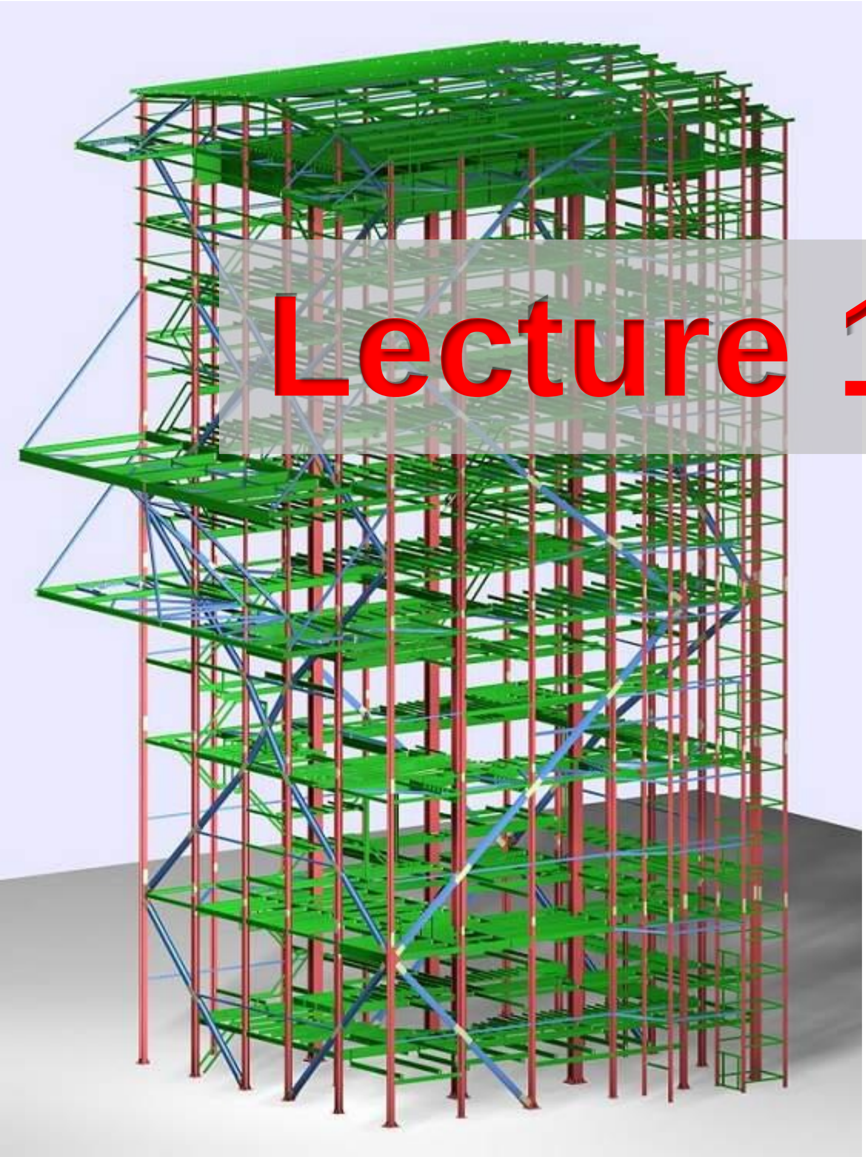
Timber and Steel Design



Lecture 10 Beam-Column

- Combined Axial & Bending
- AISC Interaction Equations
- Moment Amplification
- Design of Beam-Column

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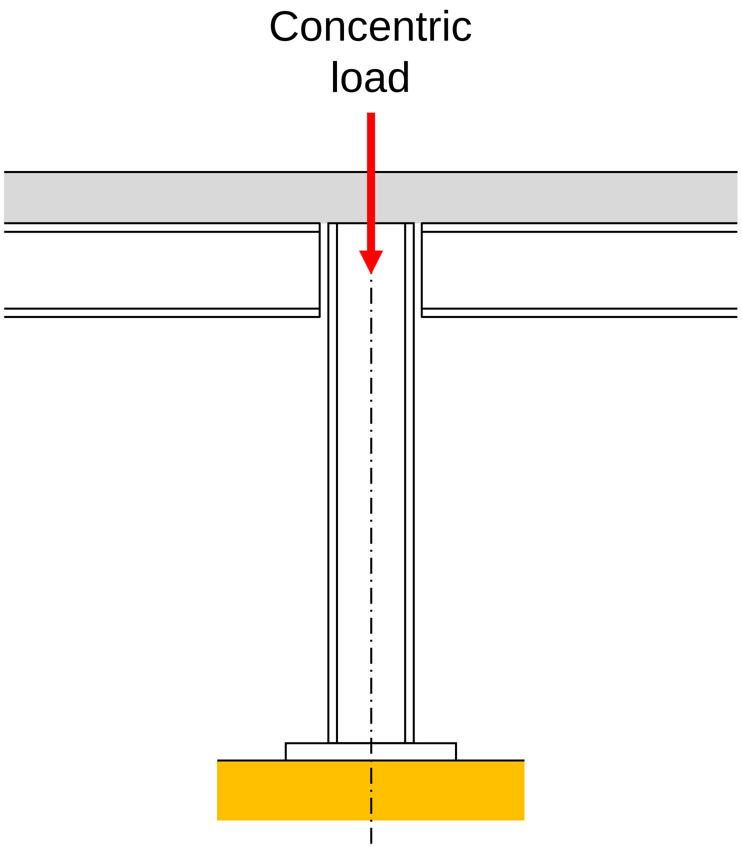
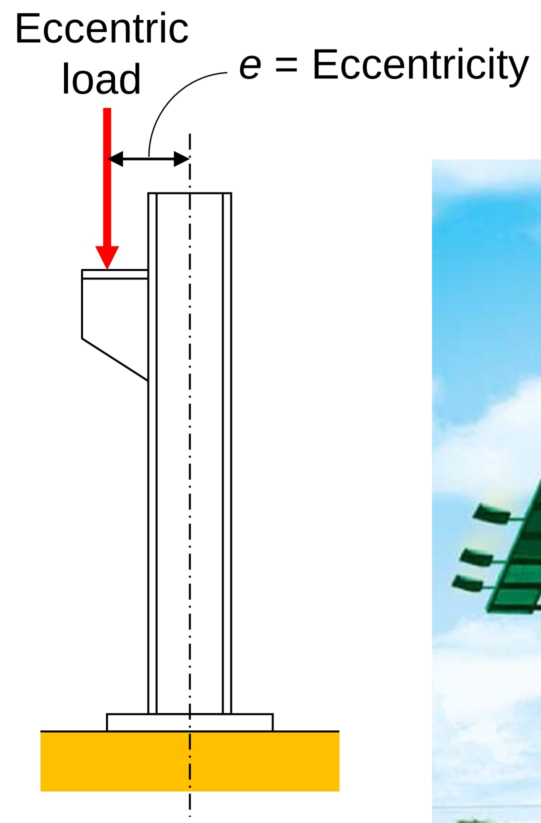
SURANAREE UNIVERSITY OF TECHNOLOGY

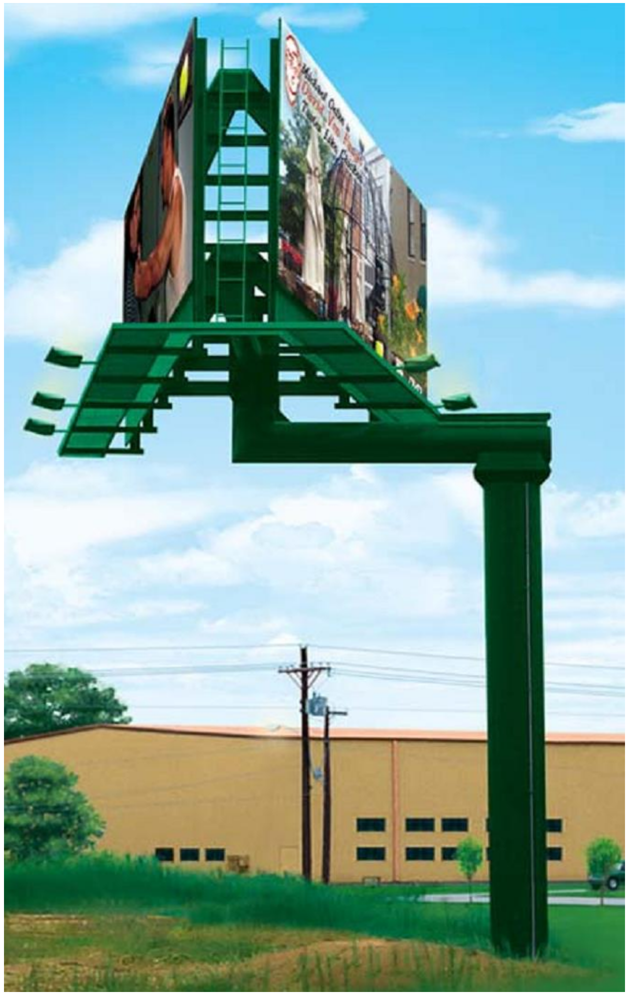
INSTITUTE OF ENGINEERING

SCHOOL OF CIVIL ENGINEERING

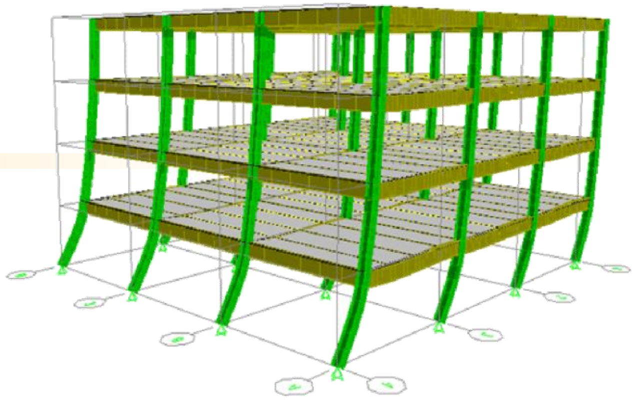
Concentric Loads vs. Eccentric Loads







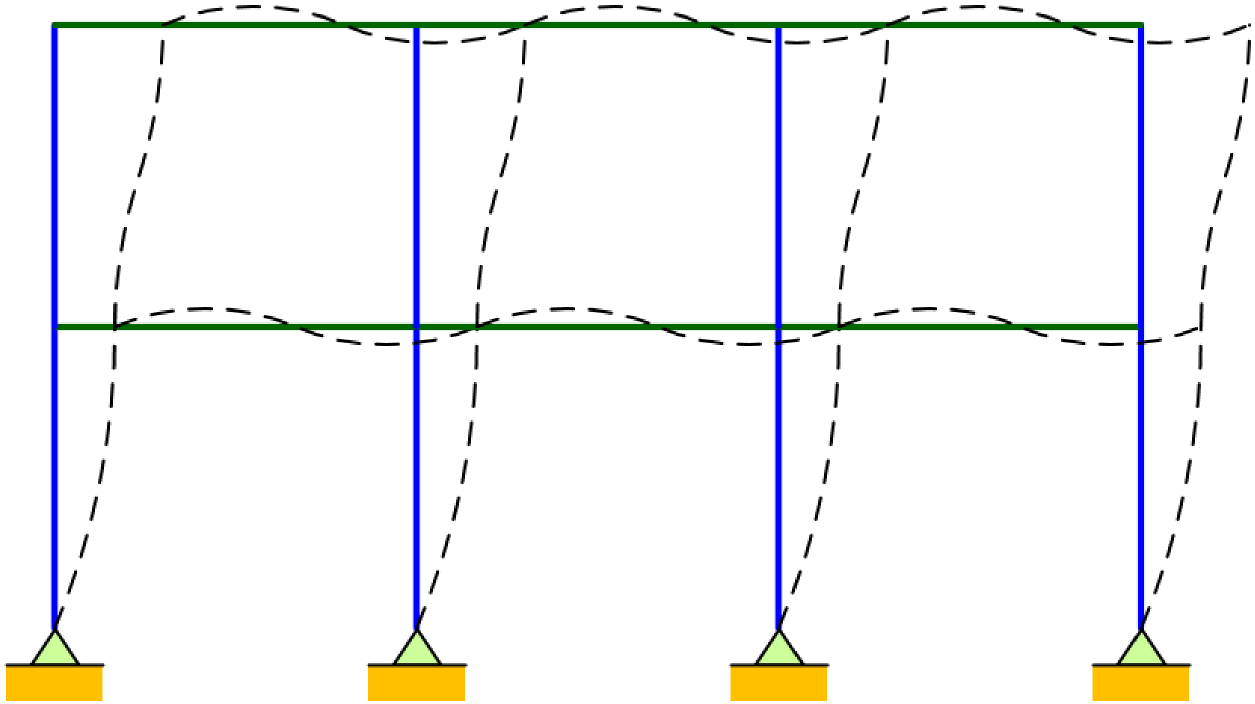
Columns in Buildings



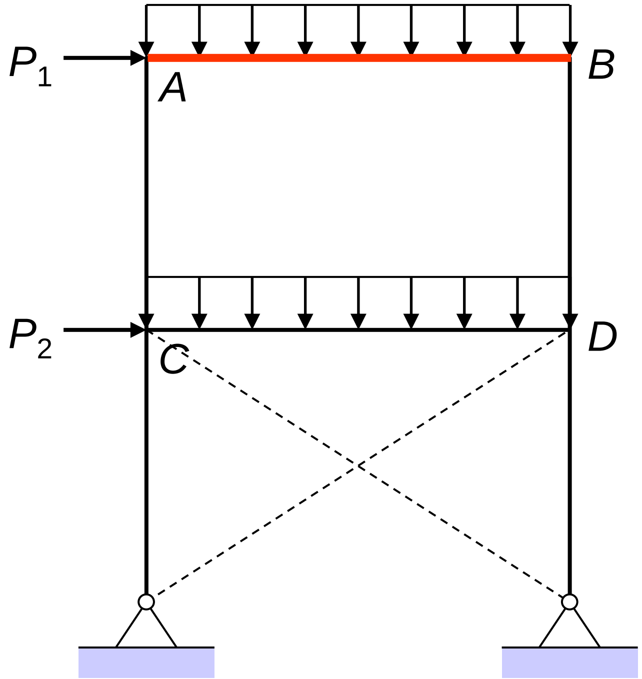
Subject to moments and axial load transferred from

Gravity loads : Dead Load & Live Load

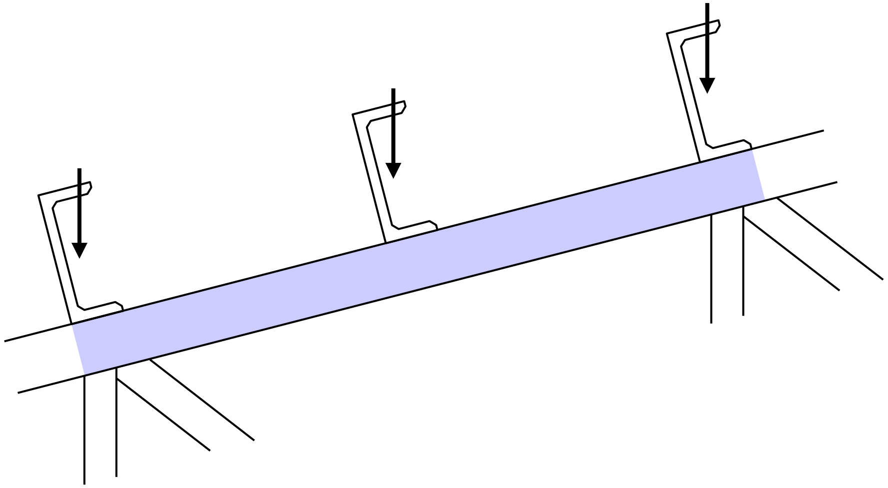
Lateral loads : Earthquake & Wind Load



คานในโครงข้อแข็งรับแรงด้านข้าง

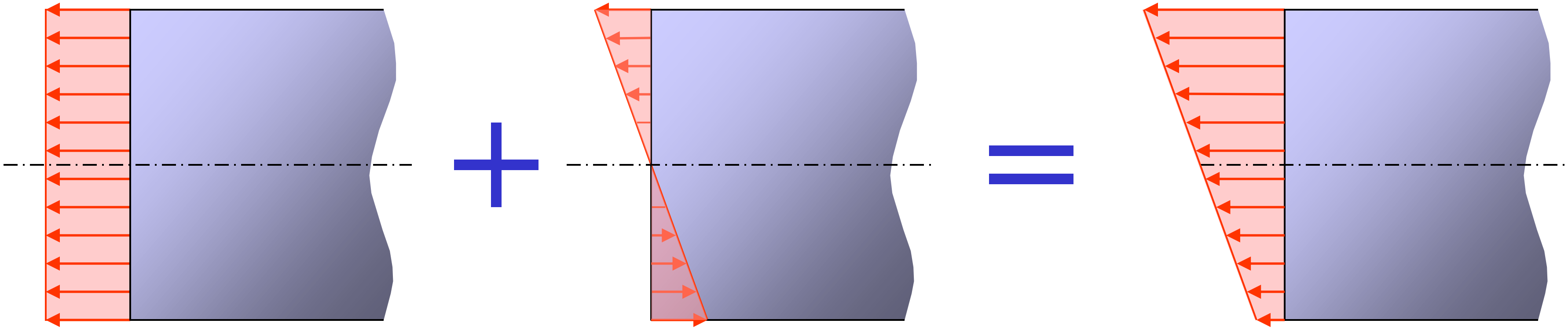


จันทันรับแปกลางช่วง



Combined Axial - Bending Stresses

Superposition of stresses from axial force and bending moment



$$\sigma_a = \frac{P}{A}$$

$$\sigma_b = \frac{My}{I}$$

$$\sigma = \frac{P}{A} + \frac{My}{I}$$

Bending about single axis: $$f = \frac{P}{A} \pm \frac{Mc}{I}$$

Bending about both axes: $$f = \frac{P}{A} \pm \frac{M_x y}{I_x} \pm \frac{M_y x}{I_y}$$

Combined Bending and Axial Strength

AISC H1

AISC provides the equation for combined bending and axial force :

(a) When $\frac{P_r}{P_c} \geq 0.2$ $$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(H1-1a)}$$

(b) When $\frac{P_r}{P_c} < 0.2$ $$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(H1-1b)}$$

where

P_r = the required axial strength under LRFD or the required allowable axial strength with ASD

P_c = the available axial strength under LRFD or the available allowable axial strength with ASD

M_r = the requiredflexural strength under LRFD or the required allowable flexural strength with ASD

M_c = the available flexural strength under LRFD or the available allowable flexural strength with ASD

Example 10-1 : A 3,500 ksc W300 × 65.4 tension member with no holes is subjected to the axial loads $P_D = 10$ tons and $P_L = 12$ tons, as well as the bending moments $M_{Dy} = 1.4$ t-m and $M_{Ly} = 3.5$ t-m. Is the member satisfactory if $L_b < L_p$?

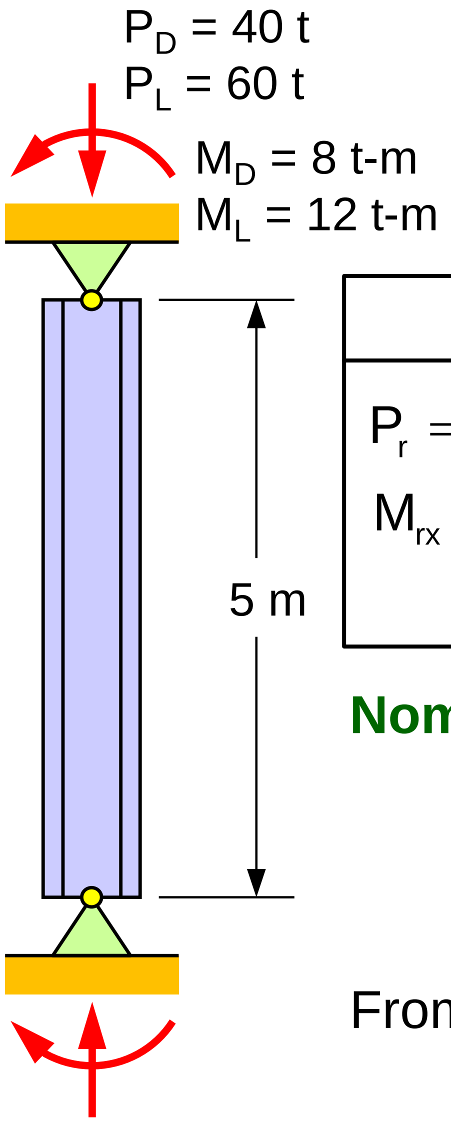
Solution : From Table A-1, C-1 : W300 × 65.4 ($A = 83.36$ cm^2, $Z_y = 288$ cm^3)

LRFD	ASD
$P_r = 1.4(10)+1.7(12) = 34.4$ tons	$P_r = P_a = 10+12 = 22$ tons
$M_{ry} = 1.4(1.4)+1.7(3.5) = 7.91$ t-m	$M_{ry} = 1.4+3.5 = 4.9$ t-m
$P_c = \phi_t F_y A_g = 0.9(3.5)(83.36)$ $= 188$ tons	$P_c = F_y A_g / \Omega_t = 3.5(83.36)/1.67$ $= 175$ tons
$M_{cy} = \phi_b F_y Z_y = 0.9(3.5)(288)/100$ $= 9.1$ t-m	$M_{cy} = F_y Z_y/\Omega_b = 3.5(288)/(1.67\times 100)$ $= 6.0$ t-m
$\dfrac{P_r}{P_c} = \dfrac{34.4}{188} = 0.183 < 0.2$	$\dfrac{P_r}{P_c} = \dfrac{22}{175} = 0.126 < 0.2$
∴ Use AISC Equation (H1-1b)	∴ Use AISC Equation (H1-1b)

When $\frac{P_r}{P_c} < 0.2$ $$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right) \leq 1.0 \qquad \text{(H1-1b)}$$

LRFD	ASD
$\frac{P_r}{2P_c} + \frac{M_{ry}}{M_{cy}} \leq 1.0$	$\frac{P_r}{2P_c} + \frac{M_{ry}}{M_{cy}} \leq 1.0$
$\frac{34.4}{2(188)} + \frac{7.91}{9.1}$	$\frac{22}{2(175)} + \frac{4.9}{6.0}$
$= 0.961 < 1.0$ **OK**	$= 0.880 < 1.0$ **OK**

Example 10-2 : Determine whether a W350×159 column can subjected to the load as shown. $F_y = 2{,}500$ ksc, $K_x = 1.92$ and $K_y = 1.0$.



Solution : W350×159 ($A_g = 202$ cm^2, $Z_x = 2{,}927$ cm^3, $r_x = 15.3$ cm, $r_y = 8.9$ cm, $L_p = 4.47$ m, $L_r = 19.64$ m, $M_p = 73.2$ t-m, $M_r = 46.7$ t-m)

LRFD	ASD
$P_r = 1.4(40) + 1.7(60) = 158$ tons $M_{rx} = 1.4(8) + 1.7(12)$ $= 31.6$ t-m	$P_r = 40 + 60 = 100$ tons $M_{rx} = 8 + 12 = 20$ t-m

Nominal axial compressive strength:

$$K_xL_x/r_x = (1.92)(500)/15.3 = 63 \leftarrow \text{Control}$$

$$K_yL_y/r_y = (1.00)(500)/8.9 = 56$$

From Table B-2 → $F_{cr} = 2{,}034$ ksc

$$P_n = F_{cr}A_g = 2.034 \times 202 = 411 \text{ tons}$$

Nominal moment strength:

$$L_p < L_b \leq L_r \rightarrow M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_p$$

$$M_p = F_yZ_x = \frac{2.5 \times 2{,}927}{100} = 73.2 \text{ t-m}$$

$$M_n = 1.0\left[73.2 - \left(73.2 - \frac{0.7(2.5)(2{,}670)}{100}\right)\left(\frac{500 - 447}{1{,}964 - 447}\right)\right]$$

$$= 72.3 \text{ t-m} < 73.2 \text{ t-m} \quad \textbf{OK}$$

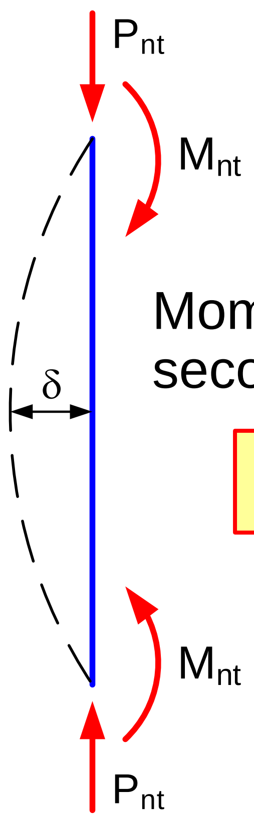
LRFD	ASD
$P_c = \phi_c P_n = 0.9(411) = 370$ tons	$P_c = P_n / \Omega_c = 411/1.67 = 246$ tons
$M_{cx} = \phi_b M_n = 0.9(72.3) = 65.1$ t-m	$M_{cx} = M_n/\Omega_b = 72.3/1.67 = 43.3$ t-m
$\frac{P_r}{P_c} = \frac{158}{370} = 0.43 > 0.2$	$\frac{P_r}{P_c} = \frac{100}{246} = 0.41 > 0.2$
$\therefore$ Use AISC Equation (H1-1a)	$\therefore$ Use AISC Equation (H1-1a)

When $\frac{P_r}{P_c} \geq 0.2$ $\quad \frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0$ (H1-1a)

LRFD	ASD
$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0$	$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0$
$\frac{158}{370} + \frac{8}{9}\left(\frac{31.6}{65.1}\right)$	$\frac{100}{246} + \frac{8}{9}\left(\frac{20}{43.3}\right)$
$= 0.858 < 1.0$ **OK**	$= 0.817 < 1.0$ **OK**

Moment Amplification

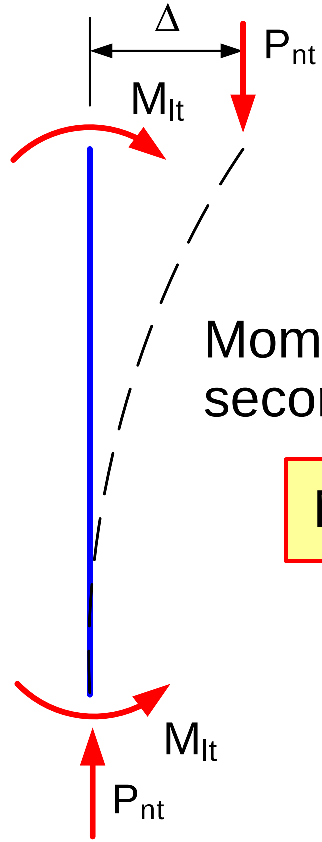
When a beam column is subjected to moment along its unbraced length, it will be displaced laterally. The moment will be increased equal to the axial force time the displacement.



Moment will be increased by second-order moment $P_{nt}\delta$

$$M_r = M_{nt} + P_{nt}\delta$$

Column in braced frame



Moment will be increased by second-order moment $P_{nt}\Delta$

$$M_r = M_{nt} + P_{nt}\Delta$$

Column in unbraced frame

Approximate Second-Order Analysis

AISC Specification Appendix 8, two amplification factors B_1 and B_2 are used for the two types of moments.

Required moment strength: $M_r = B_1 M_{nt} + B_2 M_{lt}$ (AISC Equation A-8-1)

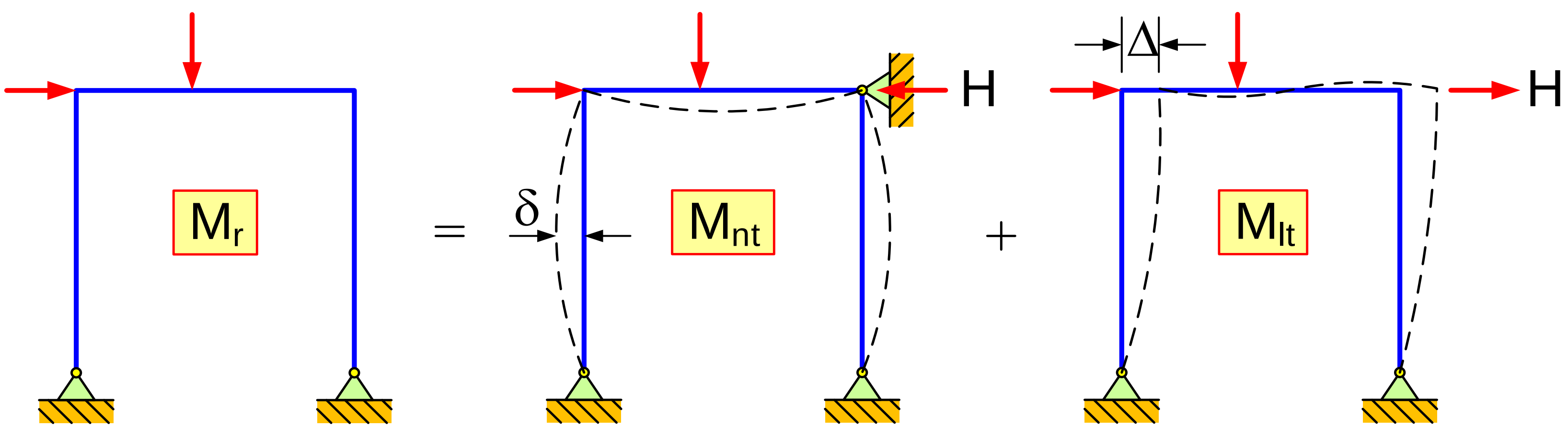
Required axial strength: $P_r = P_{nt} + B_2 P_{lt}$ (AISC Equation A-8-2)

where M_{nt} = max. moment assuming that no sidesway (no translation)

M_{lt} = max. moment caused by sidesway (lateral translation)

P_{nt} = axial load in braced condition

P_{lt} = axial load in sidesway condition



Member in Braced Frames (B_1)

The amplification factor for the moment **M_{nt}** in braced frame is

$$B_1 = \frac{C_m}{1-(\alpha P_r / P_{e1})} \geq 1.0$$ (AISC Equation A-8-3)

where P_r = required unamplified axial compressive strength ($P_{nt} + P_{lt}$)

$= P_u$ for LRFD, P_a for ASD

α = 1.00 for LRFD, 1.60 for ASD

$P_{e1} = \frac{\pi^2 EI}{L^2}$ = elastic critical buckling strength

C_m = equivalent uniform moment factor

(a) No transverse loading on member

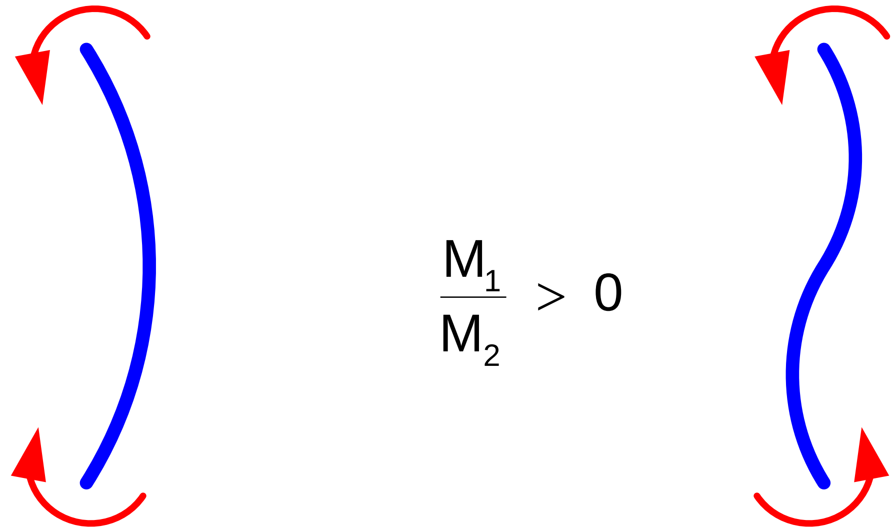
$$C_m = 0.6 - 0.4(M_1 / M_2)$$ (AISC Equation A-8-4)

M_1 = the smaller end moment

M_2 = the larger end moment

$M_1/M_2 = +$ for reverse curvature and – for single curvature

$$\frac{M_1}{M_2} < 0 \qquad\qquad \frac{M_1}{M_2} > 0$$



Single Curvature Double Curvature

(b) Transverse loading on member

For example top chord of truss with purlin between joints

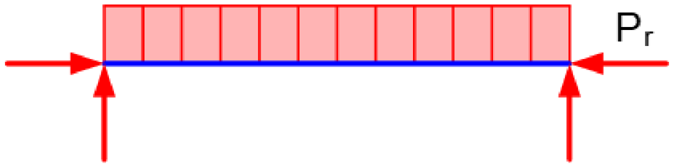
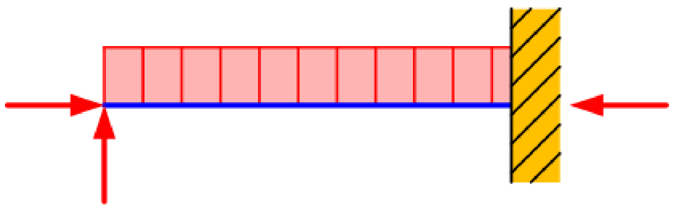
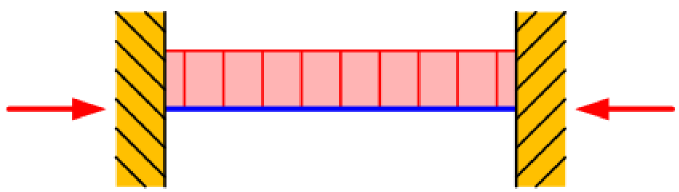
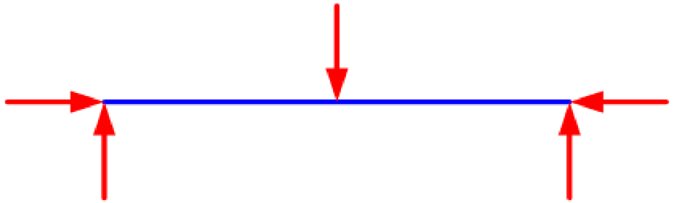
The value of C_m is determined by analysis or conservatively used equal to 1.0

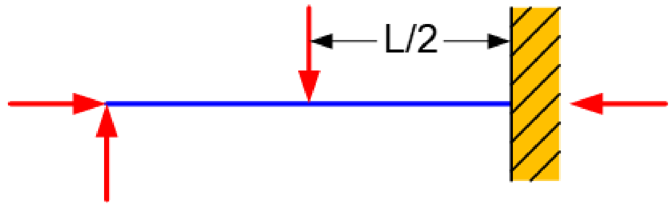
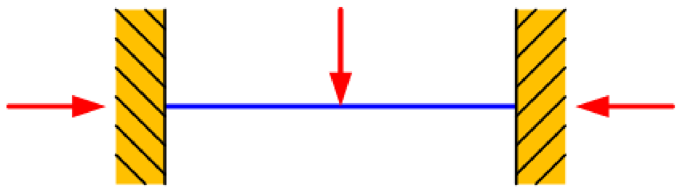
Commentary to Appendix 8 of AISC Specification provide a more refined formula of C_m.

$$C_m = 1 + \Psi\left(\frac{\alpha P_r}{P_{e1}}\right)$$

(AISC Equation C-A-8-2)

Commentary on AISC Specification Table C-A-8.1

Case	Ψ	C_m
	0	1.0
	-0.4	$1 - 0.4\frac{\alpha P_r}{P_{e1}}$
	-0.4	$1 - 0.4\frac{\alpha P_r}{P_{e1}}$
	-0.2	$1 - 0.2\frac{\alpha P_r}{P_{e1}}$

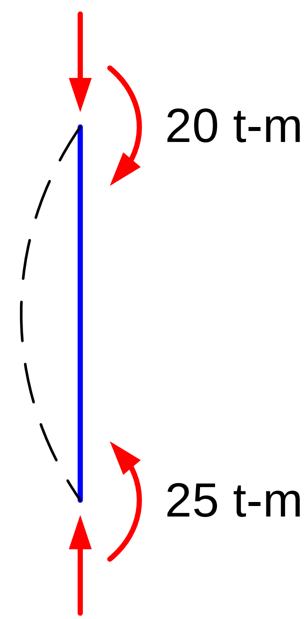
	-0.3	$1 - 0.3\dfrac{\alpha P_r}{P_{e1}}$
	-0.2	$1 - 0.2\dfrac{\alpha P_r}{P_{e1}}$

Example of C_m :

(a) No sidesway and no transverse loading

Moments bend member in single curvature

$$C_m = 0.6 - 0.4\left(-\frac{20}{25}\right) = 0.92$$

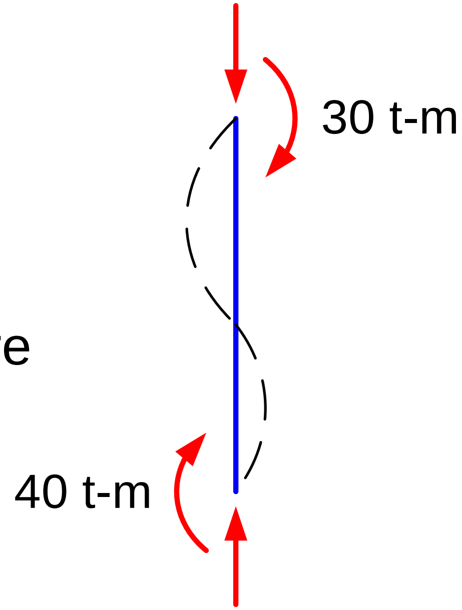


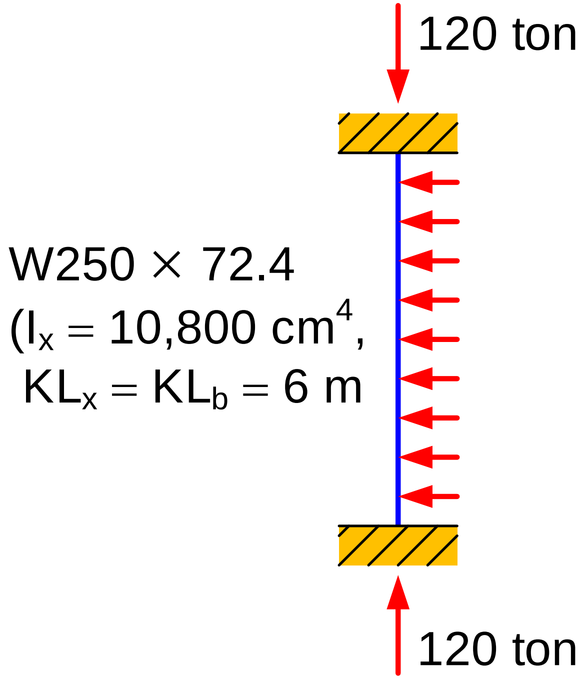
Example of C_m :

(b) No sidesway and no transverse loading

Moments bend member in reverse curvature

$$C_m = 0.6 - 0.4\left(+\frac{30}{40}\right) = 0.30$$





(c) Member has restrained ends and transverse loading and is bent about x axis

C_m is determined from Table C-A-8.1:

$$\alpha P_r = 120 \text{ ton}$$

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 (2.04\times10^6)(10,800)}{(600)^2}$$

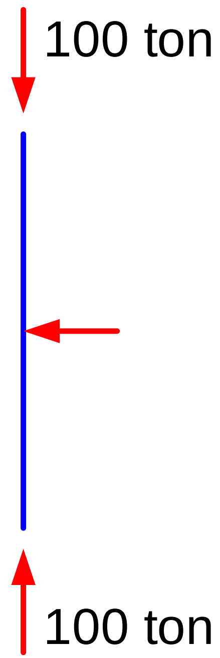
$$= 604,020 \text{ kg} = 604 \text{ ton}$$

$$C_m = 1 - 0.4\left(\frac{120}{604}\right) = 0.92$$

Example of C_m :

(d) Member has unrestrained ends and transverse loading and is bent about x axis

W300 × 87.0
($I_x = 18,800\ cm^4$,
$KL_x = KL_b = 6$ m



C_m is determined from Table C-A-8.1:

$$\alpha P_r = 100 \text{ ton}$$

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 (2.04 \times 10^6)(18,800)}{(600)^2}$$

$$= 1,051,442 \text{ kg} = 1,051 \text{ ton}$$

$$C_m = 1 - 0.2\left(\frac{100}{1,051}\right) = 0.98$$

Example 10-3 : A 3.5-m W400x232 (2,500 ksc steel) is used as a beam-column in a braced frame. It is bent in single curvature with equal and opposite end moments and is not subjected to intermediate transverse loads. Is the section satisfactory if $P_D = 70$ tons, $P_L = 90$ tons, $M_{Dx} = 8$ t-m and $M_{Lx} = 12$ t-m, $M_{Dy} = 3$ t-m and $M_{Ly} = 5$ t-m.

Solution W400×232 ($A_g = 295.4$ cm^2, $I_x = 92{,}800$ cm^4, $I_y = 31{,}000$ cm^4,
$r_x = 17.7$ cm, $r_y = 10.2$ cm, $Z_x = 4{,}954$ cm^3, $Z_y = 2{,}325$ cm^3,
$L_p = 5.13$ m, $L_r = 24.82$ m, $M_p = 124$ t-m, $M_r = 78.4$ t-m)

LRFD	ASD
$P_r = P_u = 1.4(70) + 1.7(90) = 251$ tons	$P_r = P_a = 70 + 90 = 160$ tons
$M_{ntx} = M_{ux} = 1.4(8) + 1.7(12) = 31.6$ t-m	$M_{ntx} = M_{ax} = 8 + 12 = 20$ t-m
$M_{nty} = M_{uy} = 1.4(3) + 1.7(5) = 12.7$ t-m	$M_{nty} = M_{ay} = 3 + 5 = 8$ t-m

Nominal axial compressive strength:

$$K_xL_x/r_x = (1.0)(350)/17.7 = 19.77$$

$$K_yL_y/r_y = (1.0)(350)/10.2 = 34.31$$ ← Control

From Table B-2 → $F_{cr} = 2{,}352$ ksc

$$P_n = F_{cr} A_g = 2.352 \times 295.4 = 695 \text{ tons}$$

Moment about x axis:

$$C_{mx} = 0.6 - 0.4\left(-\frac{28.8}{28.8}\right) = 1.0$$

$$P_{e1x} = \frac{\pi^2 E I_x}{(K_x L_x)^2} = \frac{\pi^2 (2.04 \times 10^6)(92{,}800)}{(1.0 \times 350)^2 \times 1{,}000} = 15{,}253 \text{ tons}$$

$$B_{1x} = \frac{C_{mx}}{1 - (\alpha P_r / P_{e1x})} = \frac{1.0}{1 - \dfrac{(1.0)(228)}{15{,}253}} = 1.02 \geq 1.0 \quad \textbf{OK}$$

$$L_b < L_p \rightarrow M_{nx} = M_{px} = F_y Z_x = \frac{2.5 \times 4{,}954}{100} = 124 \text{ t-m}$$

Moment about y axis:

$$C_{my} = 0.6 - 0.4\left(-\frac{11.6}{11.6}\right) = 1.0$$

$$P_{e1y} = \frac{\pi^2 E I_y}{(K_y L_y)^2} = \frac{\pi^2 (2.04 \times 10^6)(31{,}000)}{(1.0 \times 350)^2 \times 1{,}000} = 5{,}095 \text{ tons}$$

$$B_{1y} = \frac{C_{my}}{1-(\alpha P_r / P_{e1y})} = \frac{1.0}{1-\dfrac{(1.0)(228)}{5,095}} = 1.05 > 1.0 \quad \textbf{OK}$$

$L_b < L_p$ → $$M_{ny} = M_{py} = F_y Z_y = \frac{2.5 \times 2,325}{100} = 58.1 \text{ t-m}$$

LRFD	ASD
$P_c = \phi_c P_n = 0.9(695) = 625.5$ tons	$P_c = P_n / \Omega_c = 695/1.67 = 416$ tons
$M_{rx} = B_{1x} M_{ntx} = 1.02(31.6) = 32.2$ t-m	$M_{rx} = B_{1x} M_{ntx} = 1.02(20) = 20.4$ t-m
$M_{ry} = B_{1y} M_{nty} = 1.05(12.7) = 13.3$ t-m	$M_{ry} = B_{1y} M_{nty} = 1.05(8) = 8.4$ t-m
$M_{cx} = \phi_b M_{nx} = 0.9(124) = 112$ t-m	$M_{cx} = M_{nx} / \Omega_b = 124/1.67 = 74.3$ t-m
$M_{cy} = \phi_b M_{ny} = 0.9(58.1) = 52.3$ t-m	$M_{cy} = M_{ny} / \Omega_b = 58.1/1.67 = 34.8$ t-m
$\frac{P_r}{P_c} = \frac{251}{625.5} = 0.401 > 0.2$	$\frac{P_r}{P_c} = \frac{160}{416} = 0.385 > 0.2$
$\therefore$ Use AISC Equation (H1-1a)	$\therefore$ Use AISC Equation (H1-1a)

When $\frac{P_r}{P_c} \geq 0.2$ $$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(H1-1a)}$$

LRFD	ASD
$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0$	$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0$
$0.401 + \frac{8}{9}\left(\frac{32.2}{112} + \frac{13.3}{52.3}\right)$	$0.385 + \frac{8}{9}\left(\frac{20.4}{74.3} + \frac{8.4}{34.8}\right)$
$= 0.883 < 1.0$ **OK**	$= 0.844 < 1.0$ **OK**

Member in Unbraced Frames (B_2)

The P-Δ effect multiplier B_2 for each story and each direction

$$B_2 = \frac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1.0$$

(AISC Equation A-8-6)

where α = 1.00 for LRFD, 1.60 for ASD

P_{story} = total vertical load supported by the story (factored load for LRFD, service load for ASD)

$P_{e\,story}$ = total elastic buckling strength of the story under consideration

$$= R_M \frac{HL}{\Delta_H}$$

(AISC Equation A-8-7)

H = story shear = sum of all horizontal forces causing Δ_H

L = height of story

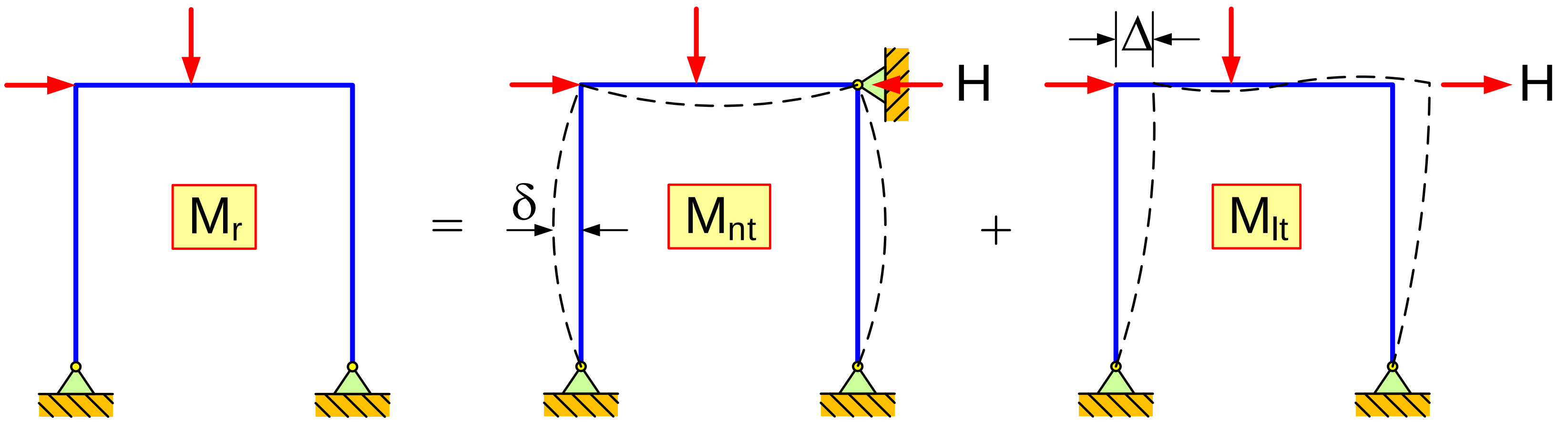
$$R_M = 1 - 0.15(P_{mf} / P_{story})$$

P_{mf} = total vertical loads in columns that are part of moment frame

Δ_H = first-order interstory drift

If there are no moment frame in the story $P_{mf} = 0$ → $R_M = 1.0$

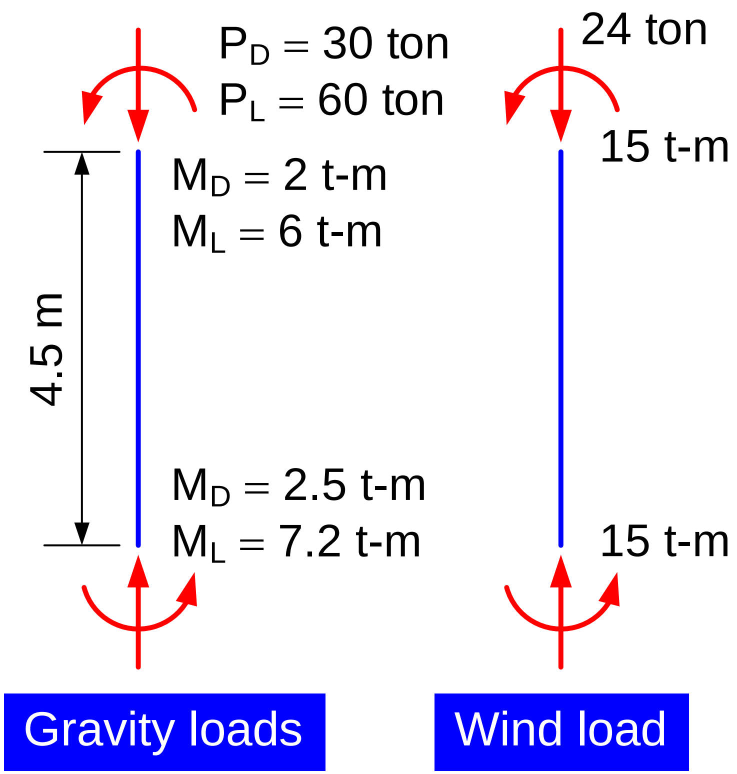
If all columns are moment frame $P_{mf} = P_{story}$ → $R_M = 0.85$



Required moment strength: $M_r = B_1 M_{nt} + B_2 M_{lt}$ (AISC Equation A-8-1)

Required axial strength: $P_r = P_{nt} + B_2 P_{lt}$ (AISC Equation A-8-2)

Example 10-4 : A W300×94 (2,500 ksc steel), 4.5 m long, is as a column in an unbraced frame. Determine whether this member is satisfactory under the loadings shown below. Use $K_x = K_y = 1.0$. All bending moments are M_x.



Solution

LRFD load combinations:

LC2: 1.4D + 1.7L

LC4: 1.4D ± 1.0W + 0.5L

ASD load combinations:

LC2: D + L

LC6: D ± 0.45W + 0.75L

Nominal axial compressive strength:

$K_xL_x/r_x = (1.0)(450)/13.1 = 34.4$

$K_yL_y/r_y = (1.0)(450)/7.51 = 59.9$ **Control**

W300×94 ($A_g = 119.8\ cm^2$, $I_x = 20{,}400\ cm^4$, $r_x = 13.1$ cm, $r_y = 7.51$ cm, $S_x = 1{,}360\ cm^3$, $Z_x = 1{,}465\ cm^3$, $Z_y = 682\ cm^3$, $L_p = 3.78$ m, $L_r = 13.83$ m, $M_p = 36.6$ t-m, $M_r = 23.85$ t-m)

From Table B-2 → $F_{cr} = 2{,}073$ ksc

$$P_n = F_{cr} A_g = 2.073 \times 119.8 = 248 \text{ tons}$$

Gravity Load Combination: LRFD use LC2, ASD use LC2

Because of symmetry, there are no sidesway moment → $M_{lt} = 0$

LRFD	ASD
$P_{nt} = P_u = 1.4(30) + 1.7(60) = 144$ tons	$P_{nt} = P_a = 30 + 60 = 90$ tons
$M_{nt1} = M_{u1} = 1.4(2) + 1.7(6) = 13$ t-m	$M_{nt1} = M_{a1} = 2 + 6 = 8$ t-m
$M_{nt2} = M_{u2} = 1.4(2.5) + 1.7(7.2) = 15.7$ t-m	$M_{nt2} = M_{a2} = 2.5 + 7.2 = 9.7$ t-m
$P_r = P_u = P_{nt} + B_2 P_{lt}$	$P_r = P_u = P_{nt} + B_2 P_{lt}$
$= 132 + 0 = 132$ tons	$= 90 + 0 = 90$ tons

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{12}{14.5}\right) = 0.269$$

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 (2.04 \times 10^6)(20{,}400)}{(1.0 \times 450)^2 \times 1{,}000} = 2{,}028 \text{ tons}$$

$$B_1 = \frac{C_m}{1-(\alpha P_r / P_{e1})} = \frac{0.269}{1-(132/2{,}028)} = 0.288 < 1.0$$ **Use 1.0**

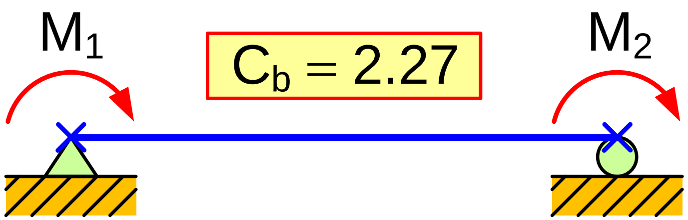
Moment magnification for braced frame:

LRFD	ASD
$M_r = B_1 M_{nt} + B_2 M_{lt}$ $= 1.0(15.7) + 0 = 15.7$ t-m	$M_r = B_1 M_{nt} + B_2 M_{lt}$ $= 1.0(9.7) + 0 = 9.7$ t-m

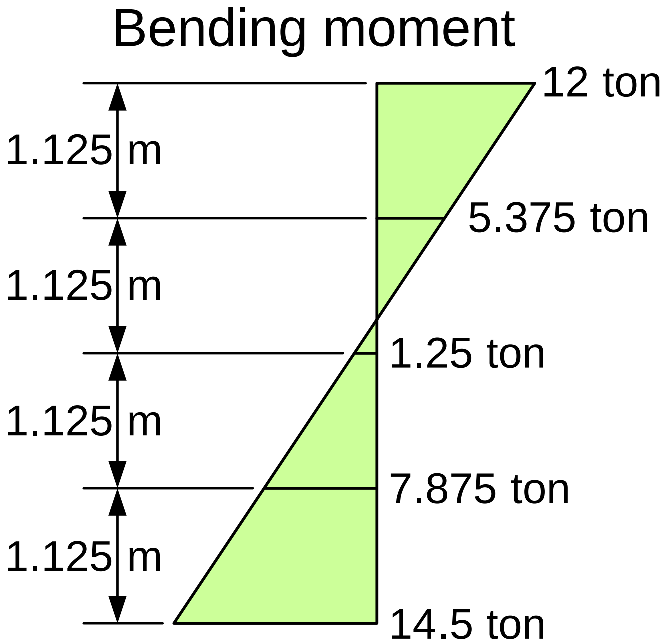
Nominal moment strength:

$$L_p < L_b \le L_r \rightarrow M_n = C_b \left[M_p - (M_p - 0.7 F_y S_x) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \le M_p$$

$$M_p = F_y Z_x = \frac{2.5 \times 1{,}465}{100} = 36.6 \text{ t-m}$$



OR . . .



$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(14.5)}{2.5(14.5) + 3(5.375) + 4(1.25) + 3(7.875)}$$

$$= 2.24$$

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right]$$

$$= 2.24\left[36.6 - (36.6 - 0.7 \times 2.5 \times 1{,}360/100)\left(\frac{450 - 378}{1{,}383 - 378}\right)\right]$$

$$= 2.24(35.71) = 80 \text{ t-m} > M_p$$

$\therefore$ **Use $M_n = M_p = 36.6$ t-m**

LRFD	ASD
$P_c = \phi_c P_n = 0.9(248) = 223$ tons	$P_c = P_n / \Omega_c = 248 / 1.67 = 149$ tons
$M_c = \phi_b M_n = 0.9(36.6) = 32.9$ t-m	$M_c = M_n / \Omega_b = 36.6 / 1.67 = 21.9$ t-m
$\dfrac{P_r}{P_c} = \dfrac{144}{223} = 0.65 > 0.2$	$\dfrac{P_r}{P_c} = \dfrac{90}{149} = 0.60 > 0.2$
$\therefore$ Use AISC Equation (H1-1a)	$\therefore$ Use AISC Equation (H1-1a)

When $\dfrac{P_r}{P_c} \geq 0.2$

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(H1-1a)}$$

LRFD	ASD
$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$	$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$
$0.65 + \dfrac{8}{9}\left(\dfrac{15.7}{32.9} + 0\right)$	$0.60 + \dfrac{8}{9}\left(\dfrac{9.7}{21.9} + 0\right)$
$= 1.07 > 1.0$ **NG**	$= 0.993 < 1.0$ **OK**

Wind Load Combination: LRFD use LC4, ASD use LC6

LRFD	ASD
$P_{nt} = 1.4(30) + 0.5(60) = 72$ tons	$P_{nt} = 30 + 0.75(60) = 75$ tons
$P_{lt} = 24$ tons	$P_{lt} = 0.45(24) = 10.8$ tons
$M_{nt1} = 1.4(2) + 0.5(6) = 5.8$ t-m	$M_{nt1} = 2 + 0.75(6) = 6.5$ t-m
$M_{lt1} = 15$ t-m	$M_{lt1} = 0.45(15) = 6.8$ tons
$M_{nt2} = 1.4(2.5) + 0.5(7.2) = 7.1$ t-m	$M_{nt2} = 2.5 + 0.75(7.2) = 7.9$ t-m
$M_{lt2} = 15$ t-m	$M_{lt2} = 0.45(15) = 6.8$ tons
For the braced condition $B_2 = 0$,	For the braced condition $B_2 = 0$,
$P_r = P_{nt} + B_2 P_{lt} = 72$ tons	$P_r = P_{nt} + B_2 P_{lt} = 75$ tons

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{5.8}{7.1}\right) = 0.273$$

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 (2.04 \times 10^6)(20,400)}{(1.0 \times 450)^2 \times 1,000} = 2,028 \text{ tons (same as before)}$$

$$B_1 = \frac{C_m}{1-(\alpha P_r / P_{e1})} = \frac{0.273}{1-(66/2{,}028)} = 0.282 < 1.0$$

Use 1.0

For unbraced condition, assume that

$$\frac{P_{story}}{P_{e\ story}} \approx \frac{P_u}{P_{e1}} = \frac{72+24}{2{,}028} = \frac{96}{2{,}028}$$

$$B_2 = \frac{1}{1+\dfrac{\alpha P_{story}}{P_{e\ story}}} = \frac{1}{1+\dfrac{1.0(96)}{2{,}028}} = 0.955 < 1.0$$

Use 1.0

LRFD	ASD
$P_r = P_{nt} + B_2 P_{lt}$ $= 72 + 1.0(24) = 96$ tons $M_r = B_1 M_{nt} + B_2 M_{lt}$ $= 1.0(7.1) + 1.0(15) = 22.1$ t-m	$P_r = P_{nt} + B_2 P_{lt}$ $= 75 + 1.0(10.8) = 86$ tons $M_r = B_1 M_{nt} + B_2 M_{lt}$ $= 1.0(7.9) + 1.0(6.8) = 14.7$ t-m

LRFD	ASD
$P_c = \phi_c P_n = 0.9(248) = 223 \text{ tons}$	$P_c = P_n / \Omega_c = 248/1.67 = 149 \text{ tons}$
$M_c = \phi_b M_n = 0.9(36.6) = 32.9 \text{ t-m}$	$M_c = M_n / \Omega_b = 36.6/1.67 = 21.9 \text{ t-m}$
$\frac{P_r}{P_c} = \frac{96}{223} = 0.43 > 0.2$	$\frac{P_r}{P_c} = \frac{86}{149} = 0.577 > 0.2$
∴ Use AISC Equation (H1-1a)	∴ Use AISC Equation (H1-1a)

When $\frac{P_r}{P_c} \geq 0.2$

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(H1-1a)}$$

LRFD	ASD
$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0$	$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0$
$0.43 + \frac{8}{9}\left(\frac{22.1}{32.9} + 0\right)$	$0.577 + \frac{8}{9}\left(\frac{14.7}{21.9} + 0\right)$
$= 1.03 < 1.0$ **NG**	$= 1.174 > 1.0$ **NG**

∴ Member not satisfy AISC requirements for **LRFD** and **ASD**

Design of Beam-Columns

To avoid complicated trial-and-error process, we will use a simplified method called **Equivale Axial Strength Method**

First, we assume that $P_r / P_c \geq 0.2$

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(H1-1a)}$$

This can be written as (multiplied both side by P_c)

$$P_r + \frac{8}{9}\left(\frac{P_c M_{rx}}{M_{cx}} + \frac{P_c M_{ry}}{M_{cy}}\right) \leq P_c$$

$$P_{ceq} = P_r + m M_{rx} + mU M_{ry}$$

where P_{ceq} = equivalent axial compressive strength

$$m = \frac{8P_c}{9M_{cx}} \quad \text{and} \quad mU = \frac{8P_c}{9M_{cy}} \rightarrow U = \frac{8P_c}{9M_{cy}} \cdot \frac{1}{m} = \frac{8P_c}{9M_{cy}} \cdot \frac{9M_{cx}}{8P_c}$$

$$= \frac{M_{cx}}{M_{cy}} \approx \frac{Z_x}{Z_y}$$

For the case $P_r / P_c < 0.2$ $$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right) \le 1.0 \qquad \text{(H1-1b)}$$

$$P_{ceq} = \frac{1}{2} P_r + \frac{9}{8} m M_{rx} + \frac{9}{8} m U M_{ry}$$

Table C-7 in Appendix C provides values of m and U for various W shapes by assuming that:

$$\left.\begin{aligned} P_c &= F_{cr} A_g \approx 2{,}000 A_g \\ M_{cx} &= M_p = F_y Z_x \end{aligned}\right\} \quad m = \frac{8 P_c}{9 M_{cx}} \quad \text{and} \quad U = \frac{Z_x}{Z_y}$$

Design Step :

(1) Select m and U (m = 6.5, U = 3)

(2) Compute P_{ceq} (equivalent axial load)

(3) Select W section base on P_{ceq} (as column under axial load only)

(4) Compute P_{ceq} from m and U of a selected W section

(5) Repeat (3) and (4) until P_{ceq} does not change (converse)

Example 10-5 : Select a W shape ($F_y = 2{,}500$ ksc) for the 5-m beam-column in the braced frame. This member is subjected to the axial load $P_D = 40$ tons and $P_L = 60$ tons and the bending moment $M_{DX} = 3$ t-m, $M_{LX} = 4$ t-m, $M_{DY} = 1.6$ t-m and $M_{LY} = 2.4$ t-m. Use $K_x = K_y = 1.0$.

Solution :

LRFD	ASD
$P_r = P_u = 1.4(40) + 1.7(60) = 158$ tons	$P_r = P_a = 40 + 60 = 100$ tons
$M_{ntx} = M_{ux} = 1.4(3) + 1.7(4) = 11$ t-m	$M_{ntx} = M_{ax} = 3 + 4 = 7$ t-m
$M_{nty} = M_{uy} = 1.4(1.6) + 1.7(2.4) = 6.32$ t-m	$M_{nty} = M_{ay} = 1.6 + 2.4 = 4$ t-m

Compute equivalent axial strength with m = 6.5 and U = 3 :

$$P_{ceq} = P_r + m M_{rx} + mUM_{ry}$$

LRFD	ASD
$P_{ceq} = 158 + 6.5(11) + 6.5(3)(6.32)$	$P_{ceq} = 100 + 6.5(7) + 6.5(3)(4)$
$= 353$ tons	$= 224$ tons

Assume $F_{cr} = 2{,}000$ ksc

LRFD	ASD
$A_{reqd} = \dfrac{P_{ceq}}{\phi_c F_{cr}} = \dfrac{353}{0.9 \times 2.0} = 196 \text{ cm}^2$	$A_{reqd} = \dfrac{\Omega_c P_{ceq}}{F_{cr}} = \dfrac{1.67 \times 224}{2.0} = 187 \text{ cm}^2$

Try section W350×156 ($A = 198 \text{ cm}^2$, $r_y = 8.53$ cm)

$KL/r = 500/8.53 = 59 \rightarrow F_{cr} = 2{,}086 \text{ ksc} > 2{,}000 \text{ ksc}$ **OK**

From Table C-7: W350x156 : $m = 5.21$ and $U = 2.19$:

LRFD	ASD
$P_{ceq} = 158 + 5.21(11) + 5.21(2.19)(6.32)$ $= 287$ tons	$P_{ceq} = 100 + 5.21(7) + 5.21(2.19)(4)$ $= 182$ tons

$\Delta P_{ceq} = 353 - 287 = 66$ tons $\rightarrow$ repeat

LRFD	ASD
$A_{reqd} = \dfrac{P_{ceq}}{\phi_c F_{cr}} = \dfrac{287}{0.9 \times 2.0} = 159 \text{ cm}^2$	$A_{reqd} = \dfrac{\Omega_c P_{ceq}}{F_{cr}} = \dfrac{1.67 \times 182}{2.0} = 152 \text{ cm}^2$

Try section W350×137 ($A = 173.6\ cm^2$, $r_y = 8.84$ cm)

$KL/r = 500/8.84 = 57 \rightarrow F_{cr} = 2{,}112\ ksc > 2{,}000\ ksc$ **OK**

From Table C-7: W350×137 : m = 4.95 and U = 2.12 :

LRFD	ASD
$P_{ceq} = 158 + 4.95(11) + 4.95(2.12)(6.32)$ $= 279$ tons	$P_{ceq} = 100 + 4.95(7) + 4.95(2.12)(4)$ $= 177$ tons

$\Delta P_{ceq} = 287 - 279 = 8$ tons $\rightarrow$ close enough **OK**

Check section

W350×137 ($A_g = 173.6\ cm^2$, $I_x = 40{,}300\ cm^4$, $r_x = 15.2$ cm, $r_y = 8.84$ cm,
$S_x = 2{,}300\ cm^3$, $S_y = 776\ cm^3$, $Z_x = 2{,}493\ cm^3$, $Z_y = 1{,}175\ cm^3$,
$L_p = 4.44$ m, $L_r = 16.82$ m, $M_p = 62.3$ t-m, $M_r = 40.3$ t-m)

Nominal axial compressive strength:

$K_xL_x/r_x = (1.0)(500)/15.2 = 33$

$K_yL_y/r_y = (1.0)(500)/8.84 = 57$ **Control**

From Table B-2 → $F_{cr} = 2{,}112$ ksc

$P_n = F_{cr}A_g = 2.112 \times 173.6 = 367$ tons

Nominal moment strength:

$$L_p < L_b \leq L_r \rightarrow M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_p$$

$$M_{px} = F_yZ_x = \frac{2.5 \times 2{,}493}{100} = 62.3 \text{ t-m}$$

$$M_{nx} = 1.0\left[62.3 - \left(62.3 - \frac{0.7(2.5)(2{,}300)}{100}\right)\left(\frac{500 - 444}{1{,}682 - 444}\right)\right]$$

$$= 61.3 \text{ t-m} < 62.3 \text{ t-m} \quad \textbf{OK}$$

$$M_{py} = F_yZ_y = \frac{2.5 \times 1{,}175}{100} = 29.4 \text{ t-m}$$

$$M_{ny} = 1.0\left[29.4 - \left(29.4 - \frac{0.7(2.5)(776)}{100}\right)\left(\frac{500 - 444}{1{,}682 - 444}\right)\right]$$

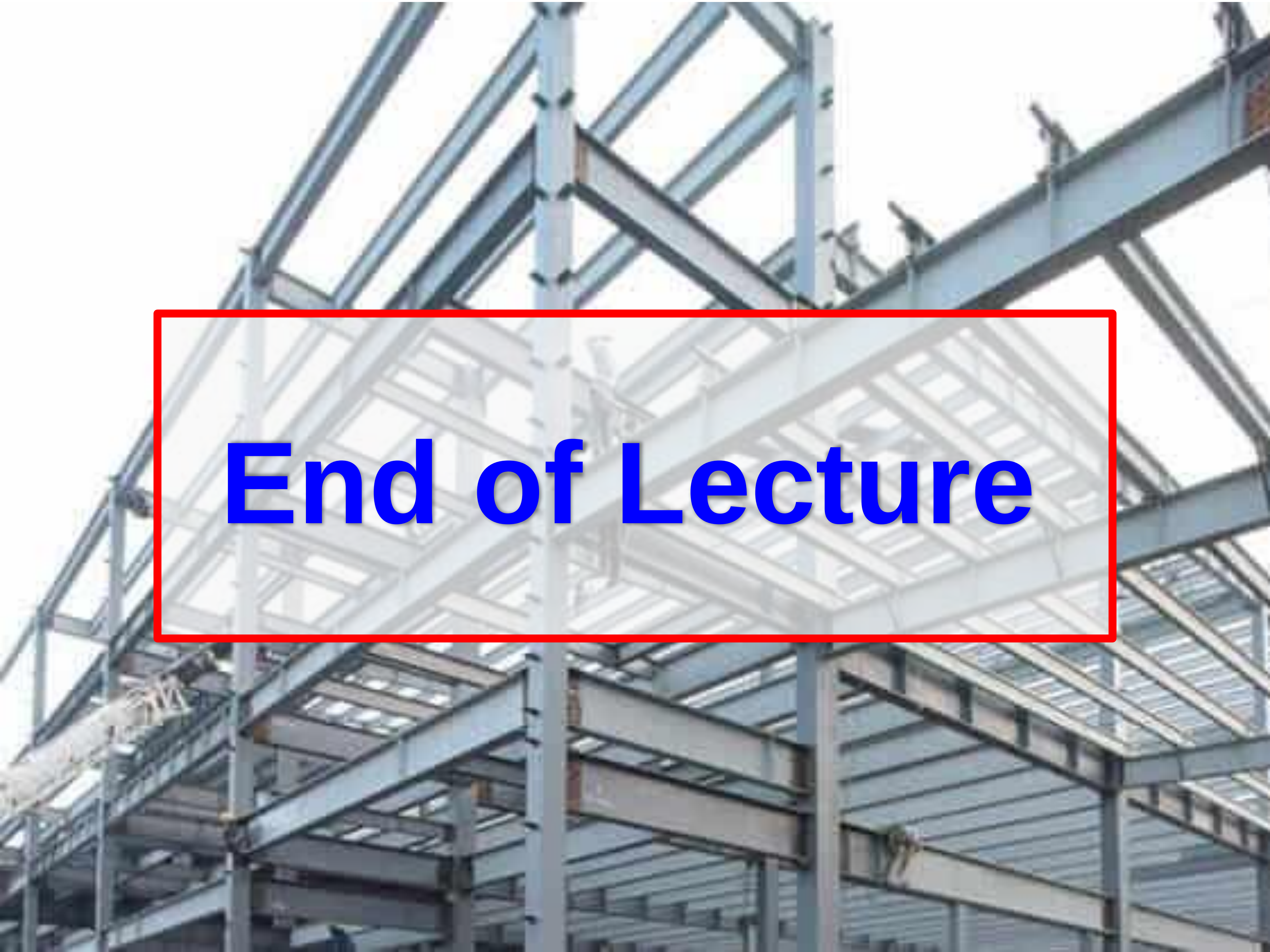
$$= 28.7 \text{ t-m} < 29.4 \text{ t-m} \quad \textbf{OK}$$

LRFD	ASD
$P_c = \phi_c P_n = 0.9(367) = 330 \text{ tons}$	$P_c = P_n / \Omega_c = 367 / 1.67 = 220 \text{ tons}$
$M_{cx} = \phi_b M_{nx} = 0.9(61.3) = 55.2 \text{ t-m}$	$M_{cx} = M_{nx} / \Omega_b = 61.3 / 1.67 = 36.7 \text{ t-m}$
$M_{cy} = \phi_b M_{ny} = 0.9(28.7) = 25.8 \text{ t-m}$	$M_{cy} = M_{ny} / \Omega_b = 28.7 / 1.67 = 17.2 \text{ t-m}$
$\frac{P_r}{P_c} = \frac{158}{330} = 0.479 > 0.2$	$\frac{P_r}{P_c} = \frac{100}{220} = 0.455 > 0.2$
$\therefore$ Use AISC Equation (H1-1a)	$\therefore$ Use AISC Equation (H1-1a)

When $\frac{P_r}{P_c} \geq 0.2$

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(H1-1a)}$$

LRFD	ASD
$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0$	$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0$
$0.436 + \frac{8}{9}\left(\frac{11}{55.2} + \frac{6.32}{25.8}\right)$	$0.455 + \frac{8}{9}\left(\frac{7}{36.7} + \frac{4}{17.2}\right)$
$= 0.831 < 1.0$ **OK**	$= 0.831 < 1.0$ **OK**


End of Lecture