Inductive Reasoning classwork

KEY

During our study of geometry, we will be using a *system of logic* to prove various mathematical concepts.

Conjecture and Inductive Reasoning

- **What is a conjecture in your own words?** *It's basically a proposal statement that has yet to be proven true.*

- **Which statement applies to inductive reasoning?** Circle one:

  (circled) using many examples to draw a general conclusion

  beginning with a general conclusion and making specific statements using it

Ex #1: Eric was driving his friends to school when his car suddenly stopped two blocks away from school. Make a list of conjectures that Eric can make as to why his car stopped.

- Ran out of gas?
- Accident ahead?
- Construction?
- Traffic?
- Car trouble?

Ex #2: The numbers represented below are called *triangular numbers*. Make a conjecture about the next triangular number.

1   3   6   10   15   21
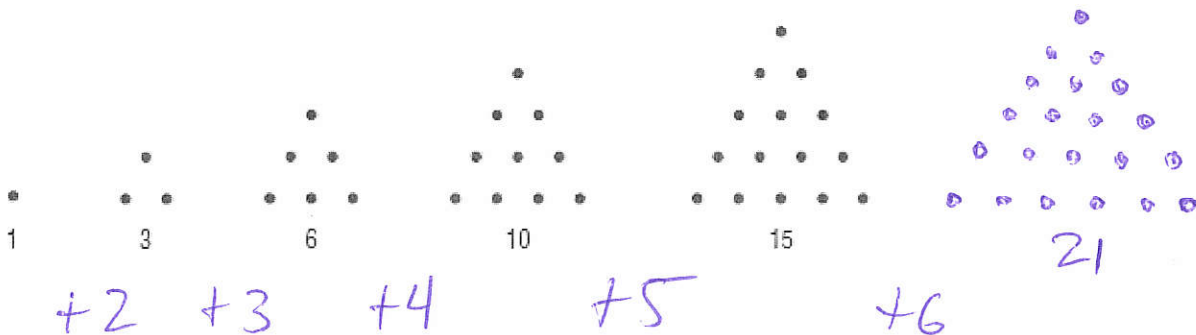
+2   +3   +4   +5   +6

Ex #3: Make a conjecture about the next item in the sequence.

a)   20, 16, 11, 5, -2, -10, -19

(-4, then -5, then -6...)

b)   2, 4, 12, 48, 240, 1440

×2 ×3 ×4 ×5 ×6

×7 = 10,080

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Ex #4: Given that points P, Q, R are collinear. Joel made a conjecture that Q is between P and R. Determine if the conjecture is true or false. If false, please explain why in simple terms.
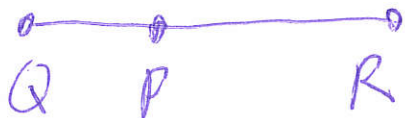
"Collinear" does not mean between.
It means on the same line, anywhere on the line.

FALSE

Counterexample

In the previous example, you may have used a <u>counterexample</u> to make your case. What is a counterexample? It is an instance that disproves a conjecture, making the conjecture false.

How many counterexamples do I need to establish, in order to prove a conjecture FALSE? 1

Ex #5: Determine whether each conjecture is true or false. Give a counterexample for any false conjecture.

a) Given: ∠1 and ∠2 are supplementary angles.
    Conjecture: ∠1 and ∠2 are both right angles.

FALSE

(They're supplementary but not right.)
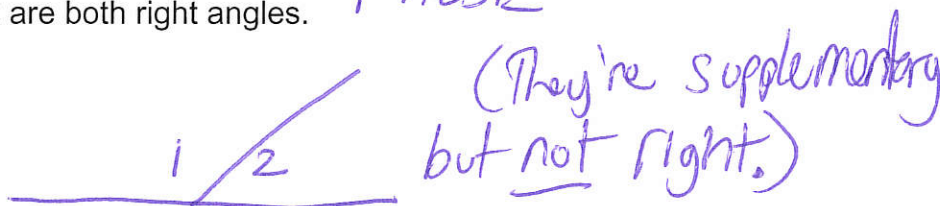
b) Given: Points A, B, and C are collinear.
    Conjecture: B is between A and C.

FALSE

(Given still true, Conjecture FALSE)

c) Given: n is negative number
    Conjecture: n^2 is a nonnegative number

true

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Now let's review the concept of creating "if-then" statements. The fancy term for an "if-then" statement is called a *conditional statement*.

Conditional statement: any statement that can be written _if-then_ form.

    Example: "Buying this car gets you $1500 cash back!"

    Example: **If** you buy this car, **then** you get $1500 cash back.

    Symbols: $p \rightarrow q$

    Read: If p, then q.   OR p implies q.

    What does "p" stand for here?   What does "q" stand for here?

    You buy the car   _You get $1500 Cash back._

Statement	If-Then Form
The sum of two odd numbers is even.	_If two numbers are odd, then their sum is even._
Two angles that have the same measure are congruent.	_If two angles have the same measure, then they are congruent._
I carry an umbrella on rainy days.	_If it rains on a certain day, then I carry an umbrella._

If-Then Statements are called *conditional statements* or *"conditionals."*

- **Hypothesis** – the portion of the statement following the word: _if_
- **Conclusion** – the portion of the statement following the word: _then_

Ex #6: Identify the hypothesis and conclusion of each statement.

    a)   If points *A*, *B*, and *C* lie on line *m*, then they are collinear.

        Hypothesis: _points A, B, C lie on line m_

        Conclusion: _they are collinear_

b)   The Tigers will play in the tournament if they win their next game.

Hypothesis: *The Tigers win their next game*

Conclusion: *They will win the tournament*

c)   If $x - 3 = 7$, then $x = 10$.

Hypothesis: *$x - 3 = 7$*

Conclusion: *$x = 10$*

Ex #7: Write each statement in if-then form.

a)   A five-sided polygon is a pentagon.

If a polygon has 5 sides, then it is a pentagon.

b)   Math teachers love to solve problems.

If a teacher is a math teacher, then s/he loves to solve problems.

Ex #8: **Take the statement: "People who live in Florida live in the U.S."**

a) Write the statement in if-then form. Identify "p" and "q" and write as a conditional statement, with the fancy → symbol.

If someone lives in Florida, then s/he lives in the U.S.
      p   *p → q*   *q*

b) What is the *inverse* of this statement, in words, and in symbols? True or False?

If someone does not live in Florida, then s/he does not live in the U.S.   ~p → ~q   FALSE (can live in Nevada)

c) What is the *converse* of this statement, in words, and in symbols? True or False?

If someone lives in the U.S., then s/he lives in Florida
      q → p   FALSE (can live in Nevada)

d) What is the *contrapositive* of this statement, in words, and in symbols? True or False?

If someone does not live in the U.S., then s/he does not live in Florida.   4   *~q → ~p   TRUE*

a) Come up with any statement of your choosing, that is already *true*. Feel free to be creative, personal, funny, anything!

Good leaders have good people-skills.

b) Write the statement in if-then form. Identify "p" and "q" and write as a conditional statement, with the fancy → symbol.

If someone is a good leader, then s/he has good people skills. $p \rightarrow q$

c) What is the *inverse* of this statement, in words, and in symbols? True or False?

If someone is not a good leader, then s/he does not have good people skills. $\sim p \rightarrow \sim q$ FALSE
(may be bad at organization or time management)

d) What is the *converse* of this statement, in words, and in symbols? True or False?

If someone has good people skills, then s/he is a good leader. $q \rightarrow p$ FALSE (may just be good with people, but lack leadership skills)

e) What is the *contrapositive* of this statement, in words, and in symbols? True or False?

If someone does not have good people skills, then s/he is not a good leader.

TRUE