Great Circle Sailings Problems

- Terms:
  - DLo is the difference in two longitudes.
  - λ1 is the starting longitude.
  - λ2 is the ending longitude.
  - L1 is the starting latitude.
  - L2 is the ending latitude.
  - Vertex is the most poleward position along the great circle route.
  - Lv is the latitude of the vertex.
  - λv is the longitude of the vertex.
  - Lx is the latitude of any spot along the great circle route.
  - DLo(v) is the difference in longitude from the vertex to a point on the great circle route.
  - Cn or C is the course/course angle. Course angle is the course measured from 0° at the reference direction clockwise or counterclockwise through 90° or 180°. It is labeled with the reference direction as a prefix and the direction of measurement from the reference direction as a suffix. For example, a true course of 330° T would be noted as N 30° W.
  - D is distance in nautical miles.
- The main formula needed are: — Bowditch Formula:

	Bowditch Formula
$Sin\ C = \frac{((Sin\ DLo)(Cos\ L2))}{Sin\ D}$	Formula 24
$Cos\ Lv = (Cos\ L1)(Sin\ C)$	Formula 25
$Sin\ DLo(v) = \frac{Cos\ C}{Sin\ Lv}$	Formula 27
$Tan\ Lx = ((Cos\mathrm{DLo(v)})(\mathrm{Tan\ Lv}))$	Formula 31
$Csc\ Lx = (CscLv)(SecDVx)$	Formula 32
$Cos\ D = ((\sin L1)(\sin L2) + (\cos L1)(\cos L2)(\cos DLo))$	Formula 36
$Tan\ C = (\frac{\sin DLo}{((\cos L1)(\tan L2))-((\sin L1)(\cos DLo))})$	Formula 37

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- Bowditch article 1016 provides background knowledge on this task and is available during Coast Guard exams. Additionally, all of Bowditch chapter X is useful for this topic.

- See the article in Bowditch or in this text regarding basic trigonometric functions and particularly how to take the inverse Sin, Cos, and Tan when solving equations.

- Great Circle Sailings questions can come in three main varieties:
  - Seeking the course and/or distance.
  - Seeking the latitude and/or longitude of the vertex.
  - Seeking a position along the great circle route.

<u>Initial Course and/or Distance</u>

GRC D1. Determine the distance and initial course along a great circle route from 33° 53.3'S, 18° 23.1'E to a position 40° 27.0'N, 73° 49.4'W.

Answer: 6762.7 nm, 304.5° T

Step 1: Convert standard latitude and longitudes to decimal form:

Departure Location

33° 53.3' S = 33.888° S

18° 23.1' E = 18.385° E

Arrival Location

40° 27.0' N = 40.450° N

73° 49.4' W = 73.823° W

Step 2: Determine the Difference in Longitude.

$\lambda 1 = 18.385°$ E

$\lambda 2 = 73.823°$ W

DLo = 18.385° + 73.823° = 92.208° to the west.

Step 3: Determine the Distance.

$$Cos\ D = ((\sin L1)(\sin L2) + (\cos L1)(\cos L2)(\cos DLo))$$

$$Cos\ D = ((\sin 33.888°)(\sin -40.450°) + (\cos 33.888°)(\cos -40.450°)(\cos 92.208°))$$

$$Cos\ D = ((0.5576)(-0.6488) + (0.8301)(0.7610)(-0.0385))$$

$$Cos\ D = ((-0.3618) + (-0.0243))$$

$$Cos\ D = -0.3861$$

$$D = 112.7° = \mathbf{6762.7}\boldsymbol{nm}$$

Step 4: Determine the Course Angle.

$$Sin\ C = \frac{((Sin\ DLo)(Cos\ L2))}{Sin\ D}$$

$$Sin\ C = \frac{((Sin\ 92.208°)(Cos\ 40.450°))}{Sin\ 112.7°}$$

$$Sin\ C = \frac{(0.9993)(0.7610)}{(.9225)}$$

$$Sin\ C = \frac{(0.7605)}{(.9225)}$$

$$Sin\ C = 0.8244$$

$$C = 55.5°$$

Because of the departure from the southern hemisphere, the elevated pole is the south pole and the correct course angle notation is S 55.5° W.

However due to the change in hemispheres, the prefix must be reversed, so the correct course angle is N 55.5° W, or **304.5° T.**

Great Circle Sailings Problems

Latitude and Longitude of the Vertex Problems

GRC D2. The great circle distance from latitude 25° 50.0’ N, longitude 077° 00.0’ W to latitude 35° 56.0’ N, longitude 006° 15.0’ W is 3616 nautical miles. The initial course is 061.7° T. Determine the latitude and longitude of the vertex.

Answer: 37° 35’ N, 025° 59’ W

Step 1: Convert standard latitude and longitudes to decimal form:
Departure Location
25° 50.0’ N = 25.833° N
77° 00.0’ W = 77.000° W
Arrival Location
35° 56.0’ N = 35.933° N
06° 15.0’ W = 06.250° W

Step 2: Determine the Latitude of the Vertex.

$$Lv = \cos^{-1}((\cos L1)(\sin C))$$
$$Lv = \cos^{-1}((\cos 25.833°)(\sin 61.7°))$$
$$Lv = \cos^{-1}(0.9000)(0.8805)$$
$$Lv = \cos^{-1}(0.7924)$$
$$Lv = 37.587°$$

Step 3: Convert the decimal longitude to standard notation.
37.587° N = **37° 35’ N**

Step 4: Determine the Difference in Longitude of the Vertex *(DLo(v))*

$$DLo(v) = \sin^{-1}\left(\frac{\cos C}{\sin Lv}\right)$$
$$DLo(v) = \sin^{-1}\left(\frac{\cos 61.7°}{\sin(37.587)}\right)$$
$$DLo(v) = \sin^{-1}\left(\frac{(0.4741)}{(0.6099)}\right)$$
$$DLo(v) = \sin^{-1}(0.7773)$$
$$DLo(v) = 51.02°$$

Step 5: Determine the Longitude of the Vertex given the initial longitude and the direction of travel (easterly in this case based on given positions).
Longitude 1 = 77.000° W
Difference of Long(vx) = 51.02°
Longitude of the Vertex = 77.00° - 51.02° = 25.98° W

Step 6: Convert the decimal longitude to standard notation.
25.98° W = **25° 59’ W**

Great Circle Sailings Problems

Positions Along the Great Circle Route

GRC D3. The great circle distance from latitude 25° 50.0' N, longitude 077° 00.0' W to latitude 35° 56.0' N, longitude 006° 15.0' W is 3616 nautical miles. The initial course is 061.7° T. The position of the vertex is 37° 35' N, 25° 59' W. Determine the latitude intersecting the great circle track 600 miles west of the vertex, along the great circle track.

Answer: 37° 09' N

Step 1: Convert standard latitude and longitudes to decimal form:

Departure Location

25° 50.0' N = 25.834° N

77° 00.0' W = 77.000° W

Arrival Location

35° 56.0' N = 35.934° N

06° 15.0' W = 06.250° W

Latitude of the Vertex

37° 35' N = 37.583° N

Longitude of the Vertex

25° 59' W = 25.983° W

Step 2: Convert 600 miles to arc.

$600 \div 60 = 10°$

Step 3: Determine the latitude at a position 600 miles (10° of arc) west of the vertex along the great circle track.

$$Tan\ Lx = \left((Cos\mathrm{DLo(v)})(\mathrm{Tan\ Lv})\right)$$

$$Tan\ Lx = \left((Cos(10°))(\mathrm{Tan}\ 37.583)\right)$$

$$Tan\ Lx = \left((0.9848)(0.7696)\right)$$

$$Tan\ Lx = (0.7579)$$

$$Lx = 37.158°$$

Step 4: Convert the decimal latitude to standard notation.

$Lx =$ 37.158° = **37° 09' N**