INTEGRATING FACTOR WORKSHEET

1. Solve the following differential equations.

   (a) $\frac{dy}{dx} + y = e^x$

   (b) $\frac{dy}{dx} - y = e^{3x}$

   (c) $(x+1)\frac{dy}{dx} - 3y = (x+1)^4$

2. Determine the particular solution for each of the following differential equations under the given conditions.

   (a) $\frac{dy}{dx} + 2y = x$; $y = 1$ at $x = 0$

   (b) $\frac{dy}{dx} + \frac{xy}{1+x^2} = x$; $y = 1$ when $x = 0$

   (c) $\frac{dy}{dx} + \frac{y}{x} = \sin 2x$; $y = \frac{2}{\pi}$ when $x = \frac{\pi}{4}$

   (d) $\frac{dy}{dx} + y\tan x = \cos^3 x$; $y = 2$ when $x = \pi$

3. (a) Show that $\tan x$ is an integrating factor for the differential equation

$$\frac{dy}{dx} + \frac{\sec^2 x}{\tan x}y = \tan x$$

   (b) Hence solve this differential equation, given that $y = 3$ when $x = \frac{\pi}{4}$.

4. By using an integrating factor, find the solution of the differential equation

$$\frac{dy}{dx} + \frac{4x}{x^2+1}y = x$$

   given that $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$.

5. (a) By using an integrating factor, find the general solution of the differential equation

$$\frac{dy}{dx} + \frac{2}{x}y = \ln x$$

   (b) Hence, given that $y \to 0$ as $x \to 0$, find the value of y when $x = 1$.

ANSWERS

1. (a) $y = \frac{1}{2}e^x + \frac{c}{e^x}$ (b) $y = \frac{1}{2}e^{3x} + ce^x$ (c) $y = (c+x)(x+1)^3$
2. (a) $y = \frac{1}{2}x - \frac{1}{4} + \frac{5}{4}e^{-2x}$ (b) $y = \frac{1}{3}(1+x^2) + \frac{2}{3\sqrt{1+x^2}}$ (c) $y = -\frac{1}{2}\cos 2x + \frac{\sin 2x}{4x} + \frac{1}{4x}$

   (d) $y = \frac{1}{4}\cos x \sin 2x + \frac{x\cos x}{2} - \frac{(\pi+4)\cos x}{2}$
3. (a) (b) $y = 1 + \left(2 - x + \frac{\pi}{4}\right)\cot x$
4. $y = \frac{1}{6}(x^2+1) + \frac{5}{6(x^2+1)^2}$
5. (a) $y = \frac{x}{3}\ln x - \frac{x}{9} + Ax^{-2}$ (b) $-\frac{1}{9}$
