T_A$
   - D $9T_A$

4. A block is hanging from a vertical spring. The block is pulled down some distance and released, and the block oscillates up and down. If the same block was attached to a new spring with a greater spring constant and it was pulled down the same distance and released, how would the maximum speed of the oscillation with the new spring compare to the maximum speed of the oscillation with the first spring?
   - A The maximum speed would be the same
   - B The maximum speed would be slower
   - C The maximum speed would be faster
   - D Cannot be determined

5. A block is oscillating up and down on vertical spring and a meterstick is placed next to the oscillation. The highest point that the center of the block reaches is 92 cm above the ground, and the lowest point that the center of the block reaches is 62 cm above the ground. What is the amplitude of the oscillation?
   - A 95 cm
   - B 65 cm
   - C 30 cm
   - D 15 cm

6. A block is attached to a horizontal spring and oscillates back and forth on a frictionless surface. At what point in the oscillation does the block have zero kinetic energy? (Select all that apply)
   - A At the equilibrium position
   - B At the maximum displacement
   - C At the point where the net force on the block is zero
   - D At the point of maximum acceleration

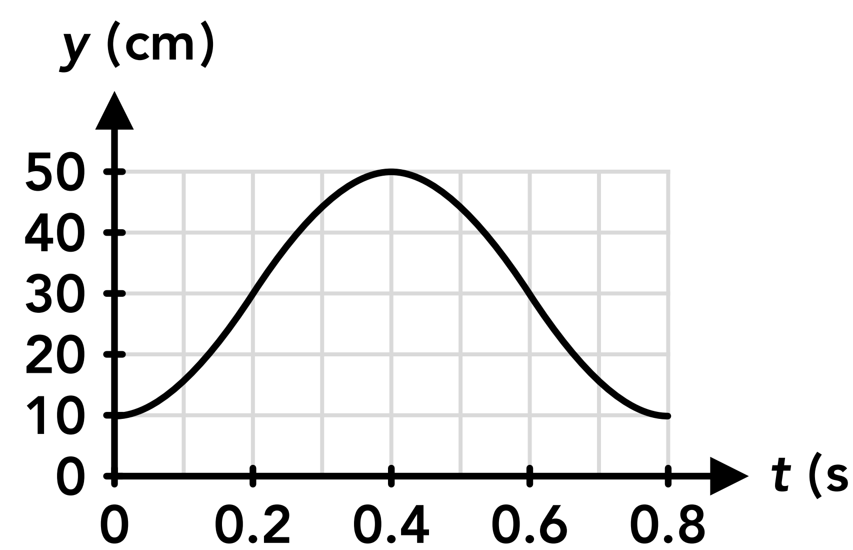
7. Two blocks are oscillating up and down on two separate but identical springs. Block A has more mass than block B. Which block oscillates up and down with a higher frequency?
   - A Block A
   - B Block B
   - C They have the same frequency
   - D Cannot be determined

8. A block is attached to a horizontal spring and oscillates back and forth between a position of 28 cm and 44 cm. At what position is the block moving the fastest?
   - A 28 cm
   - B 44 cm
   - C 36 cm
   - D 8 cm

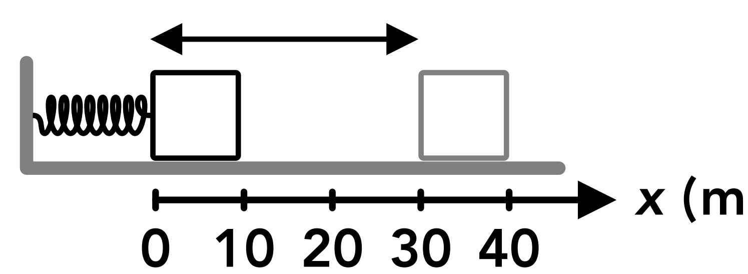
9. A block oscillates up and down on a vertical spring. At which point is the spring potential energy the greatest?
   - A At the higher amplitude
   - B At the equilibrium position
   - C At the point of maximum speed
   - D At the lower amplitude

10. For a vertical mass-spring oscillation, the spring force is always zero at which point? (Select all that apply)
   - A The equilibrium position
   - B The upper amplitude
   - C The lower amplitude
   - D None of the above

11. A mass is oscillating up and down on a vertical spring and its motion is shown in the graph on the right. At which of the following times is the mass at its maximum speed? (Select all that apply)
   - A 0 s
   - B 0.2 s
   - C 0.4 s
   - D 0.6 s



12. A block and a spring oscillate back and forth between two positions as shown on the right. What is the position of the left end of the block when the acceleration of the block is zero?
   - A 0 cm
   - B 10 cm
   - C 15 cm
   - D 30 cm



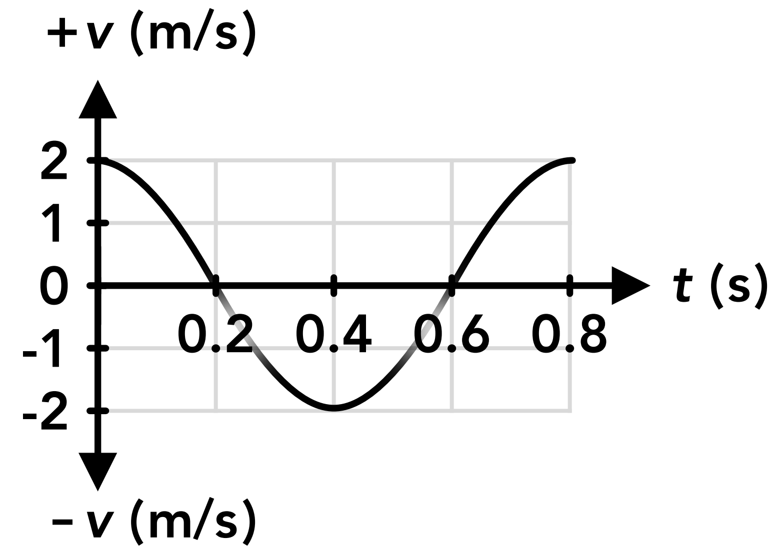
13. A block oscillates up and down on a vertical spring and its motion is shown in the graph on the right. At which of the following times is the point of maximum net force on the block? (Select all that apply)

A 0 s

B 0.2 s

C 0.4 s

D 0.6 s



14. A 2 kg block is attached to a horizontal spring with a spring constant of 200 N/m. The block is pulled 30 cm away from its equilibrium position and released from rest. What is the frequency of the resulting oscillation?

15. A 5 kg block hangs at rest on a spring with a spring constant of 300 N/m. The block is lifted 10 cm and released from rest. As the block falls, what is the maximum speed of the block?

16. A block is attached to a horizontal spring with a spring constant of 150 N/m. If it takes 0.8 seconds for the block to move from one amplitude to the other, what is the mass of the block?

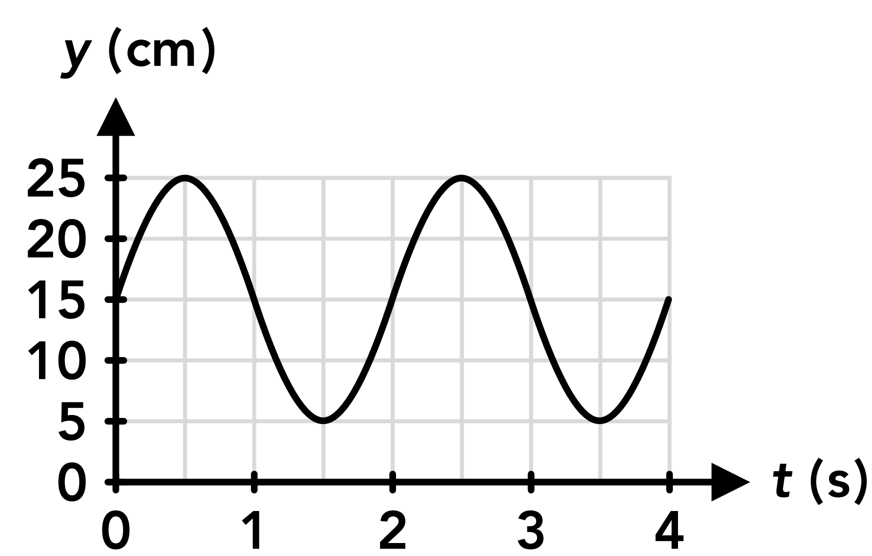
17. A block oscillates back and forth on a frictionless surface with a frequency of 0.5 Hz. It is attached to a horizontal spring with a spring constant of 100 N/m. During the oscillation, the maximum speed of the block is 1.5 m/s. How far does the block travel from the left end to the right end of the oscillation?

18. A 6 kg block oscillates up and down on a vertical spring with a spring constant of 250 N/m. The block oscillates between a height of 160 cm and 180 cm. What is the maximum kinetic energy of the block during its motion?

19. A 3 kg block oscillates back and forth on a horizontal spring with a spring constant of 120 N/m. The amplitude of the oscillation is 15 cm. What is the maximum spring potential energy of the system during the motion?

20. A 7 kg block oscillates back and forth on a horizontal spring with a frequency of 2 Hz. The distance between the left and right ends of the oscillation is 28 cm. What is the total energy of the system at any moment in time?

21. A 10 kg block oscillates up and down on a vertical spring and its motion is shown in the graph on the right. What is the spring constant of the spring?



22. A pendulum consisting of a ball attached to a string swings back and forth. As the ball swings from one end of the oscillation to the other end, the string sweeps out an angle of 12°. What is the amplitude of the oscillation?
    - A 3°
    - B 6°
    - C 12°
    - D 24°

23. Pendulum A is a 3 kg ball swinging from a 0.5 m long string, and pendulum B is a 6 kg ball swinging from a 1.5 m long string. How does the period of pendulum B compare to the period of pendulum A?
    - A $T_B = \sqrt{3}\,T_A$
    - B $T_B = 3T_A/2$
    - C $T_B = 6T_A$
    - D $T_B = \sqrt{6}\,T_A$

24. At which of the following points in a pendulum oscillation is the speed at a maximum? (Select all that apply)
    - A When the mass is at the maximum height
    - B At the amplitudes
    - C When the mass is at the minimum height
    - D At the equilibrium position

25. A pendulum consists of a 6 kg ball attached to an 80 cm long string. The ball is held so that the string makes a 5° angle with the vertical and then the ball is released and the pendulum swings back and forth. How would the period of the oscillation change if instead the ball was released from an angle of 10° from the vertical?
    - A The period would increase
    - B The period would decrease
    - C The period would be the same
    - D Cannot be determined

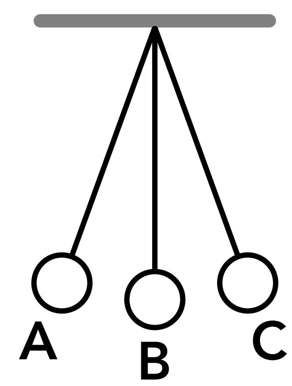
26. A 12 kg ball is attached to 40 cm long string. The ball is held so the string is 8° from the vertical and the ball is released. As the balls swings back and forth, its maximum speed is v_1. The string is then replaced with a 50 cm long string and the ball is released from the same angle. During the new oscillation, the maximum speed of the ball is v_2. How does v_2 compare to v_1?
    - A $v_1 < v_2$
    - B $v_1 > v_2$
    - C $v_1 = v_2$
    - D Cannot be determined

27. As a pendulum swings back and forth, at what point is the kinetic energy zero? (Select all that apply)
    - A At the equilibrium position
    - B When the mass is at the maximum height
    - C At the amplitudes
    - D When the mass is at the minimum height

28. A 20 kg ball swings back and forth on a 1.5 m long string. When this pendulum is near the surface of the earth it has a period of T_e. If the pendulum was brought to the surface of the moon where the strength of the gravitational field is weaker than on the surface of the earth, it would have a period of T_m. How do the two periods compare?

A $T_e > T_m$
B $T_e < T_m$
C $T_e = T_m$
D Cannot be determined

29. A snapshot of a pendulum at 3 different positions is shown on the right. Which of the following is true when the pendulum is in position B? (Select all that apply)



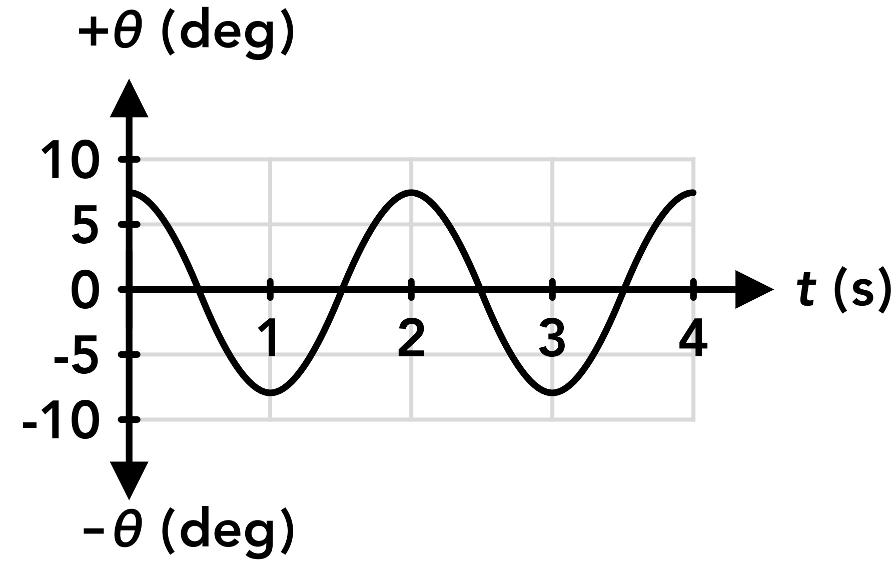
A The kinetic energy is at its maximum
B The gravitational potential energy is at its maximum
C The speed is at its maximum
D The gravitational potential energy is at its minimum

30. A pendulum consists of a 10 kg mass swinging on a 70 cm long string. What is the frequency of the oscillation?

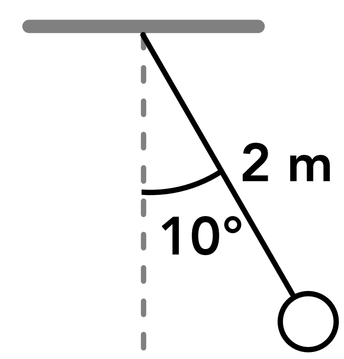
31. A ball is attached to a 1.2 m long string and held so the string makes a 7° angle with the vertical. The ball is then released. What is the speed of the ball at the lowest point in the swing?

32. A 25 kg mass is attached to a 60 cm long string. If an astronaut on the surface of Mars holds the other end of the string and lets the mass swing back and forth, what is the period of the oscillation? Use 3,390 km for the radius of Mars and 6.4×10^{23} **kg** for the mass of Mars.

33. The angle of a pendulum measured from the vertical is shown in the graph on the right. What is the length of the pendulum?



34. A 2 m long pendulum is held in the position shown on the right and released from rest. If the mass at the end of the pendulum is 12 kg, what is the total energy of the pendulum as it oscillates back and forth?



Answers

1. B, C
2. B
3. A
4. C
5. D
6. B, D
7. B
8. C
9. D
10. D
11. B, D
12. C
13. A, C
14. 1.6 Hz
15. 0.77 m/s
16. 9.73 kg
17. 0.95 m
18. 1.25 J
19. 1.35 J
20. 10.8 J
21. 98.7 N/m
22. B
23. A
24. C, D
25. C
26. A
27. B, C
28. B
29. A, C, D
30. 0.6 Hz
31. 0.42 m/s
32. 2.5 s
33. 0.99 m
34. 3.6 J

Answers - Mass-Spring Simple Harmonic Motion

1. **Answer: B, C**
   For a mass-spring system, the equilibrium position is where the net force on the mass is zero, which means the acceleration of the mass is also zero (Newton's 2nd law). For a horizontal mass-spring system, the spring force is zero at the equilibrium position. For a vertical mass-spring system, the spring force is equal to the weight force (the net force is zero).

2. **Answer: B**
   The amplitude is the distance between the equilibrium position and maximum displacement in either direction. When the block is at rest it is in the equilibrium position. The block is lifted up 10 cm and released, so the amplitude (the maximum displacement from the equilibrium position) is 10 cm.

3. **Answer: A**
   The equation for the period of a mass-spring system oscillation is given below. The spring constant is the same for both springs, and the period does not depend on the amplitude. The period is directly proportional to the square root of the mass. If the mass is multiplied by 3, the period is multiplied by $\sqrt{3}$.

   $$T_{sp} = 2\pi\sqrt{\frac{m}{k}}$$

4. **Answer: C**
   The equation for the maximum speed of a mass-spring oscillation is given below. The mass and amplitude are the same. The maximum speed is direction proportional to the square root of the spring constant. If the spring constant is increased, the maximum speed also increases.

   $$v_{max} = A\sqrt{\frac{k}{m}}$$

5. **Answer: D**
   The amplitude of the oscillation is the distance between the equilibrium position and the maximum displacement in either direction. The distance between the two maximum displacements (the two ends of the oscillation) is 30 cm, which is equal to two amplitudes, so the amplitude is 15 cm.

6. **Answer: B, D**
   The block has zero kinetic energy when it's velocity is zero. This happens at the maximum displacement when the block is reversing direction and momentarily stops. The point of maximum displacement is also where the spring force (and the net force) on the block is at its maximum, which is the point of maximum acceleration (Newton's 2nd law).

7. **Answer: B**
   The equation for the frequency of a mass-spring system is given below. The springs are identical so the spring constant is the same. The frequency is inversely proportional to the square root of the mass. Block A has a greater mass so it has a lower frequency.

8. **Answer: C**
   The maximum speed of mass-spring system occurs at the equilibrium position when the net force and acceleration is zero. The equilibrium position is half way between the two maximum displacements, 28 cm and 44 cm, which is at 36 cm. The amplitude of the oscillation is 8 cm.

9. **Answer: D**
   The spring potential energy depends on the displacement of the spring (how much the spring is stretched from its original unstretched length). For a vertical spring, the spring is stretched the most when the block is at the lowest point or the lower amplitude of the oscillation.

10. **Answer: D**
    For a vertical mass-spring system there is always a downwards weight force on the mass. There is an upwards spring force on the mass at any point where the spring is stretched some amount from its original length (with no mass attached to it). This could be at the upper amplitude but that would be a specific case. The net force is zero when the mass is at the equilibrium position, but the spring force is not zero (it's equal in magnitude to the weight force at that point).

11. **Answer: B, D**
    This is a graph of the vertical position vs time. Velocity is the slope of the position vs time graph and the magnitude of the slope (the speed) is the greatest at 0.2 seconds and 0.6 seconds. We can also see from the graph that the equilibrium position is 30 cm, half way between the two amplitudes. The maximum speed occurs when the mass is at the equilibrium position, which is at 0.2 and 0.6 seconds.

12. **Answer: C**
    The acceleration of a mass-spring system is zero when the mass is at the equilibrium position and the net force on the mass is zero. The left side of the block oscillates between 0 cm and 30 cm, and the left side is at 15 cm when the block is at the equilibrium position (half way).

13. **Answer: A, C**
    This is a graph of the velocity vs time. The speed is maximum at the equilibrium position (0 s, 0.4 s, 0.8 s) and zero when the block is at the amplitudes of the motion (0.2 s, 0.6 s). The net force on the block is maximum when the block is at the amplitudes, which happens at 0.2 seconds and 0.6 seconds. Also, the net force will be at a maximum when the acceleration is at a maximum. The slope of the velocity graph is the acceleration, and the magnitude of the slope is greatest at 0.2 seconds and 0.6 seconds.

14. **Answer: 1.6 Hz**
    The frequency does not depend on the amplitude.

    $$f_{sp} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{200\text{ N/m}}{2\text{ kg}}} = 1.6\text{ Hz}$$

15. **Answer: 0.77 m/s**

    $$v_{max} = A\sqrt{\frac{k}{m}} = (0.1\text{ m})\sqrt{\frac{300\text{ N/m}}{5\text{ kg}}} = 0.77\text{ m/s}$$

16. **Answer: 9.73 kg**
The period is the time it takes for one full cycle, which is twice the amount of time it takes to move from one amplitude to the other, so the period is 1.6 seconds.
$T_{sp} = 2\pi\sqrt{\frac{m}{k}} \qquad (1.6\text{ s}) = 2\pi\sqrt{\frac{m}{150\text{ N/m}}} \qquad m = 9.73\text{ kg}$

17. **Answer: 0.95 m**
We can find the amplitude of the motion from the frequency and the maximum speed. The distance between the left end and right end of the oscillation is twice the amplitude.
$v_{max} = 2\pi fA \qquad (1.5\text{ m/s}) = 2\pi(0.5\text{ Hz})A \qquad A = 0.477\text{ m} \qquad 2A = 0.95\text{ m}$

18. **Answer: 1.25 J**
We can find the maximum speed from the amplitude, spring constant and mass of the block, and then we can find the maximum kinetic energy from the maximum speed. The amplitude is half of the distance between the lowest and highest points in the motion, so the amplitude is 10 cm.
$v_{max} = A\sqrt{\frac{k}{m}} = (0.1\text{ m})\sqrt{\frac{250\text{ N/m}}{6\text{ kg}}} = 0.645\text{ m/s}$
$K = \frac{1}{2}mv^2 = \frac{1}{2}(6\text{ kg})(0.645\text{ m/s})^2 = 1.25\text{ J}$

19. **Answer: 1.35 J**
The spring potential energy is at a maximum when the block is at an amplitude, which is when the spring is stretched the most from its original length.
$U_{sp} = \frac{1}{2}k\Delta x^2 = \frac{1}{2}(120\text{ N/m})(0.15\text{ m})^2 = 1.35\text{ J}$

20. **Answer: 10.8 J**
During the motion, the system can have kinetic energy and spring potential energy. Energy is transformed back and forth between kinetic energy and spring potential energy, but energy is conserved and the total amount of energy remains the same. The total energy is equal to the maximum kinetic energy (which is when the spring potential energy is zero), or the total energy is also equal to the maximum spring potential energy (which is when the kinetic energy is zero). We can find the maximum speed of the block using the frequency and the amplitude, and then find the maximum kinetic energy, which is also the total energy of the system. The amplitude is half of the distance between the left and right ends of the oscillation, or 14 cm.
$v_{max} = 2\pi fA = 2\pi(2\text{ Hz})(0.14\text{ m}) = 1.759\text{ m/s}$
$K = \frac{1}{2}mv^2 = \frac{1}{2}(7\text{ kg})(1.759\text{ m/s})^2 = 10.8\text{ J}$

21. **Answer: 98.7 N/m**
We can find the period of the oscillation from the graph and use that to find the spring constant. The period is the amount of time for one cycle or one oscillation, such as the time between two high amplitudes or two low amplitudes, which is 2 seconds.
$T_{sp} = 2\pi\sqrt{\frac{m}{k}} \qquad (2\text{ s}) = 2\pi\sqrt{\frac{10\text{ kg}}{k}} \qquad k = 98.7\text{ N/m}$

Answers - Pendulum Simple Harmonic Motion

22. **Answer: B**
The amplitude of a pendulum oscillation is the angle between the vertical (the string when the pendulum is at the equilibrium position) and one end of the oscillation. The amplitude is half of the total angle between each end of the oscillation.

23. **Answer: A**
The equation for the period of a pendulum oscillation is given below. The period does not depend on the mass. The period is direction proportional to the square root of the length of the string, so if the length is multiplied by 3 the period will be multiplied by $\sqrt{3}$.

$$T_p = 2\pi\sqrt{\frac{L}{g}}$$

24. **Answer: C, D**
The maximum speed of a pendulum oscillation occurs at the equilibrium position, which is when the mass is at the minimum height and the pendulum is vertical.

25. **Answer: C**
The equation for the period of a pendulum oscillation is given below. The period does not depend on the amplitude of the oscillation, which is the initial angle of the string from the vertical.

$$T_p = 2\pi\sqrt{\frac{L}{g}}$$

26. **Answer: A**
The equation for the maximum speed of a pendulum oscillation is given below. If the maximum angle (the amplitude) is the same, increasing the length will increase the maximum speed.

$$v_{max} = \theta_{max}\sqrt{gL}$$

27. **Answer: B, C**
The kinetic energy is zero when the speed of the mass is zero. This occurs at the amplitudes (when the the mass is reversing direction) which is also when the mass is at the maximum height.

28. **Answer: B**
The equation for the period of a pendulum oscillation is given below. If the strength of the gravitational field g decreases, the period increases.

$$T_p = 2\pi\sqrt{\frac{L}{g}}$$

29. **Answer: A, C, D**
Position B is the equilibrium position of the pendulum. This is where the speed and the kinetic energy are at their maximum value. It's the lowest point in the motion so it's also the point where the gravitational potential energy is at its minimum.

30. **Answer: 0.6 Hz**

$$f_p = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{g}{0.7\text{ m}}} = 0.6\text{ Hz}$$

31. **Answer: 0.42 m/s**
The speed at the lowest point in the swing is the maximum speed which is found using the equation below. Note that the angle must be in radians, not degrees.

$$\theta_{max} = \frac{7°}{} \times \frac{2\pi\text{ rad}}{360°} = \frac{7\pi}{180}\text{ rad}$$

$$v_{max} = \theta_{max}\sqrt{gL} = \left(\frac{7\pi}{180}\text{ rad}\right)\sqrt{g(1.2\text{ m})} = 0.42\text{ m/s}$$

32. **Answer: 2.5 s**
The strength of the graviational field (or the acceleration due to gravity) g on the surface of Mars must be found first. Then the period can be found using g and the length of the pendulum. The period does not depend on the mass of the pendulum.

$$g = \frac{GM}{r^2} = \frac{(6.67 \times 10^{-11})(6.4 \times 10^{23}\ \text{kg})}{(3{,}390{,}000\ \text{m})^2} = 3.71\ \text{m/s}^2$$

$$T_p = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{0.6\ \text{m}}{3.71\ \text{m/s}^2}} = 2.5\ \text{s}$$

33. **Answer: 0.99 m**
The period of the oscillation can be found from the graph as the time between two positive amplitudes or two negative amplitudes, which is 2 seconds. The length of the pendulum can be found using the equation for the period, and we can assume g is 9.8 m/s².

$$T_p = 2\pi\sqrt{\frac{L}{g}} \qquad (2\ \text{s}) = 2\pi\sqrt{\frac{L}{g}} \qquad L = 0.99\ \text{m}$$

34. **Answer: 3.6 J**
We can use the amplitude and length to find the maximum speed of the pendulum, which can be used to find the maximum kinetic energy, which is equal to the total energy of the pendulum.

$$\theta_{max} = \frac{10°}{} \times \frac{2\pi\ \text{rad}}{360°} = \frac{\pi}{18}\ \text{rad}$$

$$v_{max} = \theta_{max}\sqrt{gL} = \left(\frac{\pi}{18}\ \text{rad}\right)\sqrt{g(2\ \text{m})} = 0.773\ \text{m/s}$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(12\ \text{kg})(0.773\ \text{m/s})^2 = 3.6\ \text{J}$$

We could also find the gravitational potential energy of the mass at the maximum height shown relative to when the mass is at the lowest height and the pendulum is vertical. The kinetic energy is zero when the mass is at the maximum height, so the maximum gravitational potential energy is equal to the total energy. Using geometry:
$U_{g\ max} = mg\Delta y = (12\ \text{kg})g(2 - 2\cos(10°)) = 3.6\ \text{J}$