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CUBE NOTES

Class 11/12 | AP Physics | IIT JEE | NEET

Rotational Variables

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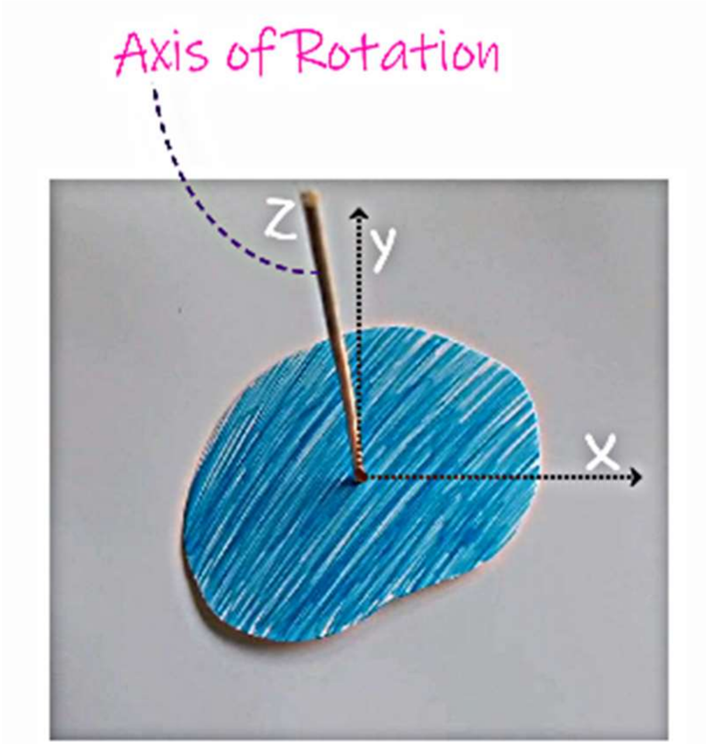
Key Idea

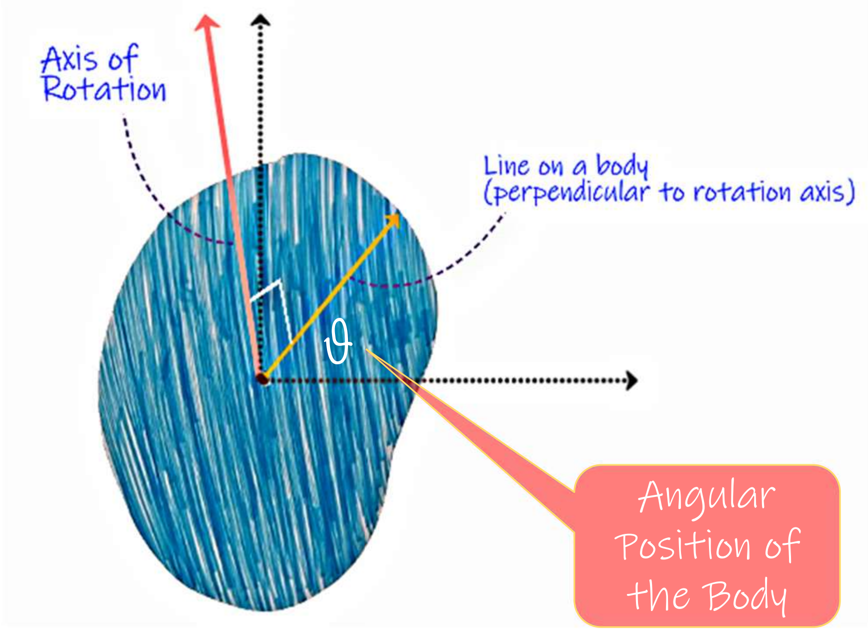
- **Translation Motion:** Movement along a straight or curved path
- **Rotation Motion:** An object turns about an axis.
- **Axis of Rotation:** The fixed line about which the body rotates (e.g., z-axis).
- **Rigid Body:** An object whose shape does not change during rotation.
- **Pure Rotation:** Every point on the body moves in a circle centered on the axis of rotation



Angular Position

Definition: Angle an imaginary line makes relative to the positive x-axis.

(The line is fixed to the body and perpendicular to the rotation axis)



- Zero Angular Position: Line lies along the x-axis.

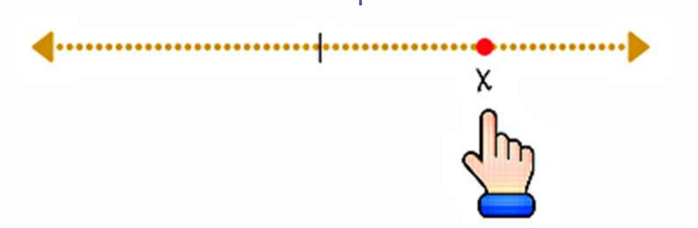
- Units: Radians (instead of degrees).

180° arc → π radians

360° arc → 2π radians

1 radian = 57.3° = 0.159 revolutions (rev)

Linear Equivalent

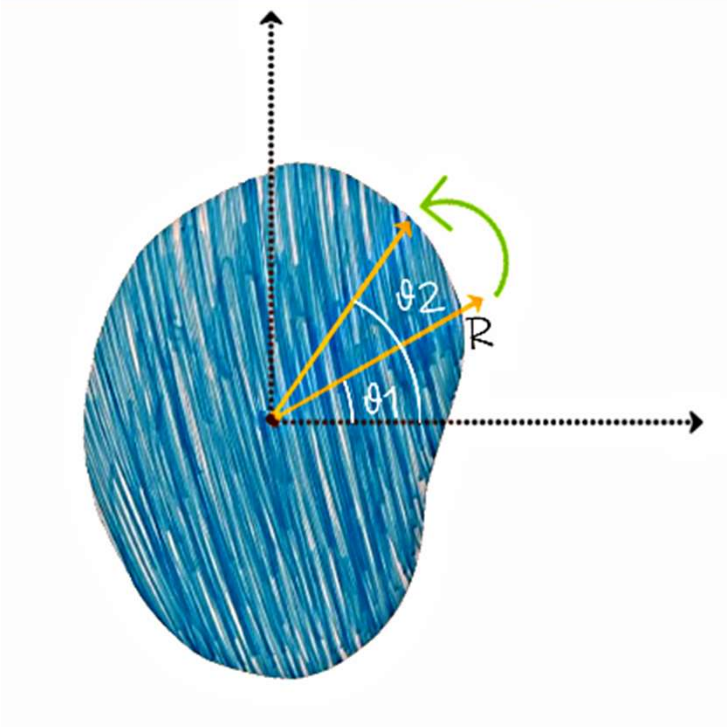


Angular Displacement

Definition: Change in angular position ($\Delta\theta = \theta_2 - \theta_1$)

- Linear Equivalent: Change in x ($\Delta x = x_2 - x_1$).

- Every point on the body experiences the same angular displacement.



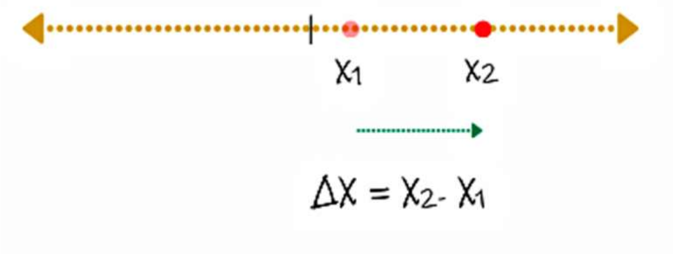
Example:

Initial position $\theta_1 = 30^o$

Final position $\theta_2 = 60^o$

Angular displacement $\Delta\theta = 60^o - 30^o = 30^o$

Linear Equivalent



Angular Velocity

Definition: Rate of change of angular displacement

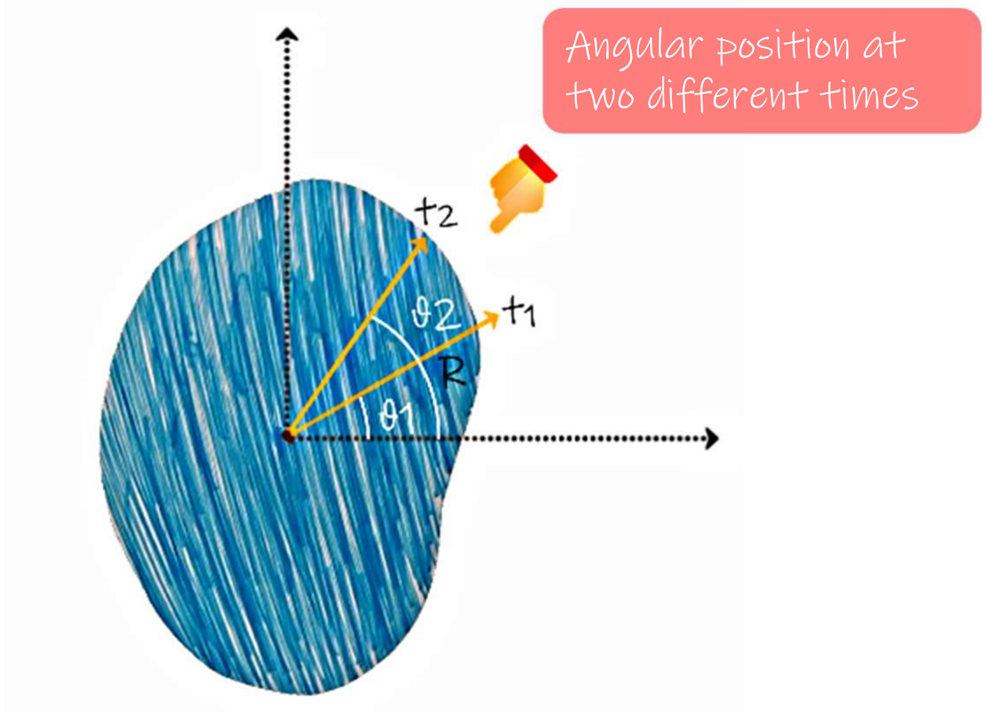
- **Average Angular Velocity:**

$$\omega_{avg} = (\theta_2 - \theta_1) / (t_2 - t_1)$$

- **Instantaneous Angular Velocity:**

$$\omega = d\theta / dt$$

- **Units:** radians /s or revolutions /s



- **Example:**

Angular position $\theta(t) = 7 \sin(3t + \pi/2)$

Differentiate to find angular velocity $\omega(t) = 21 \cos(3t + \pi/2)$

Angular Acceleration

Definition: Rate of change of angular velocity.

- Average Angular Acceleration:

$$\alpha_{avg} = (\omega_2 - \omega_1) / (t_2 - t_1)$$

- Instantaneous Angular Acceleration:

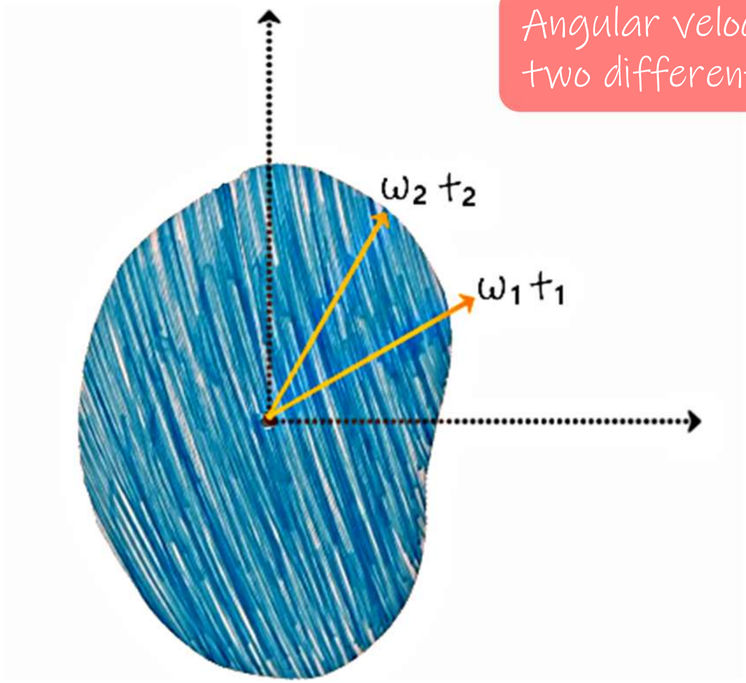
$$\alpha = d\omega / dt$$

- Units: rad/s² or revolutions/s²

- Example:

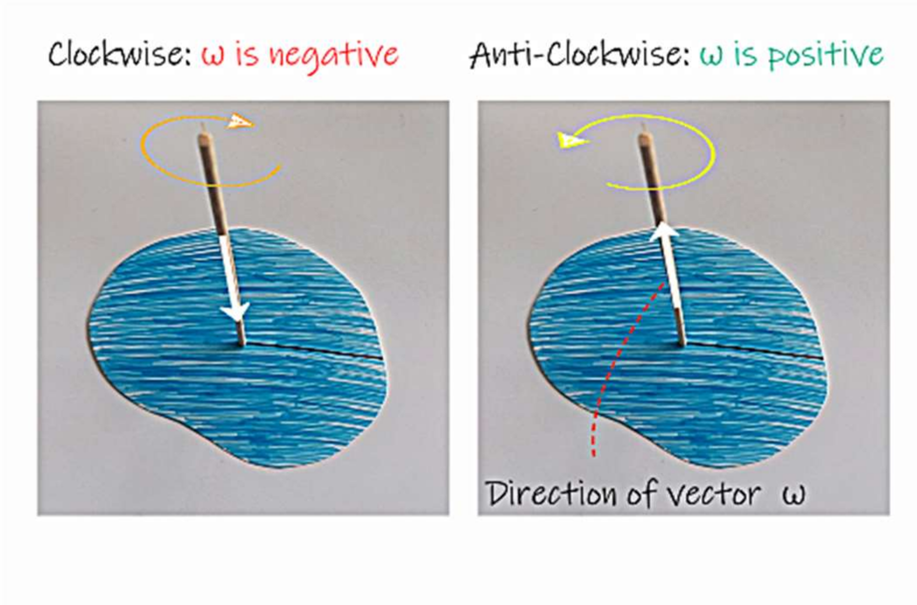
Initial angular velocity $\omega_1 = 10$ rad/s
Final angular velocity $\omega_2 = 20$ rad/s
Time interval $\Delta t = 2$ s
Angular acceleration $\alpha = (20 - 10) / 2 = 5$ rad/s²

Angular velocity at two different times



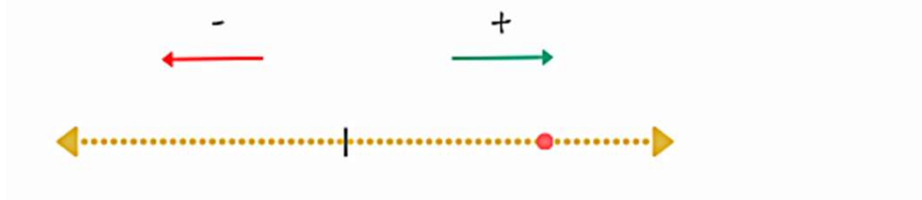
Vectors and Rotation

- Angular Quantities as Vectors: Determined by –ve sign if motion is clockwise and +ve, if anti-clockwise



- Key Point:
  - Angular (see suvelocity (ω) and angular acceleration (α) are vectors
  - Angular displacement ($\Delta\theta$) is not a vector because it does not follow vector addition rules bsequent slides)

Linear Equivalent

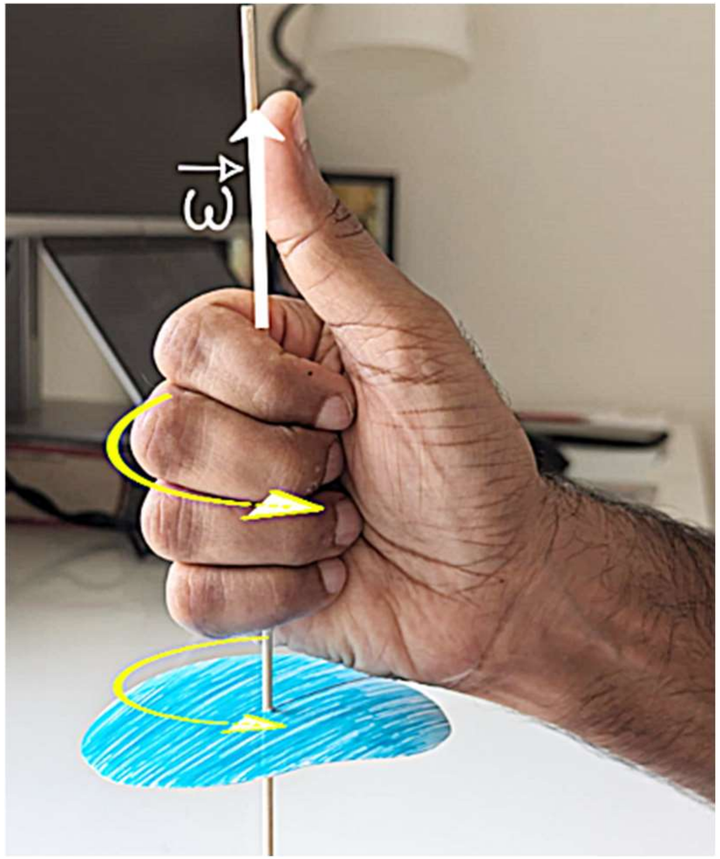


The Right Hand Rule

Direction of angular velocity determined by the Right hand Rule

- **How to Use:** Curl fingers in the direction of rotation, then thumb points in direction of angular velocity vector.

- **Example:** Counterclockwise rotation → Thumb points up.



Summary of Rotational Variables

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Variable	Symbol	Units	When to Use	Caution/Keep in Mind
Angular Position	θ	radians	Used to describe the current orientation of a rotating body.	1. Always use radians not degrees. 2. The angular position continues to increase with each rotation (e.g., after 2 complete rotations, $\theta = 4\pi$ radians).
Angular Displacement	$\Delta\theta$	radians	Used to describe the change in orientation from one point in time to another.	1. Angular displacement is the same for all points on the rotating body. 2. Not a true vector because it does not obey commutative property of vector addition.
Angular Velocity	ω	rad/s	Used to describe how quickly the angular position is changing over time.	Direction determined by right-hand rule: 1. Counterclockwise is positive, 2. Clockwise is negative. 3. The magnitude of angular velocity is angular speed.
Angular Acceleration	α	rad/s²	Used to describe how quickly the angular velocity is changing over time.	The direction of angular acceleration is 1. The same as angular velocity when speeding up and 2. Opposite when slowing down.

Why Angular Displacement is Not a Vector

Vectors follow Commutative Property:

$\vec{a} + \vec{b} = \vec{b} + \vec{a} = \vec{c}$

Angular Displacement vector does not follow commutative property, hence not a vector

Example:

Rotate book 90° around x-axis, then 90° around y-axis.

Another order gives different final orientation.