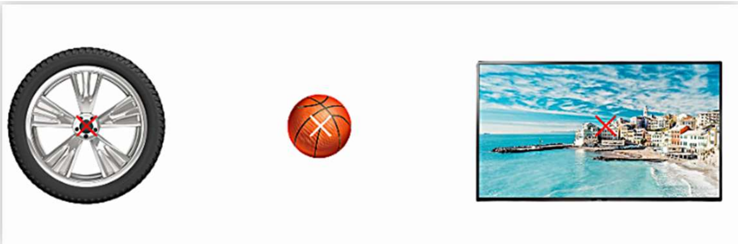
Center of Mass

Key Idea

Symmetry of objects makes it easy to find the center of mass



When dealing with objects like a ball, tire, or flat-screen television, determining the center of mass (CM) is straightforward due to their symmetry. For example, throwing a ball as a projectile allows us to predict its motion, following a parabolic path similar to that of a point mass. However, with objects like a baseball bat, where every particle moves differently, predicting motion becomes erratic.

Ball vs. Baseball Bat: A ball's symmetrical shape allows for easy identification of its COM, while a baseball bat's motion is complex because *each particle moves independently*. The bat's CM, however, follows a parabolic arc, similar to a ball's motion.

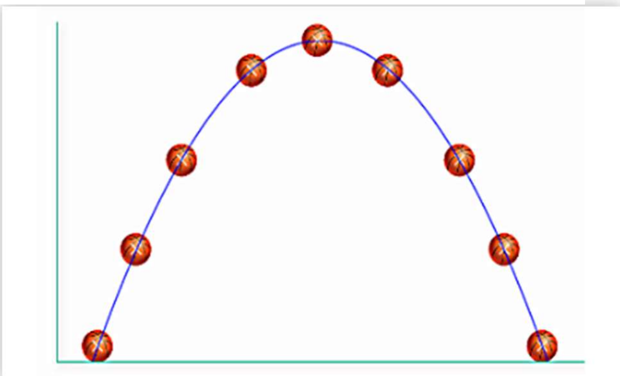


Fig 1: Balls' trajectory of CM is easy to identify

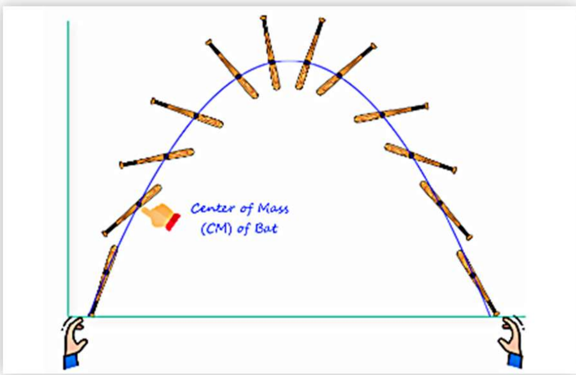
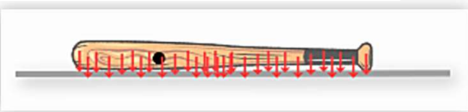


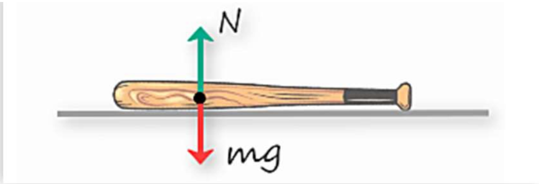
Fig. 2: Bats' trajectory of CM is not easy to identify due to lack of symmetry

Center of Mass (CM): The CM is a special point within an object that moves *as if all the object's mass were concentrated there* and all external forces were applied to it.

Force of gravity acts on every particle of the bat.

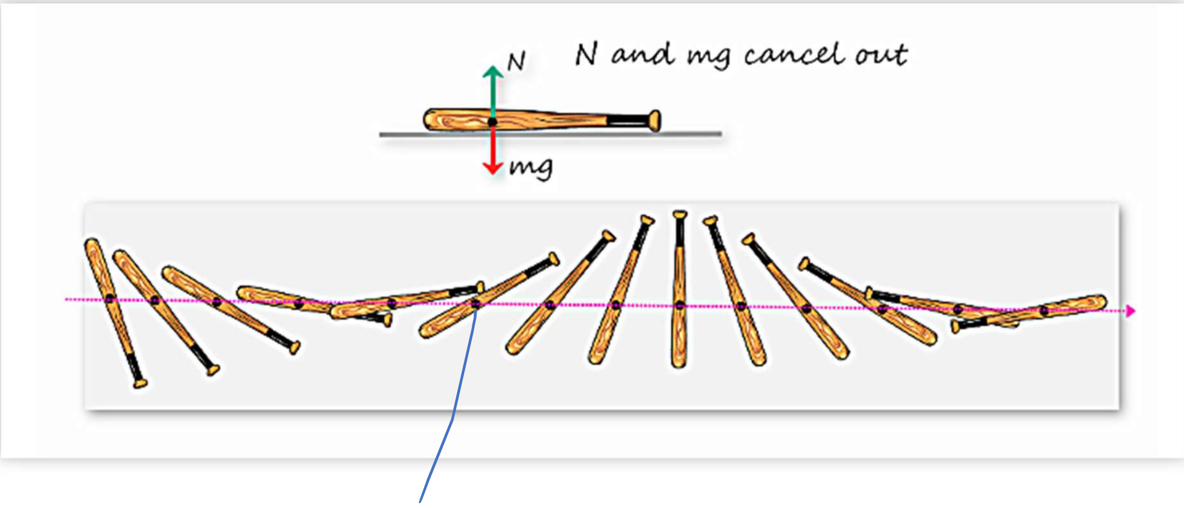


...however, while working on problems, it can be assumed that the force of gravity for the entire bat acts through the CM



For instance, when a bat moves under gravity, its CM follows a parabolic path (Fig 2, above).

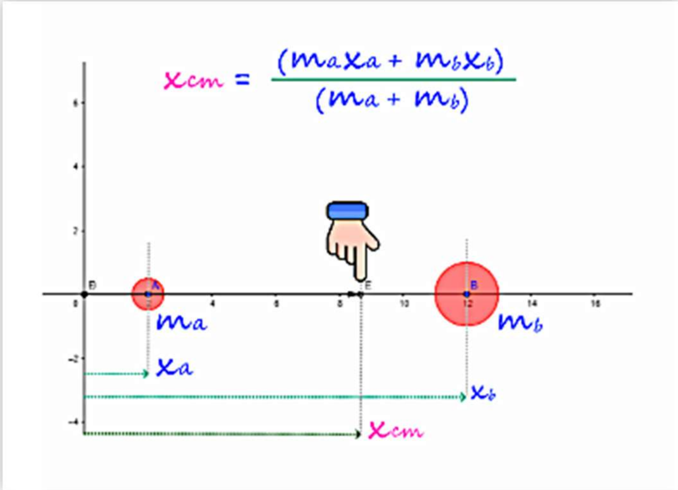
If spun on a horizontal table with no net force, the CM moves along a straight line.



CM moves along a straight line

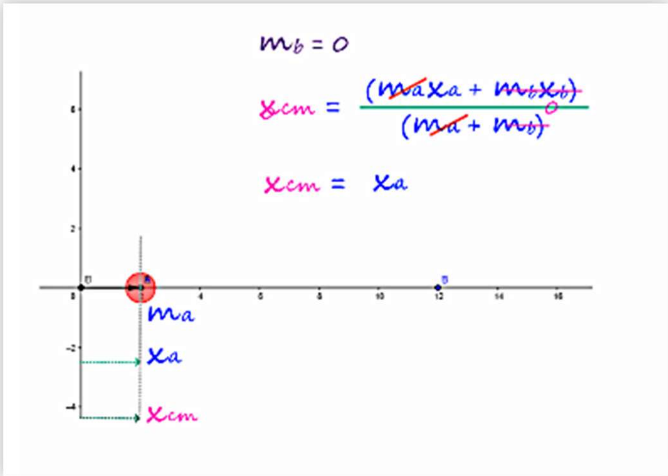
Calculating CM for a System of Particles

A. For a system of two masses, m_a and m_b, located at positions x_a and x_b respectively:

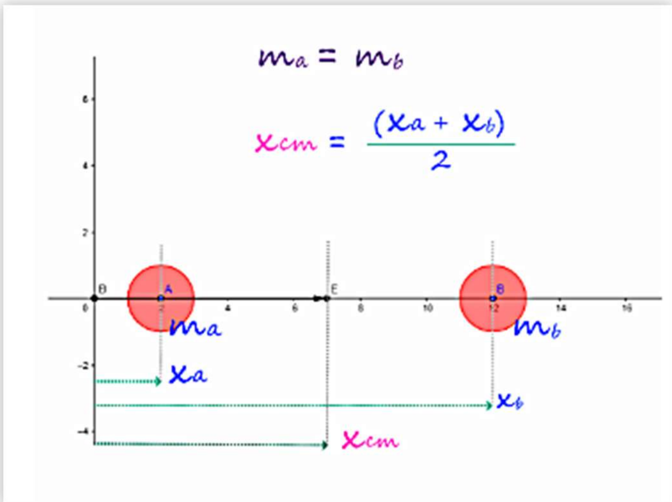


$$x_{cm} = \frac{(m_a x_a + m_b x_b)}{(m_a + m_b)}$$

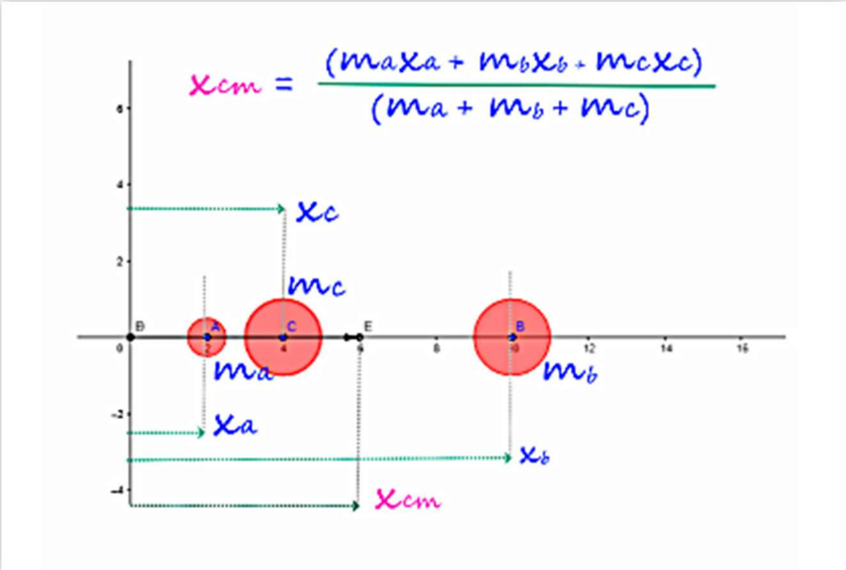
Situation 1: When $m_b = 0$, the CM coincides with x_a because the system is reduced to mass m_a only.



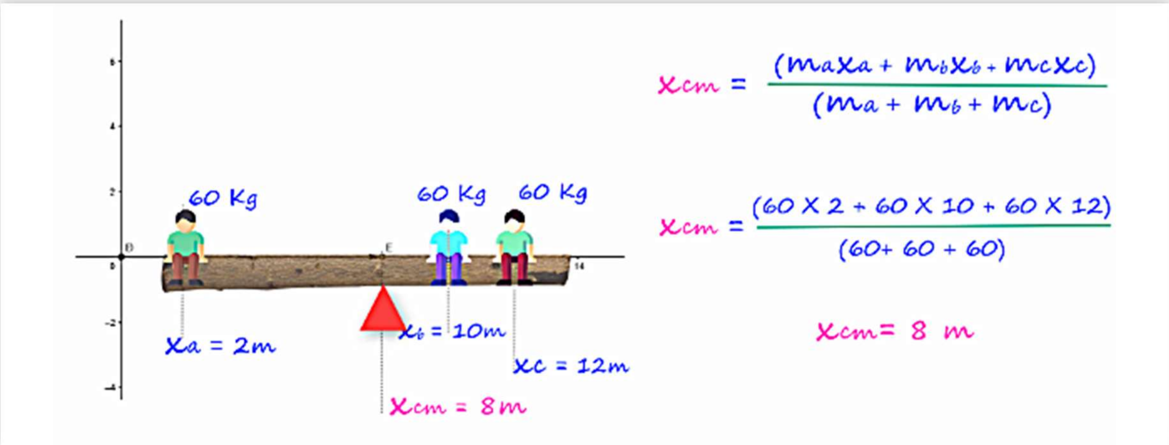
Situation 2: If $m_a = m_b$, the CM lies midway between x_a and x_b



B. Introducing a third mass m_c at position x_c, the CM of the system can be calculated by adding $m_c x_c$ in the numerator and m_c in the denominator.



Example: Consider three people of equal mass sitting on a log at positions x_a = 2, x_b = 10, and x_c = 12 meters. What is the CM (see pictorial for solution)



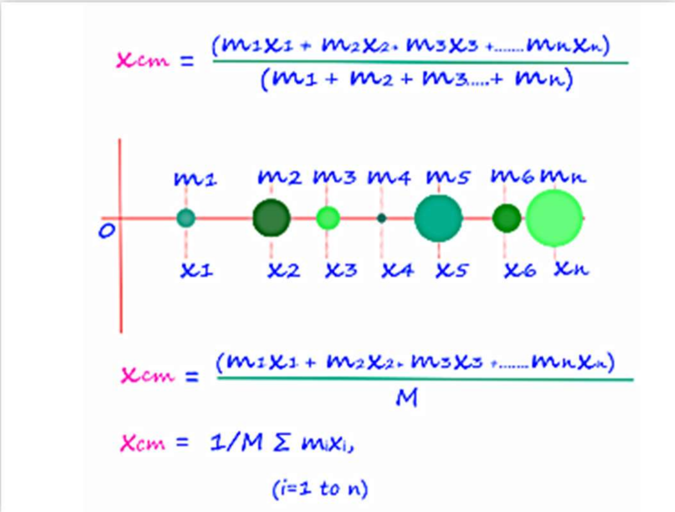
General Equation

A. For a system of n particles spread across the x-axis:

$$x_{cm} = (m_1x_1 + m_2x_2 + m_3x_3 + \ldots + m_nx_n) / (m_1 + m_2 + \ldots + m_n)$$

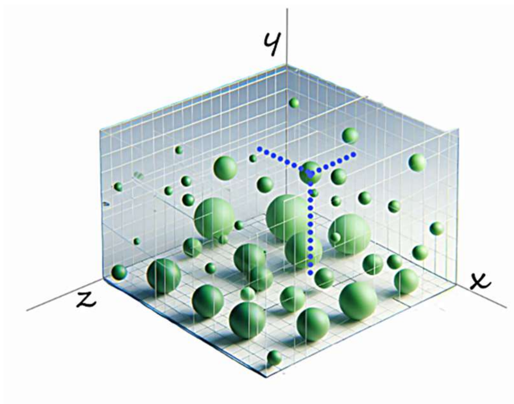
Or if $M = m_1 + m_2 + \ldots + m_n$

$$x_{cm} = (1/M) * \Sigma(m_ix_i) \qquad (i=1 \text{ to } n)$$



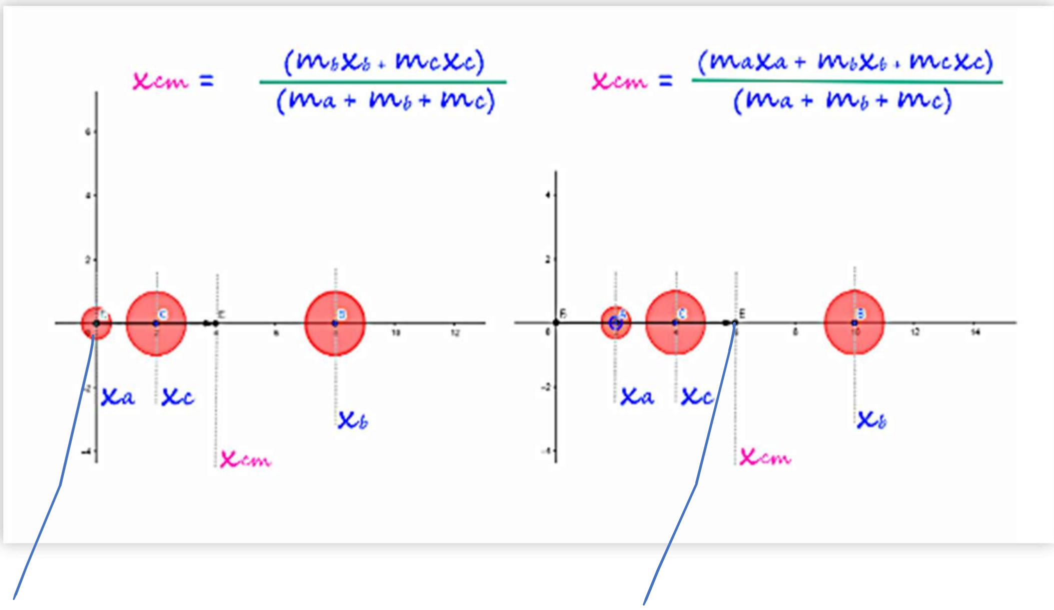
B. Similar equations can be formulated for y and z coordinates in three-dimensional space, where each coordinate represents the COM of the system.

x-coordinate of COM:	$x_{cm} = (1/M) * \Sigma(m_ix_i)$	(i=1 to n)
y-coordinate of COM:	$y_{cm} = (1/M) * \Sigma(m_iy_i)$	(i=1 to n)
z-coordinate of COM:	$z_{cm} = (1/M) * \Sigma(m_iz_i)$	(i=1 to n)



Coordinate System Selection

- Choosing a convenient coordinate system can simplify COM calculations. For instance, placing one mass at the origin eliminates certain terms from the equation, streamlining the calculation process.
- *The choice of coordinate system does not affect the COM's position relative to the particles*; it's a property of the physical particles, not the coordinate system used.



Placing A at (0,0) makes calculation a lot simpler

Distance of COM from A and B remains the same in both cases