One-sided limits

General vs. one-sided limits

When you hear your professor talking about limits, he or she is usually talking about the general limit. Unless a right- or left-hand limit is specifically specified, you're dealing with a general limit.

The general limit exists at the point $x = c$ if

1. The left-hand limit exists at $x = c$,
2. The right-hand limit exists at $x = c$, and
3. The left- and right-hand limits are equal.

These are the three conditions that must be met in order for the general limit to exist. The general limit will look something like this:

$$\lim_{x \to 2} f(x) = 4$$

You would read this general limit formula as "The limit of f of x as x approaches 2 equals 4."

Left- and right-hand limits may exist even when the general limit does not. If the graph approaches two separate values at the point $x = c$ as you approach c from the left- and right-hand side of the graph, then separate left- and right-hand limits may exist.

Left-hand limits are written as

$$\lim_{x \to 2^-} f(x) = 4$$

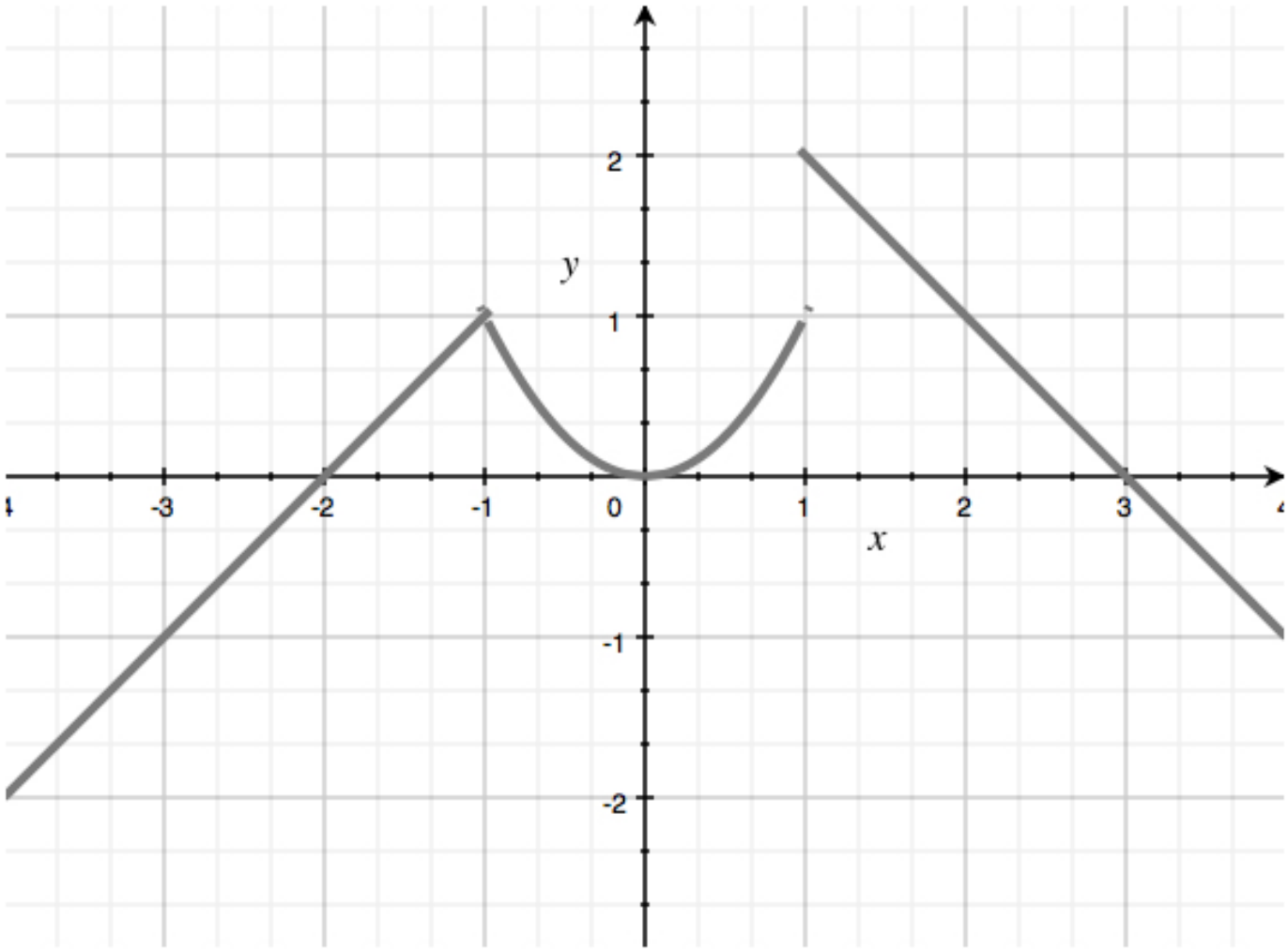
The negative sign after the 2 indicates that we're talking about the limit as we approach 2 from the negative, or left-hand side of the graph.

Right-hand limits are written as

$$\lim_{x \to 2^+} f(x) = 4$$

The positive sign after the 2 indicates that we're talking about the limit as we approach 2 from the positive, or right-hand side of the graph.

In the graph on the right, the general limit exists at $x = -1$ because the left- and right-hand limits both approach 1. On the other hand, the general limit does not exist at $x = 1$ because the left-hand and right-hand limits are not equal, due to a break in the graph.

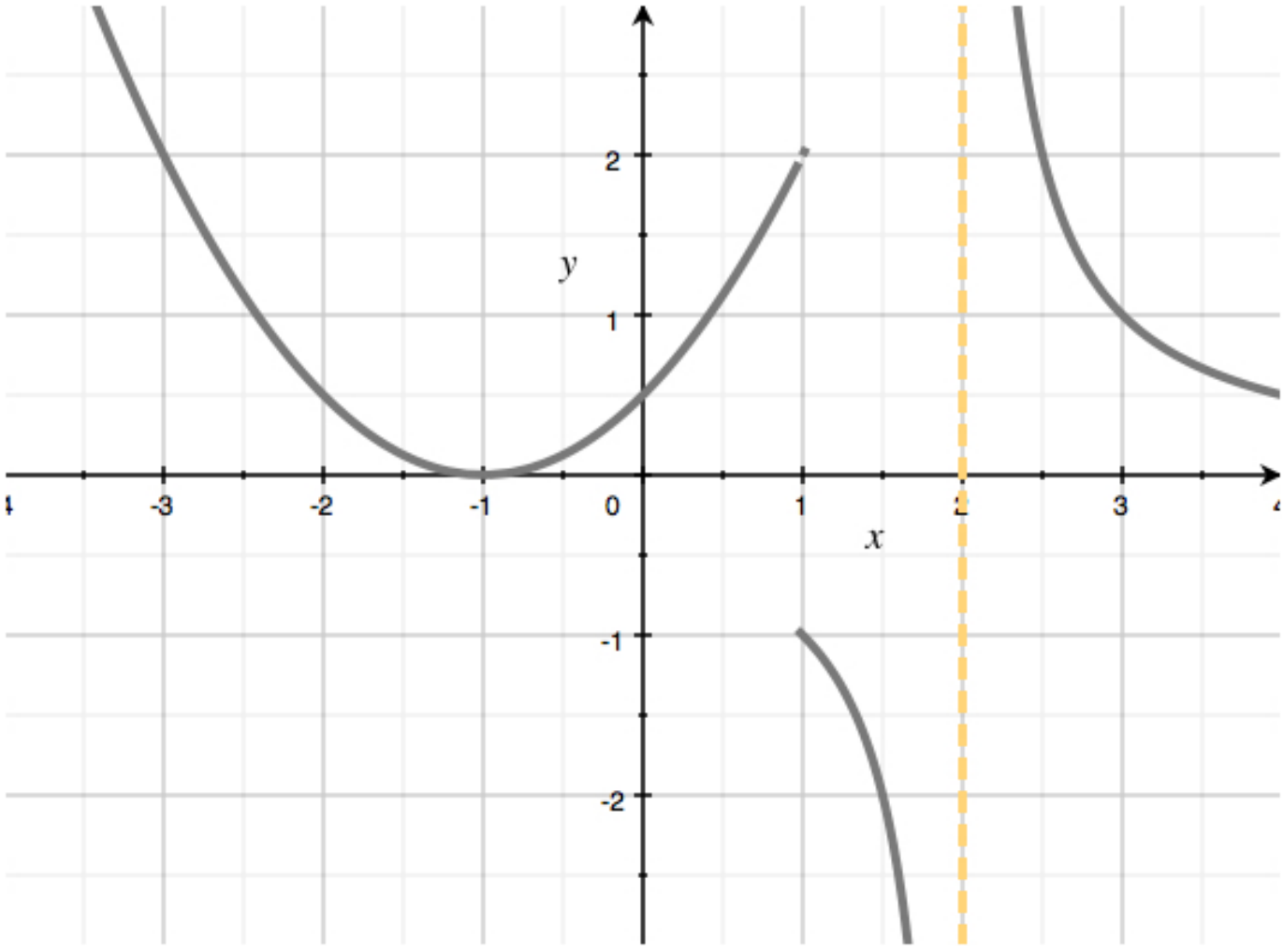


You can see from the graph that the left- and right-hand limits are equal at $x = -1$, but not at $x = 1$.

So we already know that a general limit does not exist where the left- and right-hand limits are not equal. Limits also do not exist whenever we encounter a vertical asymptote.

There is no limit at a vertical asymptote because the graph of a function must approach one fixed numerical value at the point $x = c$ for the limit to exist at c. The graph at a vertical asymptote is increasing and/or decreasing without bound, which means that it is approaching infinity instead of a fixed numerical value.

In the graph below, separate right- and left-hand limits exist at $x = 1$, so the general limit does not exist at that point. The left-hand limit is 2, because that is the value that the graph approaches as you trace the graph from left to right. On the other hand, the right-hand limit is -1, since that's the value that the graph approaches as you trace the graph from right to left.



Where there is a vertical asymptote at $x = 2$, the left-hand limit is $-\infty$, and the right-hand limit is $+\infty$. However, the general limit does not exist at the vertical asymptote because the left- and right-hand limits are unequal. So we can say that the general limit does not exist at $x = 1$ or at $x = 2$.