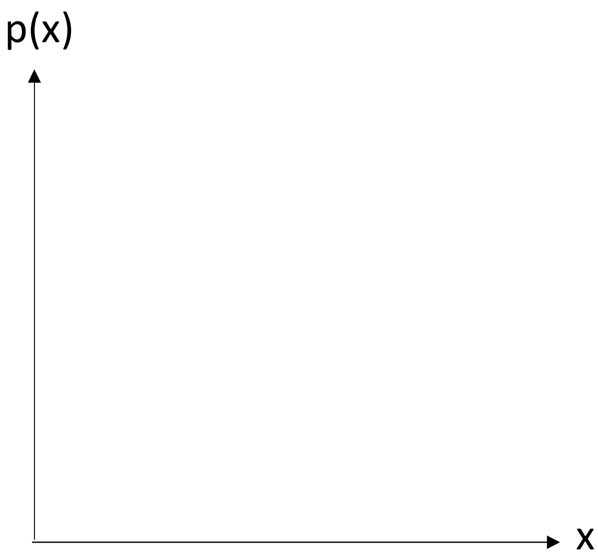
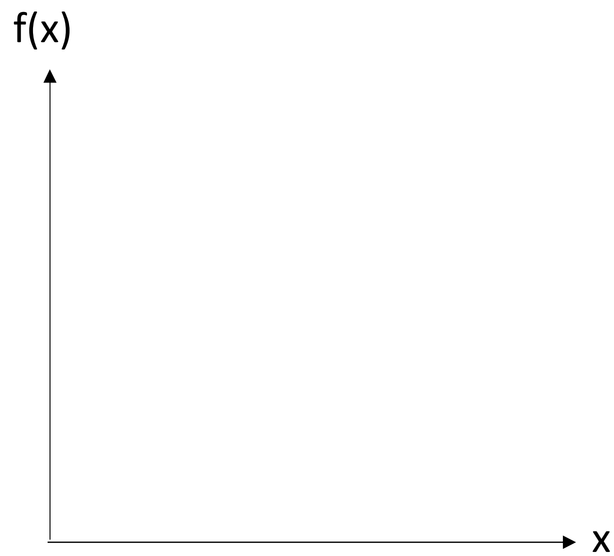
Exam P
ครั้งที่ 2

Mode

ตำแหน่งที่ prob มากที่สุด





21. An insurer’s medical reimbursements have density function *f*, where *f*(*x*) is proportional to , for 0 < *x* < 1, $\mathrm{f(x)} = xe^{x^2}$ and 0, otherwise. Calculate the mode of the medical reimbursements

22. The number of policies that an agent sells has a Poisson distribution with modes at 2 and 3. K is the smallest number such that the probability of selling more than K policies is less than 25%. Calculate K.

Moment generating functions

$$\mathrm{M_x(t)} =$$

$$\mathrm{M_x(0)} =$$

$$\mathrm{M'_x(0)} =$$

$$\mathrm{M^{(n)}{}_x(0)} =$$

$$\left.\frac{d(\ln(\mathrm{M_x(t)})}{dt}\right|_{t=0} =$$

$$\left.\frac{d^2(\ln(\mathrm{M_x(t)})}{dt^2}\right|_{t=0} =$$

23. The value of a piece of factory equipment after three years of use is $100(0.5)^{x}$ where *X* is a random variable having moment generating

$$M_x(t) = \frac{1}{1-2t}, t \le 1/2$$

Calculate the expected value of this piece of equipment after three years of use.

24. a certain class of accidents is a random variable, *X*, with moment generating functionAn actuary determines that the claim size for

$$M_x(t) = \left(\frac{1}{1-2500t}\right)^4, t \le 1/2500$$

Calculate the standard deviation of the claim size for this class of accidents

25. Let $f(x) = xe^{-x}, x \geq 0$ find moment generating function of X

26. Let *X* represent the number of policies sold by an agent in a day. The moment generating function of *X* is

$$M_x(t) = 0.45e^{t} + 0.35e^{2t} + 0.15e^{3t} + 0.05e^{4t}, \text{ for } -\infty < t < \infty$$

Calculate the standard deviation of *X*.

$M_{X+Y}(t) =$

$M_{aX+b}(t) =$

27. A company insures homes in three cities, J, K, and L. Since sufficient distance separates the cities, it is reasonable to assume that the losses occurring in these cities are mutually independent. The moment generating functions for the loss distributions of the cities are:

$$M_J(t) = (1-2t)^{-3} \qquad M_K(t) = (1-2t)^{-2.5} \qquad M_L(t) = (1-2t)^{-4.5}$$

Let *X* represent the combined losses from the three cities. Calculate . $E(X^3)$

Joint moment generating functions

$M_{X,Y}(s,t) =$

$E(X^n Y^m) =$

28. *X* and *Y* are independent random variables with common moment generating function . $M_X(t) = e^{\frac{t^2}{2}}$

Let $W = X + Y$, and $Z = X - Y$

Determine the joint moment generating function, $M(t_1, t_2)$ of *W* and *Z*.

(A) $\exp(2t_1^2 + 2t_2^2)$

(B) $\exp[(t_1 - t_2)^2]$

(C) $\exp[(t_1 + t_2)^2]$

(D) $\exp(2t_1 t_2)$

(E) $\exp(t_1^2 + t_2^2)$

Transformations

รู้ $f_X(x), Y = g(X)$ อยากหา $f_Y(y)$

$$
\begin{tikzcd}
f_X(x) \arrow[r] \arrow[d] & f_Y(y) \arrow[d] \\
F_X(x) \arrow[r] & F_Y(y)
\end{tikzcd}
$$

29. The time, *T*, that a manufacturing system is out of operation has cumulative distribution function

$$F(t) = \begin{cases} 1 - \left(\dfrac{2}{t}\right)^2, & t > 2 \\ 0, & \text{otherwise.} \end{cases}$$

The resulting cost to the company is $Y = T^2$. Let *g* be the density function for Y Determine *g*(*y*), for *y* > 4.

30. An actuary models the lifetime of a device using the random variable $Y = 10X^{0.8}$, where *X* is an exponential random variable with mean 1. Let *f*(*y*) be the density function for *Y*.

Determine *f*(*y*), for *y* > 0.

(A) $10y^{0.8}\exp(-8y^{-0.2})$

(B) $8y^{-0.2}\exp(-10y^{0.8})$

(C) $8y^{-0.2}\exp[-(0.1y)^{1.25}]$

(D) $(0.1y)^{1.25}\exp[-0.125(0.1y)^{0.25}]$

(E) $0.125(0.1y)^{0.25}\exp[-(0.1y)^{1.25}]$

Normal & Lognormal Distribution

Normal

Lognormal

31. Losses, *X*, under an insurance policy are normal distribution with parameter $\mu = 5, \sigma = 2$ and $Y = e^X$ find Prob(Y <100)

Deductible & Policy limit

Loss :

Payment , reimburse :

deductible

Policy limit

32. The owner of an automobile insures it against damage by purchasing an insurance policy with a deductible of 250. In the event that the automobile is damaged, repair costs can be modeled by a uniform random variable on the interval (0, 1500).

Calculate the standard deviation of the insurance payment in the event that the automobile is damaged.

33. An insurance policy pays for a random loss *X* subject to a deductible of *C*, where $0 < C < 1$
The loss amount is modeled as a continuous random variable with density function

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Given a random loss *X*, the probability that the insurance payment is less than 0.5 is equal to 0.64. Calculate *C*.

34. A manufacturer’s annual losses follow a distribution with density function

$$f(x) = \begin{cases} \dfrac{2.5(0.6)^{2.5}}{x^{3.5}}, & x > 0.6 \\ 0, & \text{otherwise.} \end{cases}$$

To cover its losses, the manufacturer purchases an insurance policy with an annual deductible of 2. Calculate the mean of the manufacturer’s annual losses not paid by the insurance policy.

35. A company buys a policy to insure its revenue in the event of major snowstorms that shut down business. The policy pays nothing for the first such snowstorm of the year and 10,000 for each one thereafter, until the end of the year. The number of major snowstorms per year that shut down business is assumed to have a Poisson distribution with mean 1.5.

Calculate the expected amount paid to the company under this policy during a one-year period.

36. An insurance policy reimburses a loss up to a benefit limit of 10. The policyholder’s loss, *Y*, follows a distribution with density function:

$$f(y) = \begin{cases} 2y^{-3}, & y > 1 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the expected value of the benefit paid under the insurance policy.