3. Mind Map: Dot Product of 2 Vectors

Dot Product

- **Types of Products**
  - Scalar × Vector — Changes magnitude → vector result
    - *(Same direction if scalar > 0, opposite if scalar < 0)*
  - Vector × Vector
    - Dot Product → scalar result
    - Cross Product → vector result
- **Dot Product**
  - Definition: $a \cdot b = |a||b|\cos(\theta)$
  - Alternate Views:
    1. $a \cdot b = |a|$ × component of b along a
    2. $a \cdot b = |b|$ × component of a along b
- **Example**
  - Work = Force × Displacement (simple multiplication when F & d are in the same direction)
  - F and d at an Angle — Work = $F \cdot d \cdot \cos(\theta)$
- **Unit Vector Notation**
  - Formula: $a \cdot b = A_xB_x + A_yB_y + A_zB_z$
  - (Use: $i \cdot i = 1$; $i \cdot j = 0$, etc)

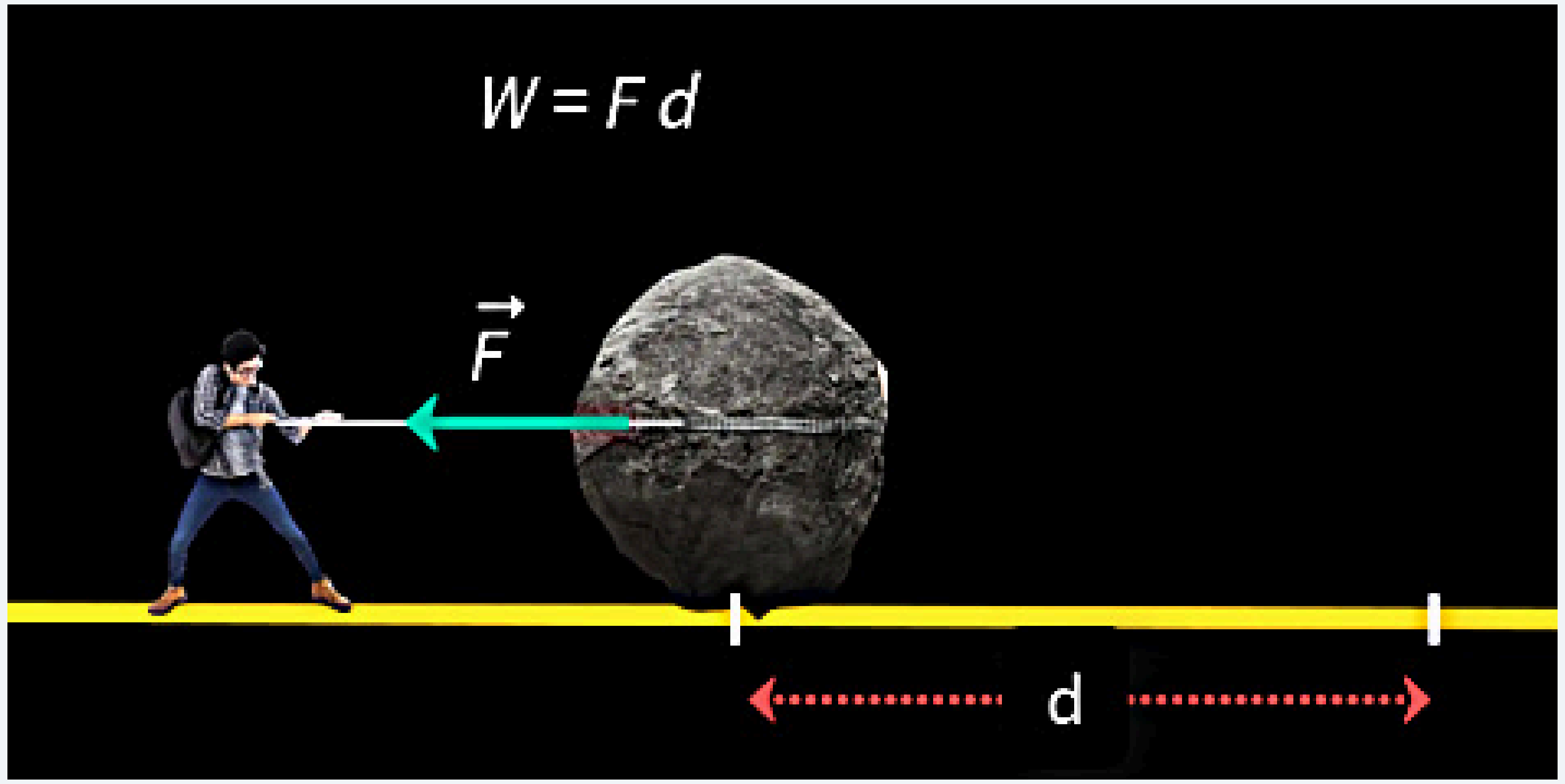
1. $\theta = 0°$ → *maximum dot product.*
2. $\theta = 90°$ → *zero dot product*
3. $a \cdot b = b \cdot a$ → *order doesn't matter*

- *Only horizontal component contributes when force is angled.*
- *No work in vertical direction if no vertical movement.*
- *Notice, $F\cos(\theta)$ is the horizontal component of force that appears in the dot product*

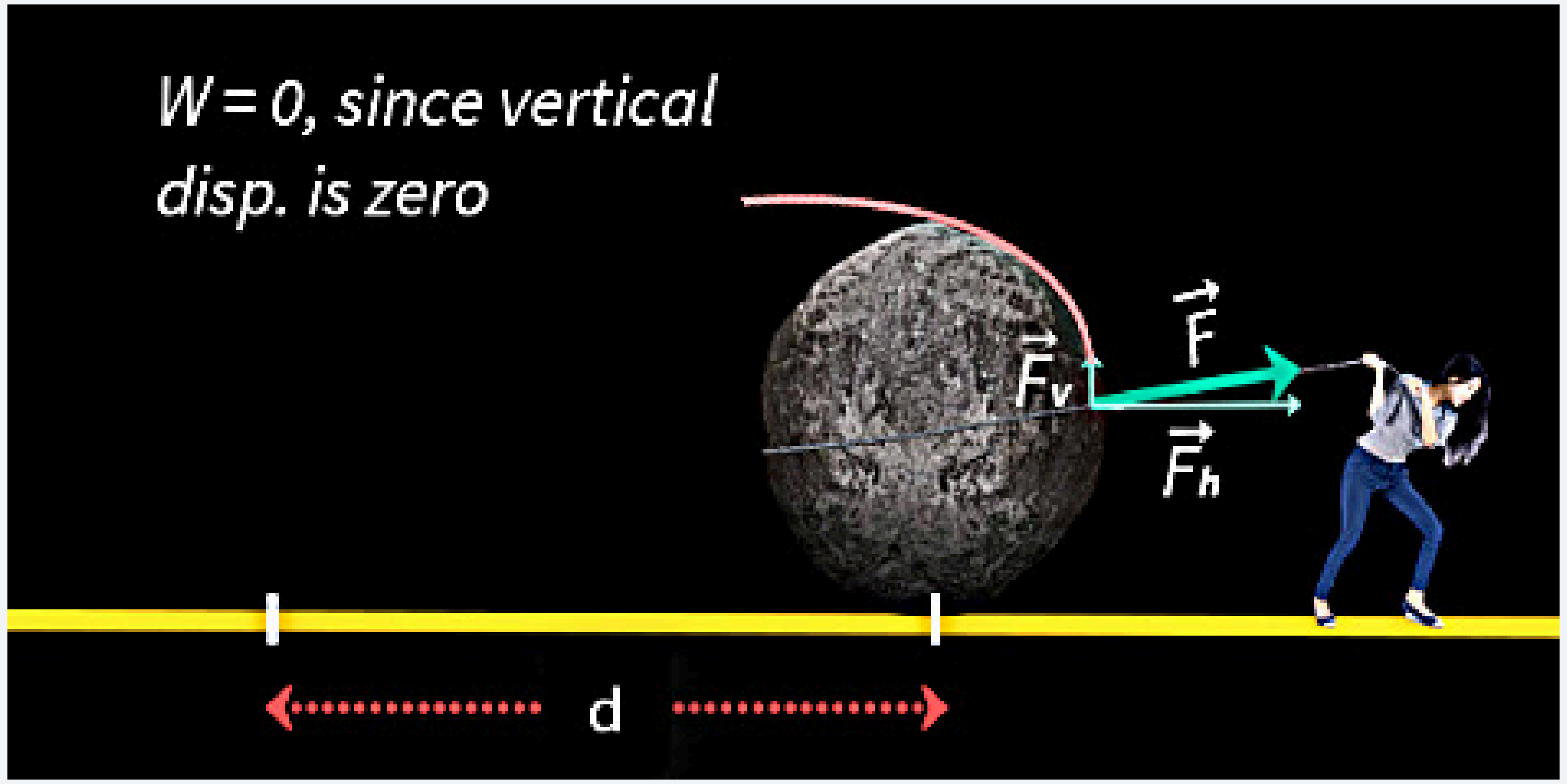
<u>Problem solving</u>

Method 1: when angle is known	$a \cdot b = \|a\|\,\|b\| \cos(\theta)$
Method 2: when components are known	$a \cdot b = A_xB_x + A_yB_y + A_zB_z$
To find θ, use	$\cos\theta = a \cdot b / \|a\|\|b\|$

Work Done by a Force: Aligned vs. Angled



Work is easy to calculate when force and displacement are in the same direction — just multiply the two.
$W = F \times d$



When force is applied at an angle, only the horizontal component — $F \cdot \cos(\theta)$ — contributes to the work. The vertical component, $F \cdot \sin(\theta)$, does no work since there's no displacement in the vertical direction

Why Only the Horizontal Component Does Work

When a force is applied at an angle to the direction of displacement, only the horizontal component of the force contributes to the work done.

The force is resolved into two components:

$F_h = F \cdot \cos(\theta)$

→ does work (aligned with displacement)

$F_v = F \cdot \sin(\theta)$

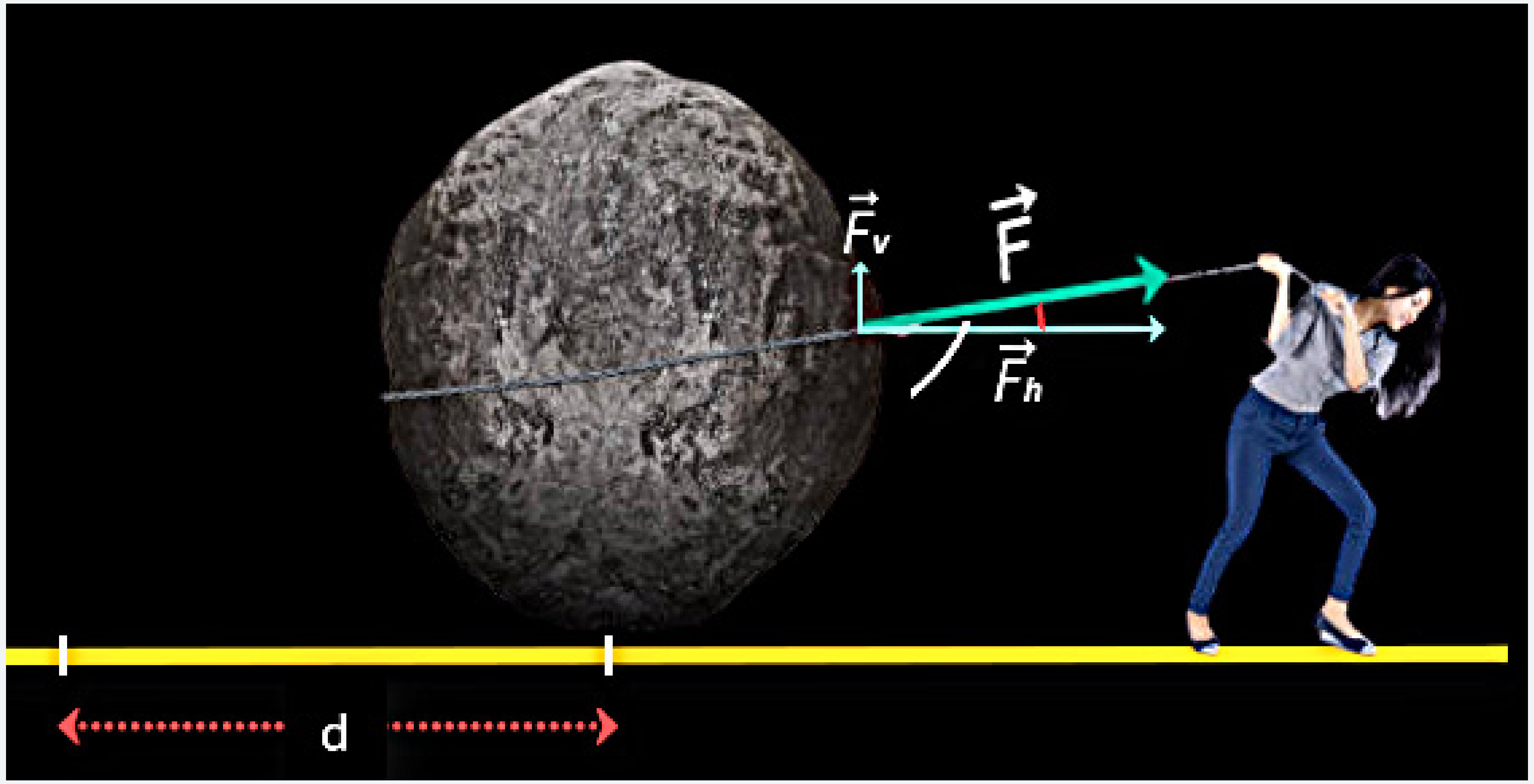
→ does no work (no vertical displacement)

So, the work done is:

$W = F_h \times d = F \cdot \cos(\theta) \cdot d$

This is precisely the dot product of the force and displacement vectors:

$$W = F \cdot d = |F||d|\cos(\theta)$$



Note: Whether the angle is θ or 360° − θ, the value of cos(θ) remains the same. That's why the dot product is unaffected by the direction in which the angle is measured.