CENTRIPETAL FORCE

Centripetal Force

1. If an object is moving in uniform circular motion, which of the following must be true?
   - A There is a single force acting on the object which points towards the center of the circle
   - B There is a net force acting on the object which points away from the center of the circle
   - C There is a single force acting on the object which points away from the center of the circle
   - D There is a net force acting on the object which points towards the center of the circle

2. Which of the following could be a centripetal force? (Select all that apply)
   - A Tension force
   - B Gravitational force
   - C Normal force
   - D None of the above, a centripetal force is a different type of force

3. An object with mass m is in uniform circular motion and experiences a centripetal acceleration of a_c. Which of the following represent the centripetal force acting on the object?
   - A m / a_c
   - B $a_c m$
   - C a_c / m
   - D $m a_c^2$

4. A car is driving around a flat track in uniform circular motion. At any point around the track, the car's velocity vector and net force vector…
   - A point in the same direction
   - B point in opposite directions
   - C are perpendicular to each other
   - D are at an angle to each other but not perpendicular

5. A ball is attached to the end of a string which swings in a horizontal circle so that the speed of the ball is constant. As the ball moves around the circle, which of the following are constant? (Select all that apply)
   - A The magnitude of the tension in the string
   - B The magnitude of the net force on the ball
   - C The gravitational force on the ball
   - D The magnitude of the acceleration of the ball

6. A net force of F is acting on an object in uniform circular motion. If the object's speed doubles and the radius of the circular path stays the same, what is the new net force on the object in terms of F?
   - A $4F$
   - B $2F$
   - C F
   - D $F/4$

7. A car is driving around a flat circular track at a constant speed. What type of force is acting as the centripetal force in this scenario?

   A A gravitational force
   B A kinetic friction force
   C A normal force
   D A static friction force

8. A ball is attached to the end of a string which swings around in a vertical circle in uniform circular motion. As the ball moves around the circle, which of the following are constant? (Select all that apply)

   A The weight of the ball
   B The tension in the string
   C The magnitude of the net force on the ball
   D The speed of the ball

9. An object is moving in uniform circular motion. If the radius of the circular path doubles but the net force acting on the object remains the same, what is the new speed of the object in terms of the original speed, $\boldsymbol{v_0}$?

   A $2\boldsymbol{v_0}$
   B $\sqrt{2}\,\boldsymbol{v_0}$
   C $\boldsymbol{v_0}/2$
   D $\boldsymbol{v_0}/4$

10. A person is riding in a fast-moving car when the driver makes a left turn at a constant speed, and the car follows a circular path for a short time. During the turn, the rider feels like they are being pushed against the right side of the car. In this scenario, which of the following is true?

   A There is a net force acting on the person which pushes them to the right
   B There is a net force acting on the person which pushes them in the direction of the car's velocity
   C There is a net force acting on the person which pushes them to the left
   D There is no net force acting on the person

11. A ball is attached to the end of a rope which swings around in outer space (assume there are no gravitational forces involved). The ball follows a circular path, and the rope and ball rotate around with an angular speed of $\boldsymbol{\omega}$. The tension in the rope is $\boldsymbol{T}$, the rope has negligible mass and the mass of the ball is $\boldsymbol{m}$. Which of the following represent the radius of the circular path of the ball?

   A $m\omega$
   B $\dfrac{m\omega^2}{T}$
   C $\dfrac{\omega^2}{m}$
   D $\dfrac{T}{m\omega^2}$

12. A person is on a swing which moves back and forth in a circular arc. When they are at the lowest point in the motion (the person is directly below the pivot point of the swing), which of the following is true?

   A The person's apparent weight is greater than their true weight
   B The person's apparent weight is less than their true weight
   C The person's apparent weight is the same as their true weight
   D The relationship between their apparent weight and true weight cannot be determined

13. An object is in uniform circular motion with a period of T. If the centripetal force acting on the object is held constant but the radius of the circle is decreased to half of the initial radius, what is the new period of the motion?

A $2T$

B $\frac{T}{2}$

C $\sqrt{\frac{1}{2}}T$

D $\frac{T}{4}$

14. When it rains, the coefficient of static friction between car tires and the road decreases which can sometimes cause the tires to slip. Which of the following is true when a car is driving in the rain? (Select all that apply)

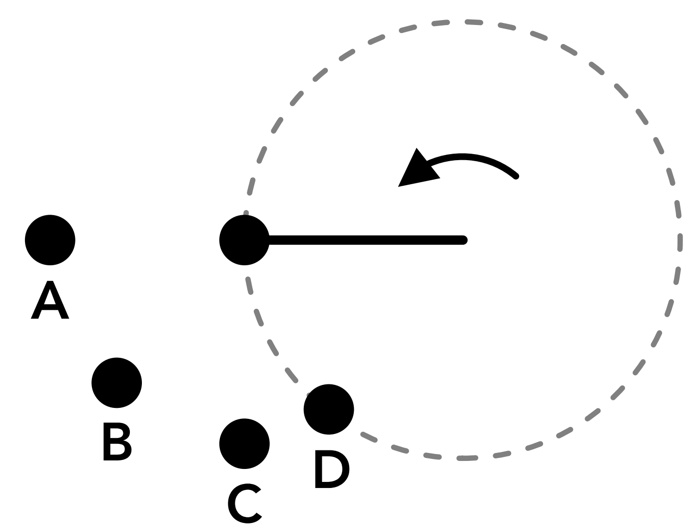
A When driving at a certain speed, the minimum radius of a curved road that the car can drive around decreases

B For a curved road with a certain radius, the maximum speed the car can drive around the curve decreases

C When driving at a certain speed, the minimum radius of a curved road that the car can drive around increases

D For a curved road with a certain radius, the maximum speed the car can drive around the curve increases

15. A ball is attached to the end of a rope and moves in horizontal uniform circular motion as shown on the right. When the ball and the rope are in the position shown, the rope is instantly cut. Which point shows where the ball would be at a period of time later?



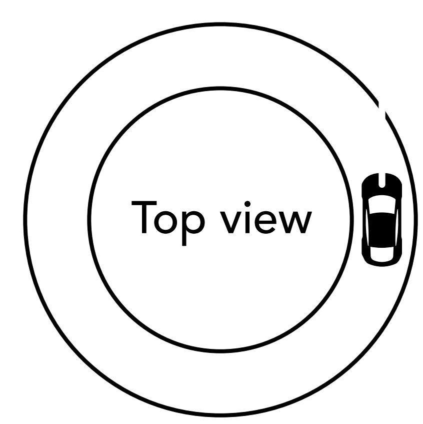
A Position A

B Position B

C Position C

D Position D

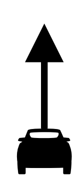
16. A car is driving around a circle at a constant speed as shown on the right which is a top-down view of the car and the road. Which of the following diagrams show the net force acting on the car as viewed from the back of the car?



A

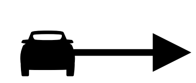
Back view

B



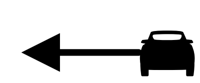
Back view

C



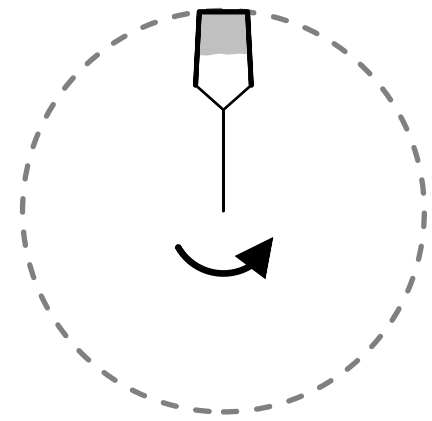
Back view

D



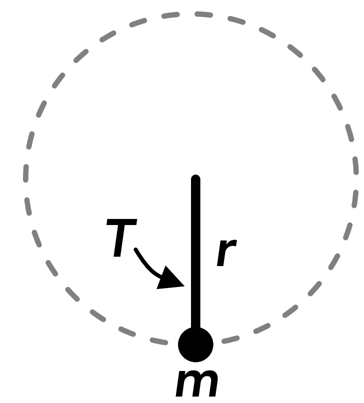
Back view

17. A bucket of water swings around in a vertical circle at a constant speed. When the bucket is at the top of the circle the buck is upside down, but the water in the bucket does not fall out. Which of the following explains why that is?



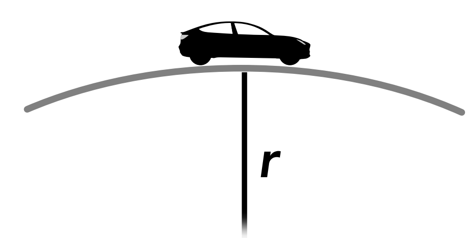
- A The bucket is in circular motion but the water is not
- B The water moves to the bottom of the bucket because of its inertia
- C The water has no weight at that moment
- D There is a force pushing the water away from the center of the circle

18. A ball of mass ***m*** is attached to a string with negligible mass and swung in a vertical circle with radius ***r*** at a constant speed. When the ball is in the position shown on the right, the tension in the string is ***T***. Which of the following represent the speed of the ball at that moment?

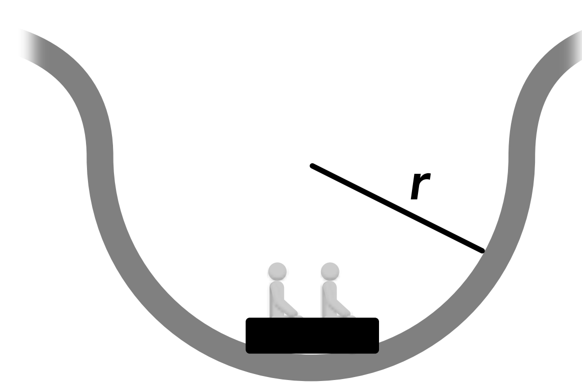


- A $\sqrt{\dfrac{r(T - mg)}{m}}$
- B $\sqrt{rT}$
- C $\sqrt{\dfrac{rT}{m}}$
- D $\sqrt{r(T + mg)}$

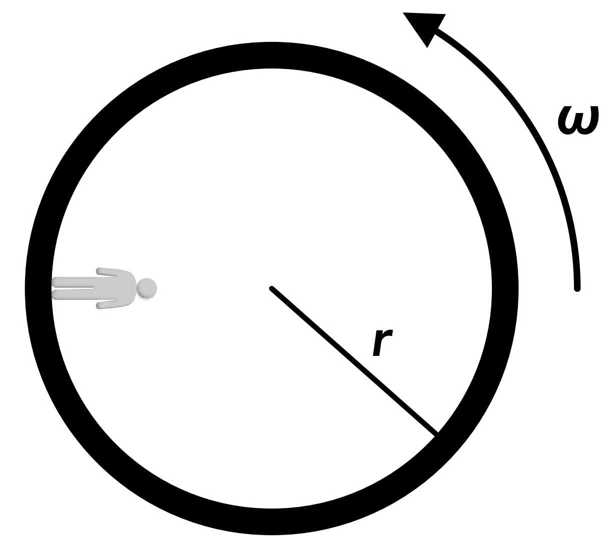
19. A car with a mass of 1000 kg is driving at a constant speed over a circular hill as shown on the right. If the car drives too fast it cannot maintain contact with the ground and its path will be above the ground. The radius of the circular hill is 50 m. What is the maximum speed the car can be driving when at the top of the hill so that it does not lose contact with the ground?



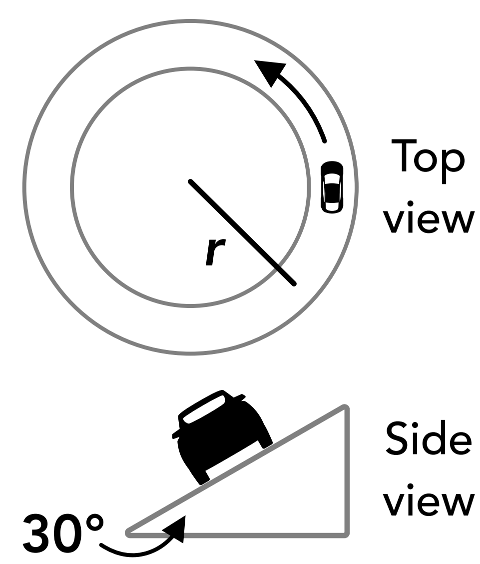
20. An 80 kg person is riding on a roller coaster that goes through a dip with a circular section which has a radius of 20 m. If the person is moving at a speed of 15 m/s at the bottom of the dip, what is their apparent weight?



21. An astronaut is in a space station deep in outer space where the gravitational force from any planet or other body is negligible. The space station is a cylinder which rotates with an angular speed of $\boldsymbol{\omega}$ in order to create artificial gravity when the astronaut is standing on the edge of the cylinder as shown. If the radius of the station is 200 m, what angular speed should it rotate so the astronaut's apparent weight is the same as their true weight on earth?



22. A car is driving around a banked circular track at a constant speed. The road is inclined at 30° relative to the ground and the coefficient of static friction between the tires and the road is 0.4. If the radius of the car's circular path is 15 m, what is the maximum speed the car can drive before it slips up the incline?



Answers

1. D	6. A	11. D	16. D	21. 0.22 rad/s
2. A, B, C	7. D	12. A	17. B	22. 12 m/s
3. B	8. A, C, D	13. C	18. A	
4. C	9. B	14. B, C	19. 22.1 m/s	
5. A, B, C, D	10. C	15. C	20. 1684 N	

Answers - Centripetal Force

1. **Answer: D**
   An object in uniform circular motion must have a net force acting on it towards the center of the circle, which is called the centripetal force. That may be a single force or the net force of several different forces.

2. **Answer: A, B, C**
   The centripetal force is just the net force acting on an object in circular motion, so any type of force can act as the centripetal force, and it could be more than one type of force resulting in the net force.

3. **Answer: B**
   The centripetal force is the centripetal acceleration multiplied by the mass (Newton's 2nd law).

4. **Answer: C**
   When an object is in uniform circular motion, its velocity vector always points tangent to the circular path and the net force is the centripetal force which always points towards the center of the circle, so the velocity vector and the net force vector are perpendicular to each other.

5. **Answer: A, B, C, D**
   The gravitational force on the ball doesn't depend on its motion so it's always the same. The ball is in uniform circular motion so the magnitude of the acceleration (the centripetal acceleration) is constant, although the direction is changing. The magnitude of the centripetal force is constant, which is the net force on the ball, although the direction of the net force is changing. The horizontal component of the tension force acting on the ball is the centripetal force, which is constant, so the tension in the string is also constant.

6. **Answer: A**
   The centripetal force (the net force acting on an object in uniform circular motion) is proportional to the square of the speed, shown in the equation below. If the speed is multiplied by 2 the force must be multiplied by 4.

   $$F_c = \frac{mv^2}{r}$$

7. **Answer: D**
   As the car moves in a circle, a static friction force acts sideways on the tires from the ground (perpendicular to the car's direction of motion) which prevents the car from sliding outwards away from the center of the circle.

8. **Answer: A, C, D**
   The weight of the ball is the gravitational force acting on the ball which does not change. The magnitude of the net force on the ball is the centripetal force which is constant. The ball is in uniform circular motion so the speed of the ball is constant. The net force on the ball is the sum of the weight force and the tension force, which act in the same direction at the top of the circle and opposite directions at the bottom of the circle, so the magnitude of the tension force must be changing.

9. **Answer: B**
   The relationship between the centripetal force (the net force), the speed and the radius is shown in the equation below. If the radius is multipled by 2 the speed must be multiplied by $\sqrt{2}$ if the force is the same.

   $$F_c = \frac{mv^2}{r}$$

10. **Answer: C**
    During the turn, the person is in uniform circular motion so there must be a net force acting on the person which points towards the center of the circular path, which in this example is to the left. This is the centripetal force, which is caused by the seat or the side of the car pushing the person to the left to keep them moving in a circle. If the car suddenly disappeared, the person would move in a straight line tangent to the circular path, away from the center of the circle because of their inertia (Newton's 1st law of motion).

11. **Answer: D**
    The variable for linear velocity in the centripetal force or acceleration equation can be replaced with other variables that correspond to the speed. The linear speed is equal to the radius times the angular speed.

    $$v = r\omega \qquad F_c = \frac{mv^2}{r} = \frac{mr^2\omega^2}{r} = mr\omega^2 \qquad r = \frac{F_c}{m\omega^2} = \frac{T}{m\omega^2}$$

12. **Answer: A**
    As the person swings back and forth they are following a circular path. At the lowest point in the path there is a normal force acting upwards on the person from the swing and a weight force acting downwards. The person is moving in circular motion and at the lowst point in the path their net acceleration is upwards (towards the center of the circlular path). If their net acceleration is upwards then the net force on the person is upwards, so the upwards normal force (which is their apparent weight) must be greater than the downwards weight force.

13. **Answer: C**
    The centripetal force is related to the period and radius in the equation below, where the linear speed is replaced with a term that includes the period of the motion. In this equation, the centripetal force is proportional to the radius and inversely proportional to the square of the period. If the radius is multipled by 1/2, the period must be multipled by $\sqrt{1/2}$ if the centripetal force is the same.

    $$F_c = m\left(\frac{2\pi}{T}\right)^2 r \qquad F_c \propto \frac{r}{T^2} \propto \frac{(1/2)r}{(\sqrt{1/2}\,T)^2}$$

14. **Answer: B, C**
    When a car is driving around a curved road, the sideways static friction force between the tires and the road is acting as the centripetal force preventing the car from sliding sideways, away from the center of the curved path. If the coefficient of static friction decreases when it rains, the maximum static friction force that can be exerted on the tires decreases, so the maximum possible centripetal force decreases. The centripetal force is related to the car's speed and the curved road's radius in the equation below. If the centripetal force decreases, the speed must also decrease if the radius is constant, or the radius must increase if the speed is constant.

    $$F_c = \frac{mv^2}{r}$$

15. **Answer: C**
    When an object is in uniform circular motion, the object's velocity is tangent to the circular path. When the rope is cut, there is no longer a centripetal force on the ball (the tension force disappears). If there is no force acting on the ball it will move in a straight line at a constant speed (Newton's 1st law of motion).

16. **Answer: D**
The car is in uniform circular motion so there is a net horizontal force acting on the car which points towards the center of the circle, which is the centripetal force (in this case it's a static friction force). The car is not accelerating up or down vertically so the net force in the vertical direction is zero.

17. **Answer: B**
The water and the bucket are both moving in circular motion. If at any moment the bucket disappeared, the water would move in a straight line tangent to the circle at a constant speed because of its inertia (Newton's 1st law of motion). The water "wants" to move away from the center of the circle, which is towards the bottom of the bucket. There is a normal force acting on the water from the bucket which points towards the center of the circle, which is the centripetal force keeping the water in circular motion. The water always has weight, and there is no actual force pushing the water away from the center of the circle (the fictitious "centrifugal force").

18. **Answer: A**
The ball is swinging in a vertical circle so there is an upwards tension force and a downwards weight force acting on the ball at that moment. The net force on the ball, which is the tension force minus the weight force, is acting as the centripetal force.

$$\sum F = F_c = \frac{mv^2}{r} \qquad T - mg = \frac{mv^2}{r} \qquad v = \sqrt{\frac{r(T - mg)}{m}}$$

19. **Answer: 22.1 m/s**
When the car is at the top of the hill, there is an upwards normal force and a downwards weight force acting on the car. The centripetal force keeping the car in circular motion (following the curve of the hill and not losing contact with the ground) is the weight force minus the normal force. As the car's speed increases, the centripetal force required to keep the car on the ground increases. The maximum possible centripetal force is when the net force on the car is only the weight force and the normal force is zero, which is when the car will lose contact with the ground. The maximum speed is when the centripetal force is equal to the weight force. The mass of the car is not relevant to the maximum speed because it can be divided out on both sides of the equation.
Below the maximum speed: $F_c = F_g - F_n$
At the maximum speed: $F_n = 0 \qquad F_c = F_g \qquad \frac{mv^2}{r} = mg \qquad v = \sqrt{rg} = \sqrt{(50\text{ m})g}$ = 22.1 m/s

20. **Answer: 1684 N**
When the person is at the bottom of the circle, there is an upwards normal force and a downwards weight force acting on the person. The person's apparent weight is the upwards normal force that they experience. Since they are in circular motion, the net force towards the center of the circle is the centripetal force, which is the normal force minus the weight force.

$$\sum F = F_c \qquad F_n - F_g = \frac{mv^2}{r} \qquad F_n - (80\text{ kg})g = \frac{(80\text{ kg})(15\text{ m/s})^2}{(20\text{ m})} \qquad F_n = 1684\text{ N}$$

21. **Answer: 0.22 rad/s**
When the person stands on the inside surface of the cylinder, they are in uniform circular motion. There is a normal force acting on the person from the surface which points towards the center of the circle, acting as the centripetal force. That normal force is also the apparent weight that the person experiences, which in this case is equal to their true weight on earth.

$$F_c = F_n = F_{g\text{ earth}} \qquad m\omega^2 r = mg \qquad \omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{g}{(200\text{ m})}} = 0.22\text{ rad/s}$$

22. **Answer: 12 m/s**

When the car is in the position shown, there is a weight force acting downwards, a normal force acting perpendicular to the surface of the road (up and left) and a static friction force pointing down the incline (parallel to the incline). If the static friction force was not acting in that direction (or if it wasn't there) the car would slip up the incline because of its inertia while in circular motion. The centripetal force is the net horizontal force acting on the car pointing towards the center of the circle, which is the sum of the horizontal components of the normal force and the friction force. The vertical component of the normal force is equal to the weight force:

$$F_n \cos(30°) = F_g = mg \qquad F_n = \frac{mg}{\cos(30°)}$$

$$f_{s\,max} = \mu_s F_n = \mu_s \frac{mg}{\cos(30°)}$$

$$\Sigma F_x = F_c \qquad F_n \sin(30°) + f_s \cos(30°) = \frac{mv^2}{r} \qquad \frac{mg}{\cos(30°)} \sin(30°) + (0.4) \frac{mg}{\cos(30°)} \cos(30°) = \frac{mv^2}{(15\ \text{m})}$$

v = 12 m/s