IB Math Minds

Integration Practice – 2

No calculator allowed on these 12 exercises.

content includes: definite & indefinite integrals, integration by substitution, area under a curve, and anti-differentiation with a boundary condition to determine the constant of integration (Q #10)

1. Find an expression for y given that $\frac{dy}{dx} = \frac{2}{3x-5}$.

2. Find $\int \frac{x^4+3x^2-6}{x^2}\,dx$.

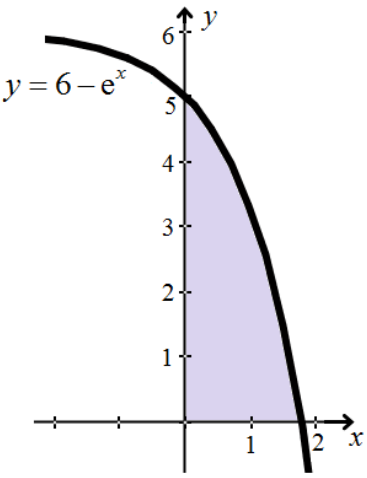
3. Evaluate $\int_{\pi/2}^{2\pi} \sin\left(\frac{x}{2}\right)dx$.

4. Find $\int \frac{e^{2x}}{1+e^{2x}}\,dx$.

5. (a) Using an appropriate substitution, show that $\int \sin x \cos x\,dx = \frac{1}{2}\sin^2 x + C$

(b) Apply the trigonometric identity $\sin 2x = 2\sin x\cos x$ to use an alternative substitution to find a different result for $\int \sin x \cos x\,dx$.

(c) Explain why the results for $\int \sin x \cos x\,dx$ in (a) and (b) are equivalent.



6. Find the **exact** area of the region bounded by the curve $y = 6 - e^x$, the x-axis and the y-axis, as shown in the diagram.

7. Evaluate $\int_0^1 x\sqrt{4-3x^2}\,dx$.

8. Without performing any computation or integration, explain why $\int_{-c}^{c} \sin x\,dx = 0$ for any real number c.

9. For real constants a and b, show that $\int x\left(ax^2+b\right)^4 dx = \frac{\left(ax^2+b\right)^5}{10a} + C$

10. The graph of the function f passes through the point $(2, 4)$. Given that $\frac{dy}{dx} = \frac{x}{x^2-3}$, find a specific expression for $f(x)$.

11. Using the substitution $u = 3x-5$, find $\int \frac{x}{3x-5}\,dx$.

12. Find the **exact** value of $k > 0$ such that the area under the graph of $y = e^{3x}$ from $x = 0$ to $x = k$ is 5 square units.