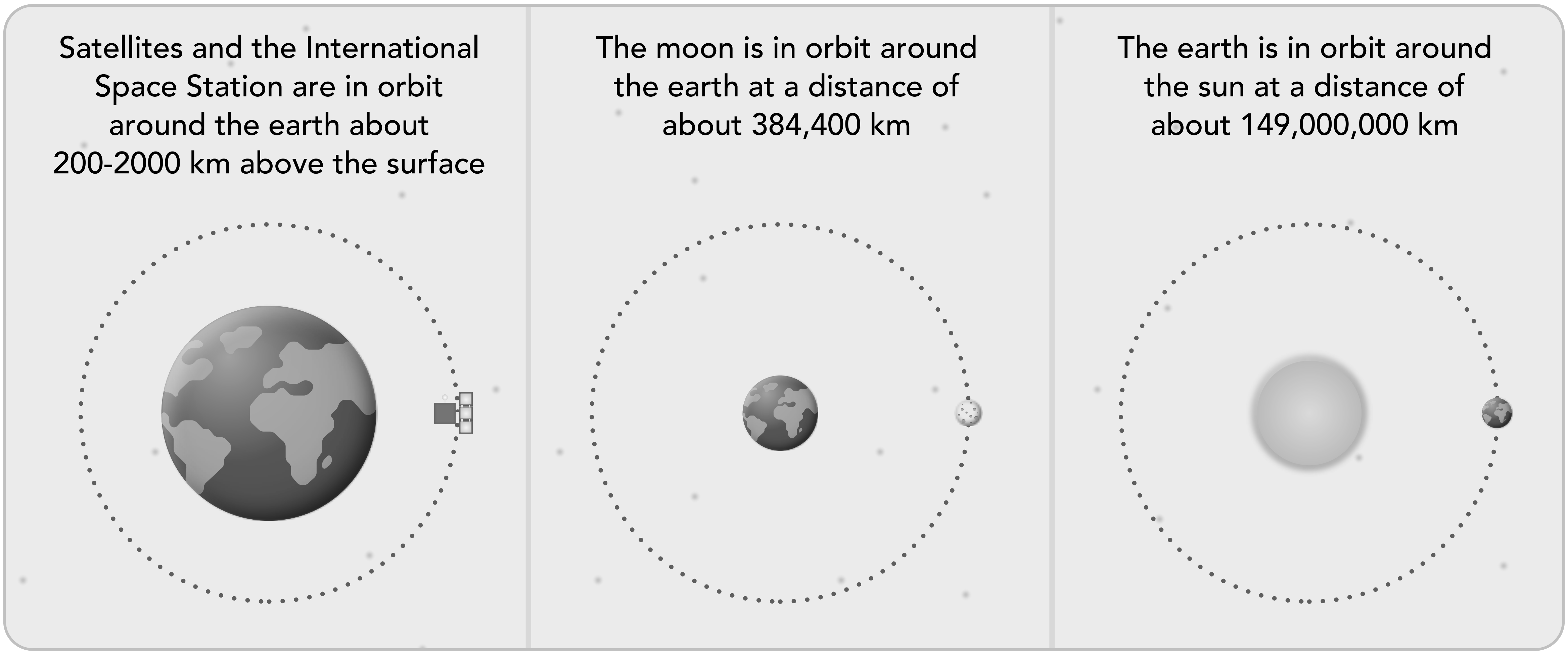
STUDY GUIDE

ORBITAL MOTION

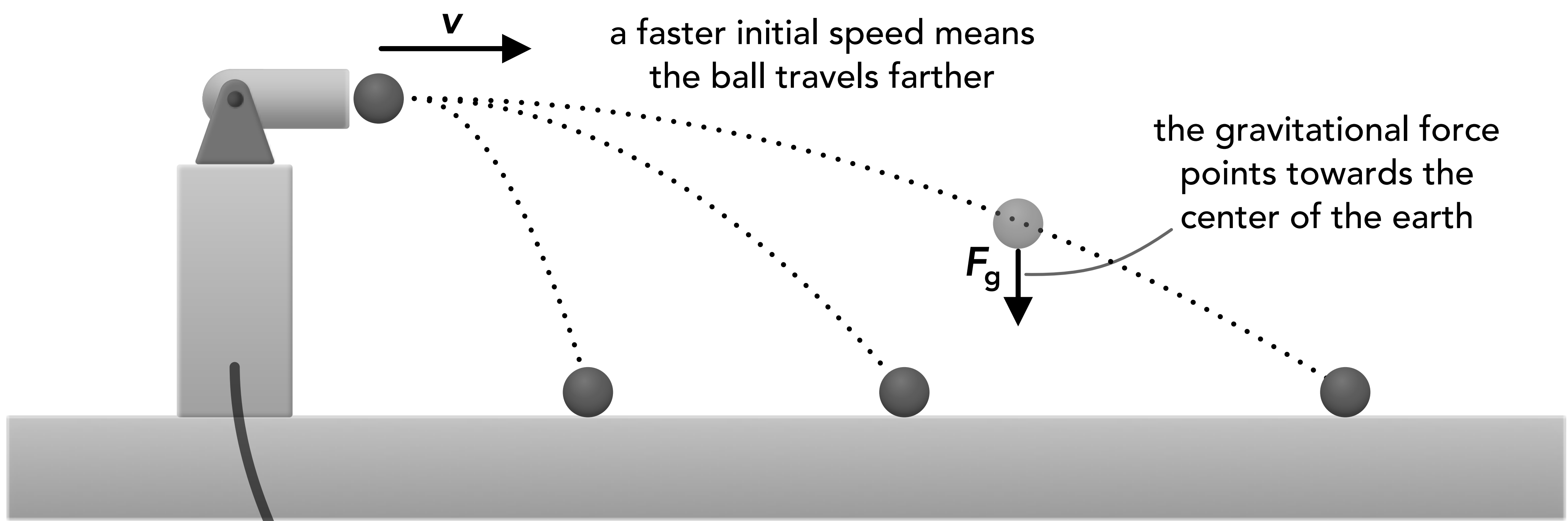
Orbital Motion

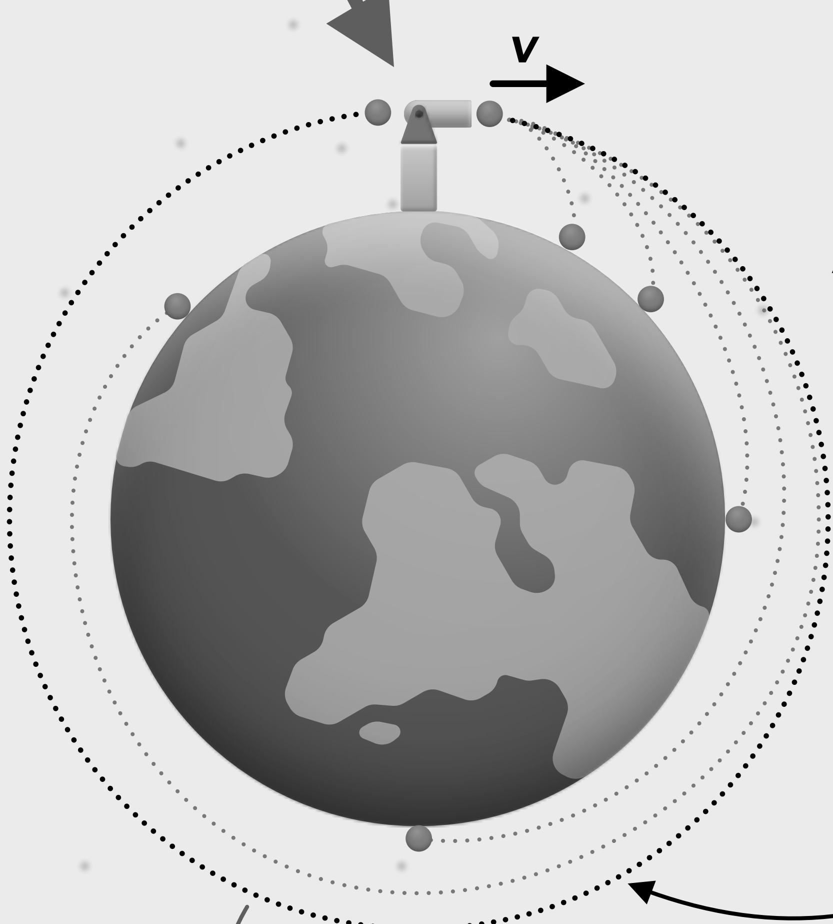
- **Orbital motion** is when an object follows a circular or elliptical path (an "orbit") around another object, where the only force acting on the object is gravity.
- Technically the two objects are both in orbit around a common point called a "barycenter" but we'll focus on a simplified version for now, where one of the objects has much more mass than the other and the barycenter is approximately at the center of the larger object.



- An object in orbital motion around a planet is actually in **projectile motion** or **free fall**.
- The only force acting on the object is the **gravitational force** from the planet which always points towards the center of the planet. It may seem like some force is required to keep the object moving, but we know from Newton's 1st law of motion that an object in motion will continue moving on its own unless a net force is applied to stop it from moving.
- We're only going to focus on the continuous orbital motion itself, not the cause of the initial velocity that started the orbital motion or changes to the orbital motion.

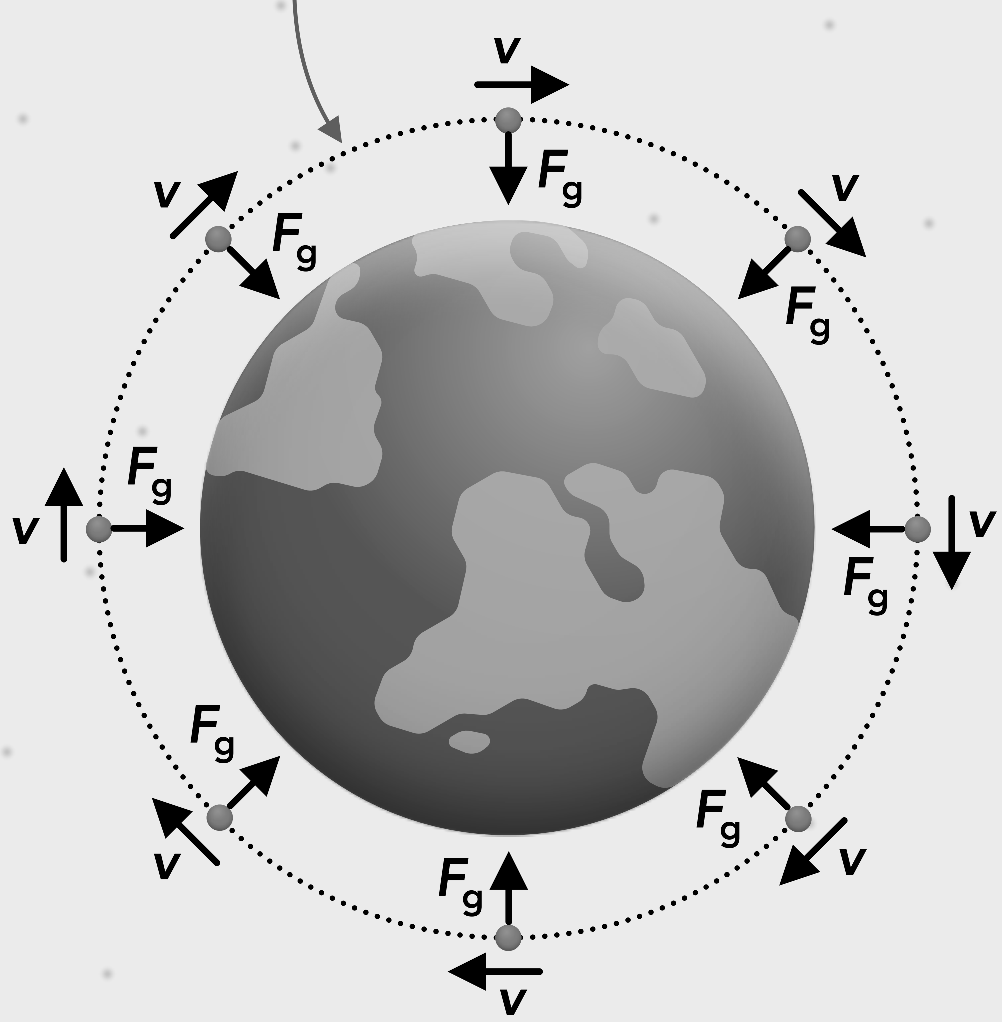
In projectile motion (or free fall) the only force acting on the object is the gravitational force (ignoring air resistance)





As the speed and the range increase to a larger scale, the curvature of the earth becomes relevant and the ground begins to “fall away” below the path of the ball.

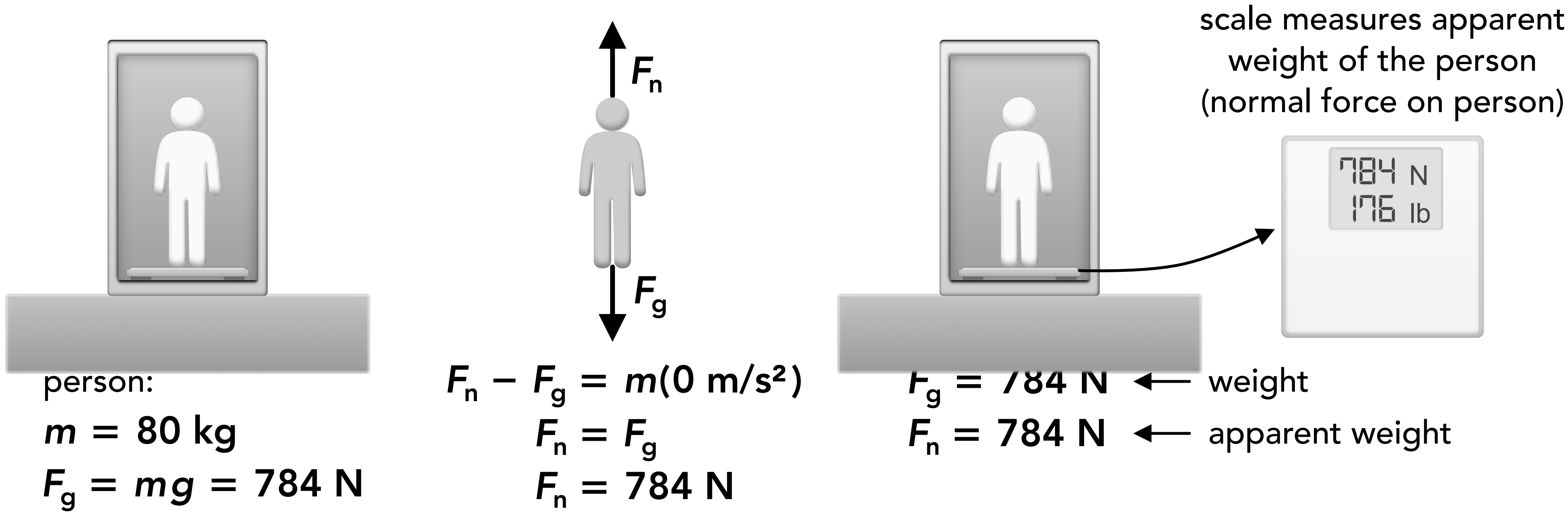
If the initial speed is fast enough (~ 7,900 m/s) the ball will never hit the ground, the projectile motion completely circles the earth and the ball ends up where it started. At this velocity the ball is in orbital motion around the earth.



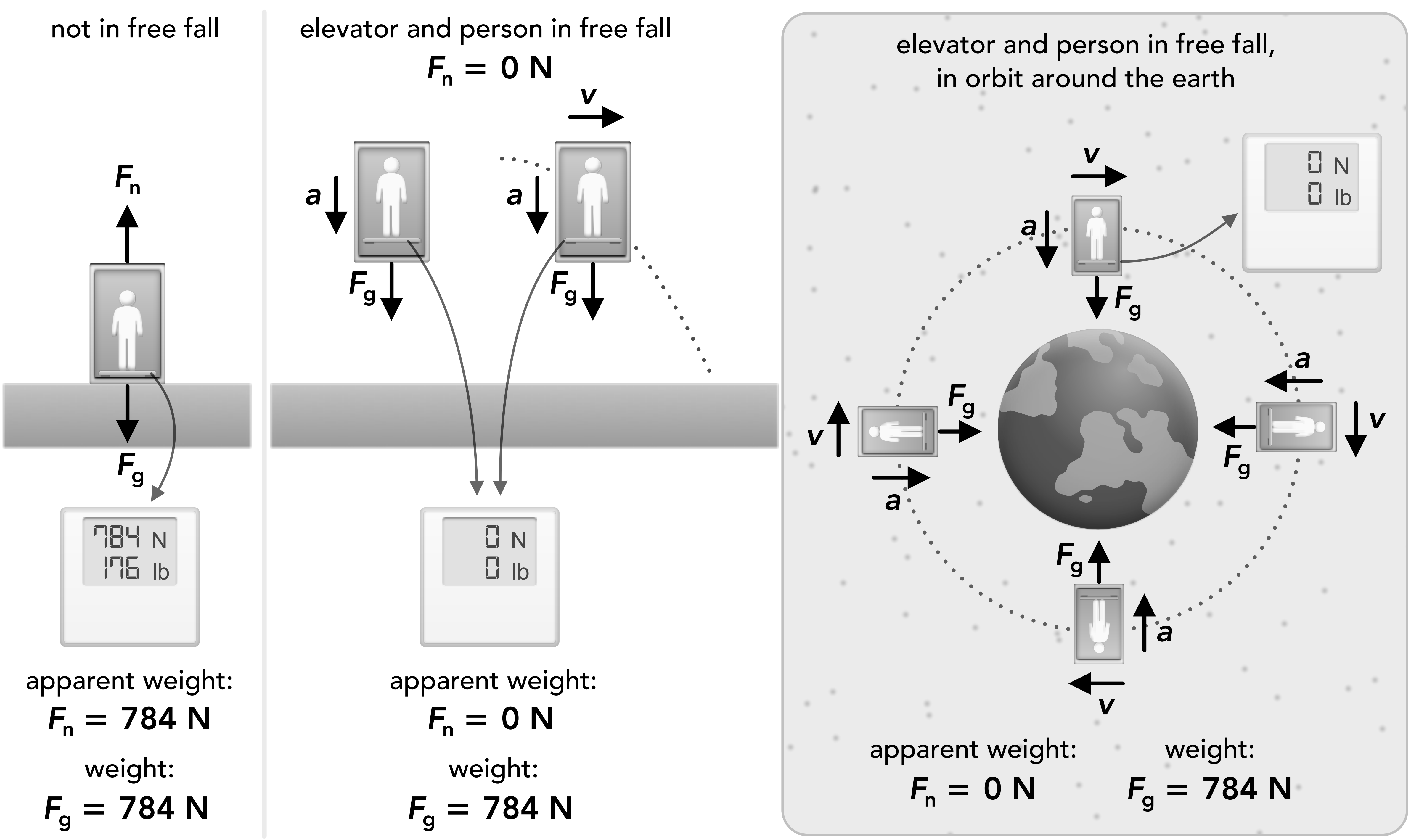
Once the ball is in orbit its speed is constant and the only force acting on it is the gravitational force (assuming no air resistance). The gravitational force is perpendicular to the velocity and changes its direction, but no force is required for the ball to keep moving.

- Since an object in orbital motion is in projectile motion or free fall, and the only force acting on it is gravity, **the object has no apparent weight** and it experiences "weightlessness".
- Gravity is still acting on an object in orbital motion. For example, the International Space Station (ISS) is in orbit around the earth at an altitude of about 400 km. At that distance from the earth, the acceleration due to gravity g is still about 8.7 m/s² or 89% of the acceleration due to gravity at the surface of the earth.
- An astronaut in the ISS orbiting the earth will feel like they're falling, because they are. It's the same thing as being in an elevator that is in free fall where you appear to be weightless. The difference is that an object in orbital motion is also moving sideways very fast, and the direction of the gravitational force keeps rotating.

Your apparent weight is the normal force supporting you from below, which is equal to the actual weight if the net force and acceleration are zero



When you're in free fall and the only force acting on you is the gravitational force, you have no apparent weight and you experience "weightlessness", even if you have some velocity. This is the case for an object in orbital motion.

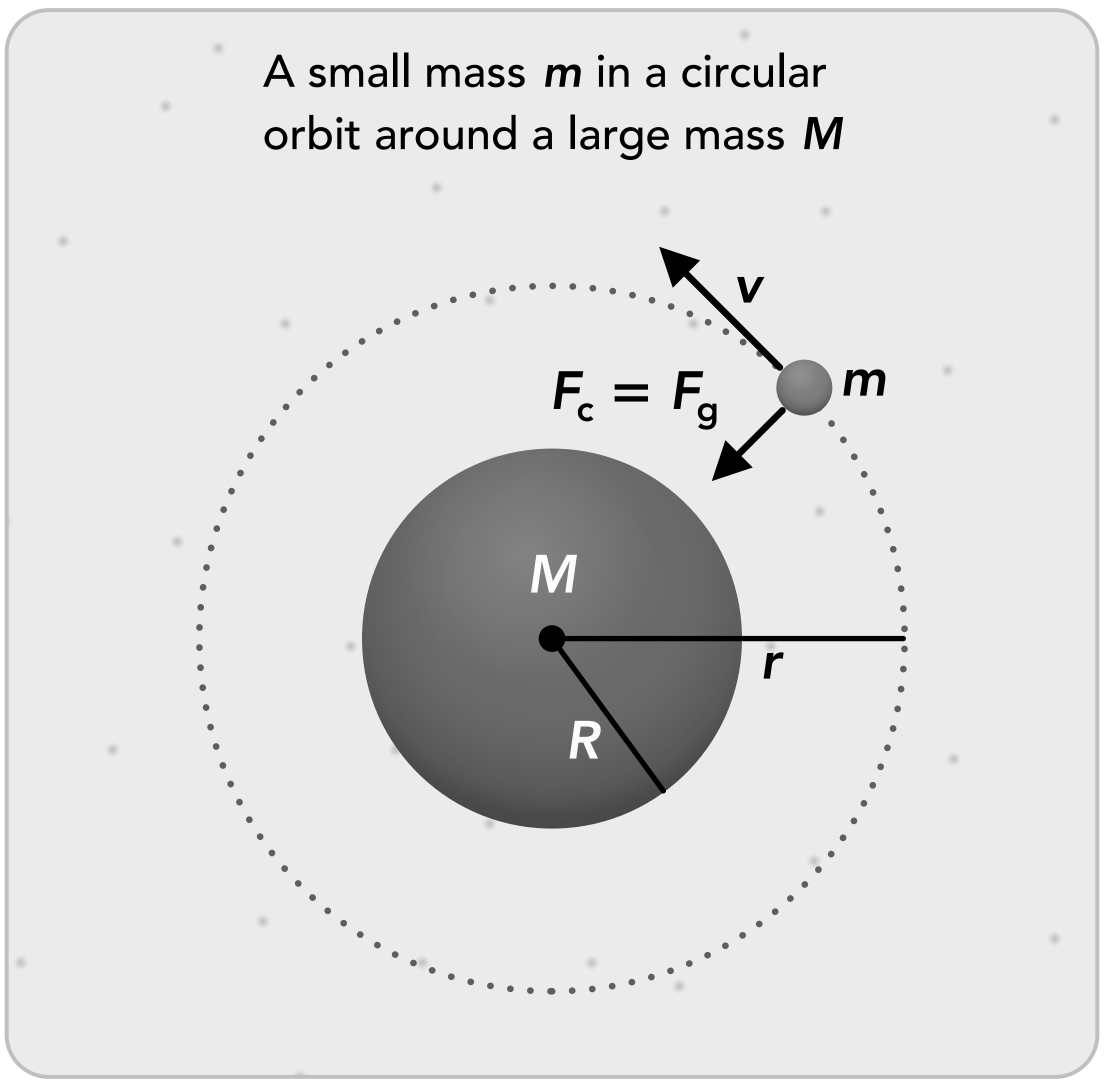


Circular Orbits

Constants		Unit	Name
G	6.67×10^{-11}	$\frac{m^3}{kg\cdot s^2}$	gravitational constant

Variables		SI Unit
M	planet mass	kg
m	object mass	kg
R	planet radius	m
r	orbital radius	m
v	orbital speed	$\frac{m}{s}$
T	orbital period	s
F_g	gravitational force	N
F_c	centripetal force	N

- The path of an object in orbit around a planet can be a circular orbit or an elliptical orbit. The same laws which are covered in the elliptical orbit section also apply to circular orbits.
- An object in a **circular orbit** is in **uniform circular motion** around the planet. Remember that an object in circular motion must have a centripetal force acting on it which always points towards the center of the circle.
- For circular orbits, **the centripetal force is the gravitational force** and the centripetal acceleration is the gravitational acceleration at that distance from the center of the planet. This means we can equate some concepts from uniform circular motion and Newton's law of universal gravitation.



The centripetal force for a circular orbit is the gravitational force acting on the small mass so we can combine circular motion and gravitation

$$F_c = F_g$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v^2 = \frac{GM}{r} \qquad T = \frac{2\pi r}{v}$$

Orbital velocity

$$v = \sqrt{\frac{GM}{r}}$$

Orbital period

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

- Using the equation above for the orbital velocity we can derive an equation for the kinetic energy K of an object in a circular orbit.
- The gravitational potential energy U_g of the object in orbit is just the gravitational potential energy of the two-mass system, regardless of the motion of either mass (so this is not specific to orbital motion).
- The total energy of an object in a circular orbit is the sum of kinetic energy and the potential energy.

Variables		SI Unit
E	total energy	J
K	kinetic enerrgy	J
U_g	potential energy	J

Kinetic energy of object in a circular orbit

$$K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

Gravitational potential energy of two-mass system

$$U_g = -\frac{GMm}{r}$$

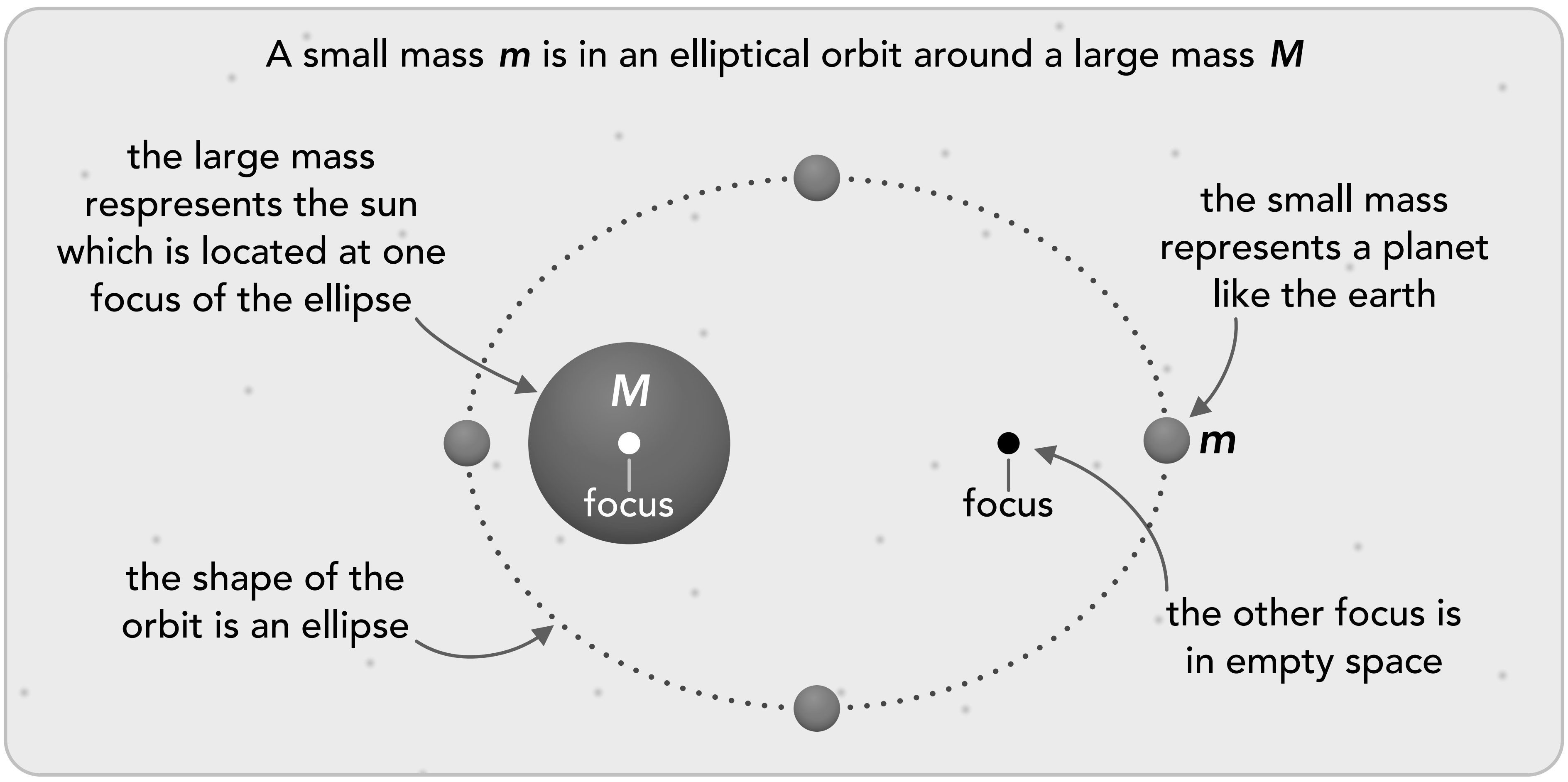
Total energy of object in a circular orbit

$$E = K + U_g = -\frac{GMm}{2r}$$

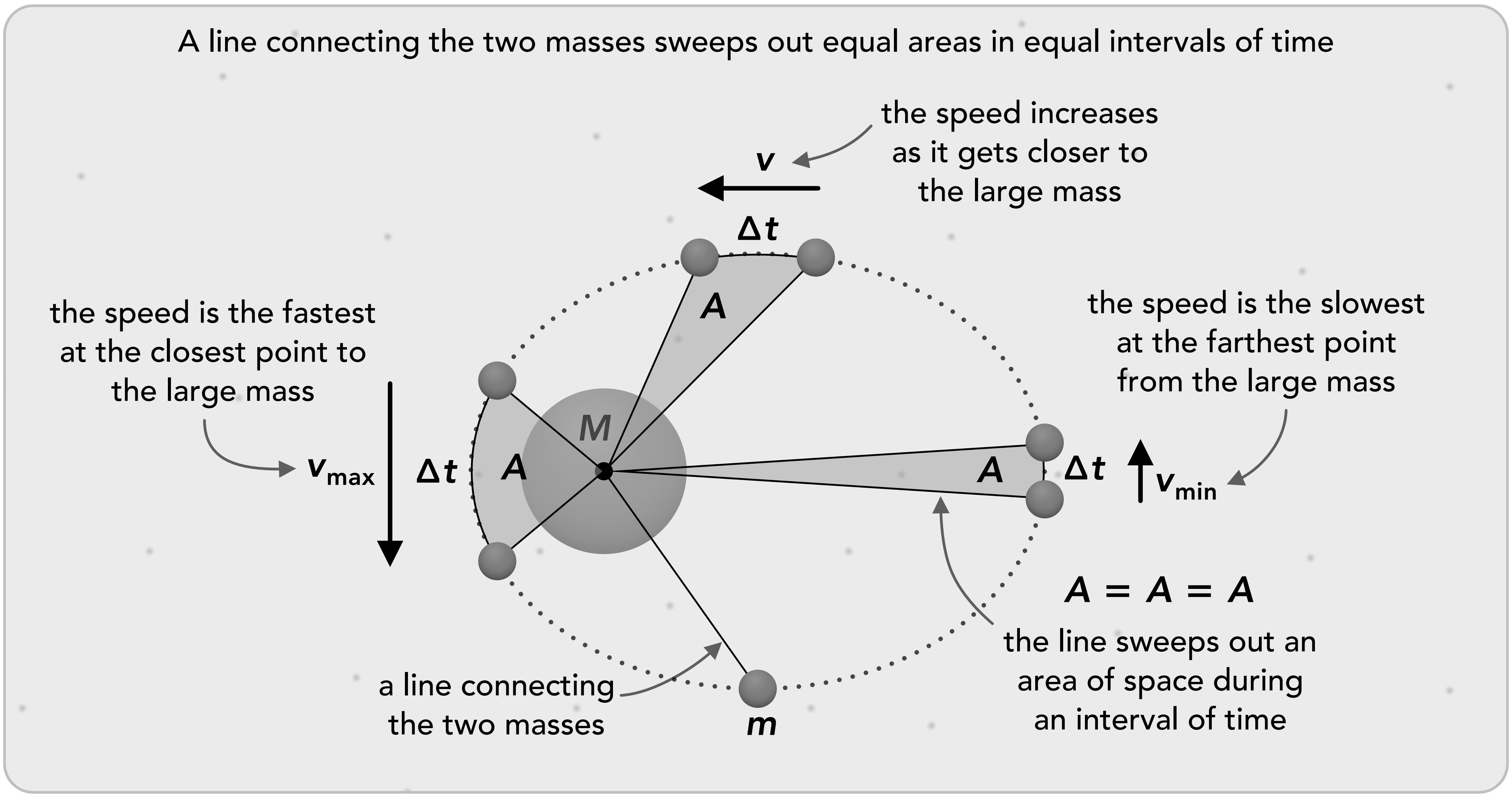
Elliptical Orbits

- Most real orbits are **elliptical orbits** which means the path is an ellipse instead of a perfect circle. A circular orbit is a special case of an elliptical orbit where the eccentricity is zero and the two focal points are at the center. The laws governing elliptical orbits also apply to circular orbits.
- In the early 1600's Johannes Kepler described the orbits of the planets around the sun. **Kepler's laws of planetary motion** are given below.

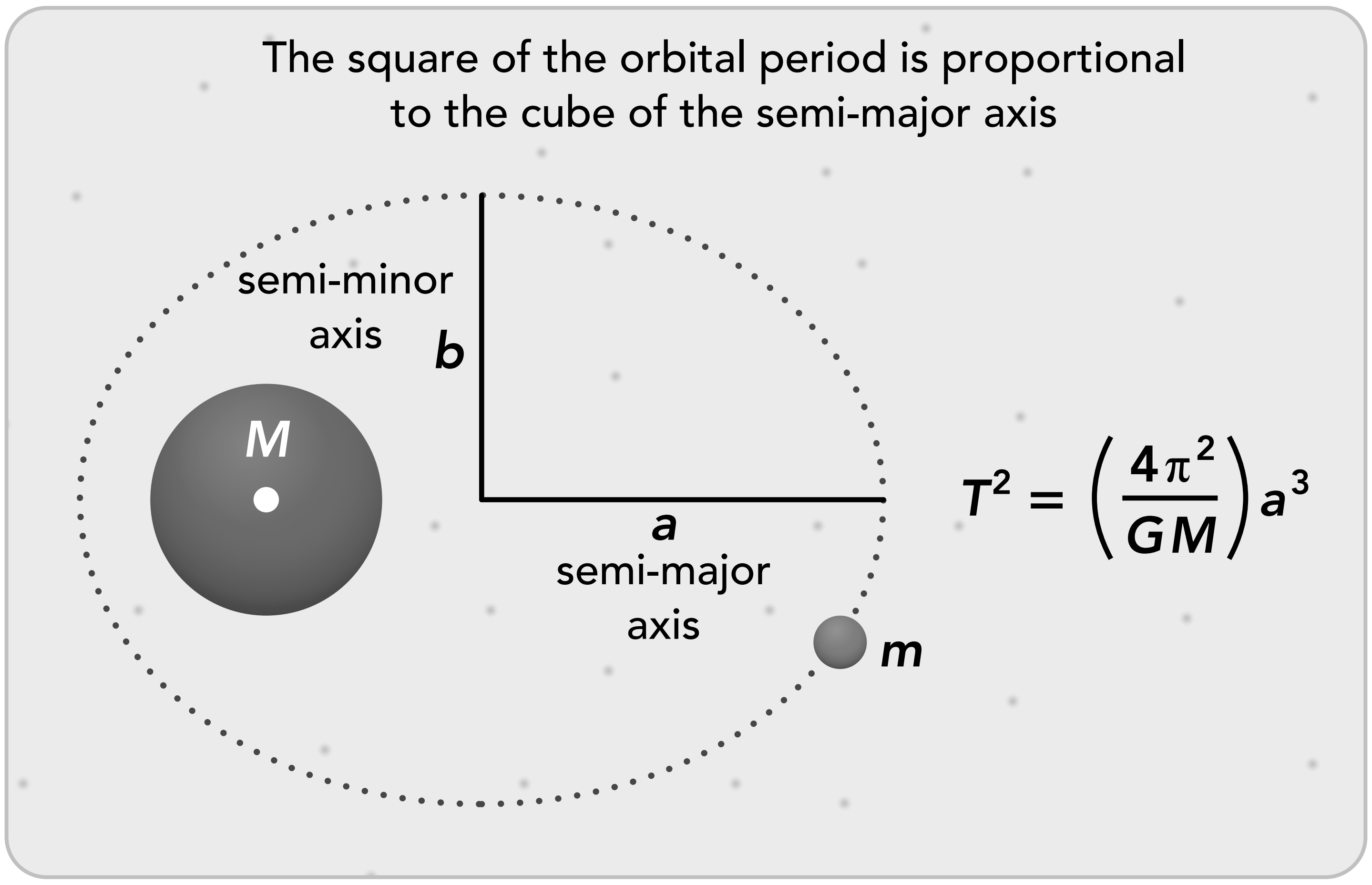
- **Law 1**: The orbit of a planet is an ellipse with the sun at one of the two foci.
- An ellipse has two foci or focal points. If one of the masses (the sun) is much larger than the other (a planet) then the center of the larger mass aligns with one focus of the ellipse according to Kepler's 1st law.



- **Law 2**: A line connecting the planet and the sun sweeps out equal areas during equal intervals of time.
- Imagine a line connecting the two masses which follows the orbital motion. During any 1 second interval (or maybe 1 month on a planetary scale) that imaginary line will sweep out or cover the same amount of area, regardless of where the planet is in the orbit. This law relates to the orbital speed and the orbital period.
- **The planet (small mass) will move faster when it's closer to the sun (large mass), and slower when it's farther away from the sun**. From Newton's law of gravitation, the gravitational force between the two masses is stronger when they are closer together. Also, because the path is elliptical and not circular, the gravitational force on the small mass is not perpendicular to its velocity (except at the left and right ends of the orbit shown). This means the gravitational force will have a component that's parallel to the velocity and it will cause the small mass to accelerate and its speed will change throughout the orbit.



- **Law 3**: The square of a planet's orbital period is proportional to the cube of the semi-major axis of its orbit.
- Unlike a circle which has a single radius, an ellipse has a semi-major axis and a semi-minor axis which are the longer "radius" and the shorter "radius" (the longest and shortest distances from the center to the perimeter).
- The equation for the period of an elliptical orbit is similar to the period of a circular orbit, but the semi-major axis is used instead of the radius.



Orbital period for elliptical orbit

$$T = 2\pi\sqrt{\frac{a^3}{G(M + m)}}$$

Orbital period for elliptical orbit (assuming M is much larger than m)

$$T = 2\pi\sqrt{\frac{a^3}{GM}}$$

- Kepler's laws describe the elliptical orbit of a small mass around a large mass. In reality, **both masses orbit the shared center of mass of the system (called the barycenter) in elliptical orbits**. That center of mass is located at one focus of each elliptical orbit.
- When one mass is much larger than the other (as in the case of the sun and the earth) the system's center of mass is within the larger mass and is very close its center. So it's a fair approximation that a focus of the elliptical orbit of the smaller mass is located at the center of the larger mass, instead of at the system's center of mass.
- As the two masses become more similar in size, the system's center of mass moves towards the middle of the two masses. If the masses are equal, the center of mass is directly between them and they appear to orbit each other.