MOMENTUM & IMPULSE

Momentum

1. An object's momentum depends on which of the following? (Select all that apply)
   - A The mass of the object
   - B The speed of the object
   - C The size of the object
   - D The direction the object is moving

2. What is the SI unit for momentum?
   - A kg
   - B m/s
   - C kg·m/s
   - D N·m/s

3. Two objects with the same mass are in free fall. Object A is falling at 8 m/s and object B is falling at 16 m/s. How do their momentums compare?
   - A Object A has twice the momentum of object B
   - B Object B has twice the momentum of object A
   - C Object A has four times the momentum of object B
   - D Object B has four times the momentum of object A

4. Two blocks with the same mass are sliding in opposite directions with the same speed. Which of the following is true? (Select all that apply)
   - A The momentum of each block is the same
   - B The magnitude of the momentum of each block is the same
   - C The momentum of each block has opposite signs (positive and negative)
   - D None of the above

5. A ball is thrown vertically into the air with an initial speed and then falls back down to the same height where its speed is the same as the initial speed. Which of the following is true about the ball? (Select all that apply)
   - A Its momentum is different at the start and end of the motion
   - B Its kinetic energy is different at the start and end of the motion
   - C It has the same momentum at the start and end of the motion
   - D It has the same kinetic energy at the start and end of the motion

6. Two objects have the same mass but are moving at different speeds. Object A has a speed of v_A and object B has a speed of $3v_A$. How to the momentums of each object compare?
   - A $p_A = 3p_B$
   - B $p_A = p_B/3$
   - C $p_A = 9p_B$
   - D $p_A = p_B/9$

7. A ball of mass m is dropped from rest. Which of the following is the momentum of the ball after it falls a distance of Δy?

   A $m\Delta y$

   B $\frac{1}{2}m\Delta y^2$

   C $mg\Delta y$

   D $m\sqrt{2g\Delta y}$

8. Which of the following objects have angular momentum? (Select all that apply)

   A A ball rolling down an incline

   B A pulley rotating at a constant speed

   C A ball falling through the air

   D A wheel rolling without slipping

9. What is the SI unit for angular momentum?

   A kg·m/s

   B N·s

   C $kg \cdot m^2$

   D $kg \cdot m^2/s$

10. A solid cylinder is rolling along the ground. Its angular momentum depends on which of the following? (Select all that apply)

    A Its mass

    B Its radius

    C Its angular velocity

    D None of the above

11. Two wheels with the same size, shape and mass are rolling in opposite directions with the same linear speed. Which of the following is true? (Select all that apply)

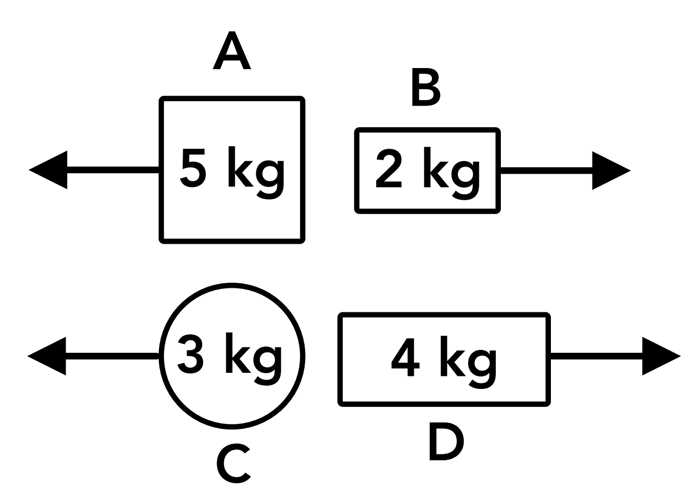
    A The magnitudes of their angular momentums are the same

    B The relationship between the magnitudes of their angular momentums cannot be determined without more information about their angular speeds

    C The signs (positive/negative) of their angular momentums are the same

    D The signs (positive/negative) of their angular momentums are different

12. 4 objects and the directions of their velocities are shown on the right. Object A is moving at 3 m/s, object B is moving at 4 m/s, object C is moving at 2 m/s and object D is moving at 2 m/s. Rank the magnitudes of the momentum of each object.



    A $p_B < p_C < p_D < p_A$

    B $p_C = p_D < p_A < p_B$

    C $p_C < p_B = p_D < p_A$

    D $p_B < p_C < p_A < p_D$

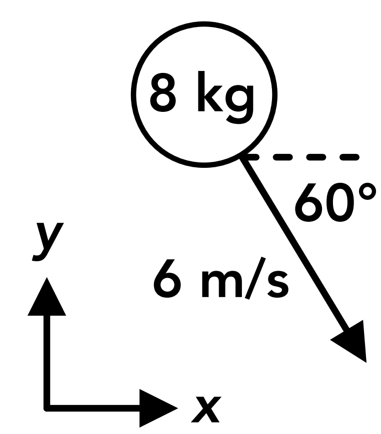
13. How much momentum does a 1,800 kg car have if it's driving at 72 km/h?

14. What speed would a 60 kg person have to run to have the same momentum as an 90 kg person running 4 m/s?

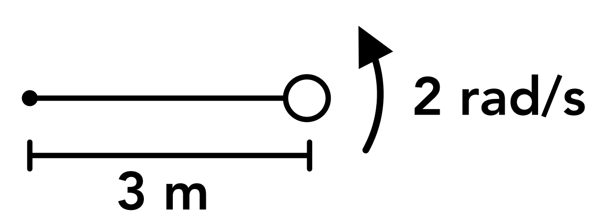
15. A 5 kg block slides down a 2 m high frictionless incline. If the block starts at the top of the incline at rest, what is the momentum of the block at the bottom of the incline?

16. A solid cylinder with a radius of 20 cm is spinning at 150 rpm. If the cylinder has an angular momentum of 8 kg·m²/s, what is the mass of the cylinder?

17. What are p_x and p_y (the momentum components) for the object shown on the right?



18. A 10 kg sphere is attached to the end of a rod with negligible mass which is rotating about its left end as shown on the right. What is the angular momentum of the rod-sphere system?



Impulse

19. Which of the following is equal to the impulse exerted on an object? (Select all that apply)
    - A The change in the object's velocity
    - B The average force applied to the object multiplied by the velocty of the object
    - C The change in the object's momentum
    - D The average force applied to the object multiplied by the period of time

20. What is the SI unit for impulse?
    - A kg
    - B kg·m
    - C kg·m/s
    - D kg·m/s²

21. Impulse has the same direction as which of the following? (Select all that apply)
    - A The direction of the applied force
    - B The direction of the object's velocity
    - C The direction of the object's momentum
    - D The opposite direction from the object's momentum

22. As a block slides down a frictionless incline, what happens to the block's momentum?
    - A It increases
    - B It decreases
    - C It stays the same
    - D Cannot be determined

23. A block slides across a frictionless surface and contacts a spring which exerts an impulse on the block and changes its momentum. The magnitude of the spring force exerted on the block increases as the spring compresses. Which of the following would we use to calculate the impulse on the block?
    - A The spring force when the block first makes contact
    - B The average spring force while the block is in contact with the spring
    - C The maximum spring force while the block is in contact with the spring
    - D Half of the average spring force while the block is in contact with the spring

24. An object is at rest when a force is exerted on the object for a period of time. After that period the velocity of the object is v. If the force had half the magnitude but it was applied for twice as long, what would have been the final velocity of the object in terms of v?
    - A $4v$
    - B $2v$
    - C v
    - D $v/2$

25. Two astronauts are floating next to each other in space. Astronaut A has a greater mass than astronaut B. Astronaut A pushes astronaut B so they move away from each other. Which astronaut experiences a greater impulse?

A Astronaut A
B Astronaut B
C Neither one experiences an impulse
D They experience the same impulse

26. If a net torque is applied to an object for a period of time, which of the following things would change? (Select all that apply)

A The object's momentum
B The object's angular momentum
C The object's rotational inertia
D The object's angular speed

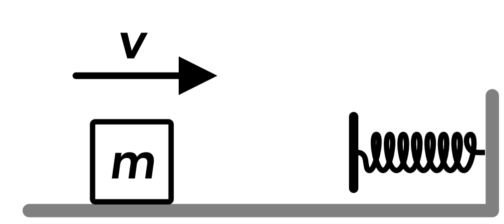
27. A pulley is rotating with an angular speed of ω. A torque is then applied to the pulley which brings the pulley to rest after a period of time Δt. If the torque had three times its magnitude instead, how long would it have taken to stop the pulley in terms of Δt?

A Δt
B $\Delta t/3$
C $3\Delta t$
D $9\Delta t$

28. If an object is spinning counterclockwise, which of the following would increase its angular momentum?

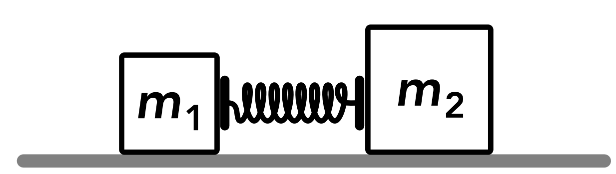
A A counterclockwise torque
B A clockwise torque
C Either a clockwise or a counterclockwise torque
D None of the above

29. A block of mass m slides across a frictionless surface with a speed of v when it collides with a spring which exerts a force on the block and reverses its direction. The block then moves away from the spring with the same speed v but in the opposite direction. If right is the positive direction, which of the following is equal to the impulse on the block?



A mv
B $-mv$
C $-2mv$
D The impulse is zero

30. Two blocks are pushed together with a spring between them so that the spring compresses some amount. The blocks are then released from rest and the spring causes them to slide away from each other on a frictionless surface. If mass m_2 is greater than mass m_1, which block moves faster after losing contact?



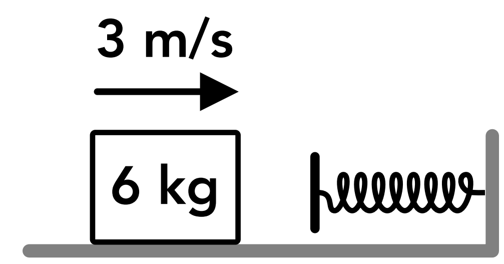
A m_1
B m_2
C The move at the same speed
D Cannot be determined

31. A ball is thrown directly upwards and it takes 2 seconds to reach its maximum height. What was the initial speed of the ball (not using kinematics)?

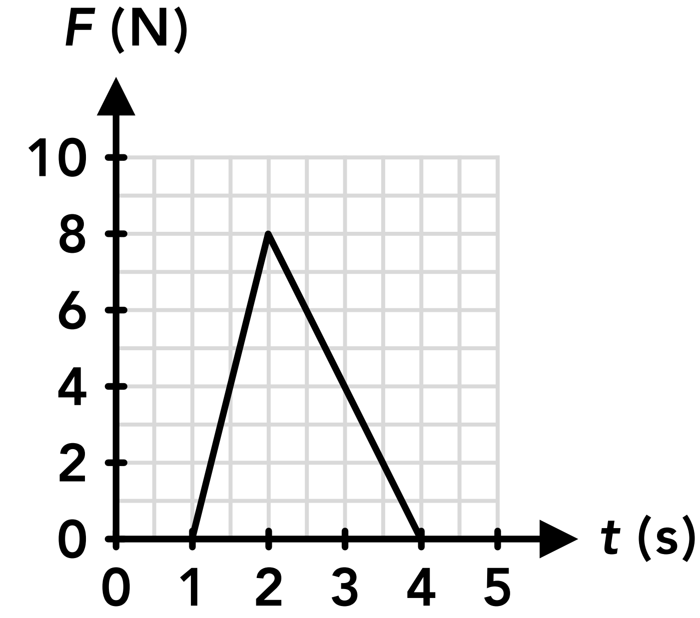
32. A person pushes an 18 kg curling stone (initially at rest) for 3 seconds with a force of 10 N. How fast is the stone moving after that time (not using kinematics)?

33. During a crash test a 1,600 kg car is brought to a speed of 25 m/s before it crashes into a wall, coming to a stop in 0.4 seconds. What is the magnitude of the average force exerted on the car from the wall during the crash (not using kinematics)?

34. A block slides on a frictionless surface and collides with a spring which has a spring constant of 100 N/m. Once the block contacts the spring, how long does it take for the block to momentarily come to rest? Note that the average spring force is equal to half of the maximum spring force.



35. A 2 kg rubber ball is dropped on the ground and it bounces back up. A graph of the force exerted on the ball from the ground is shown on the right and up is the positive direction. If the ball is moving 4 m/s at the moment it contacts the ground, how fast is it moving at the moment it loses contact with the ground?



Answers

1. A, B, D
2. C
3. B
4. B, C
5. A, D
6. B
7. D
8. A, B, D
9. D
10. A, B, C
11. A, D
12. C
13. 36,000 kg·m/s
14. 6 m/s
15. 31.3 kg·m/s
16. 25.5 kg
17. p_x = 24 kg·m/s, p_y = -41.6 kg·m/s
18. 180 kg·m²/s
19. C, D
20. C
21. A
22. A
23. B
24. C
25. D
26. B, D
27. B
28. A
29. C
30. A
31. 19.6 m/s
32. 1.67 m/s
33. 50,000 N
34. 0.49 s
35. 2 m/s

Answers - Momentum

1. **Answer: A, B, D**
   Momentum is a vector quantity. An object's momentum is equal to its mass multiplied by its velocity so the momentum depends on the mass, the speed (magnitude of the velocity) and the direction of the velocity.

2. **Answer: C**
   The SI unit for linear momentum is kg·m/s (the units for mass multipled by velocity).

3. **Answer: B**
   The momentum of an object can be found using the equation below. Momentum is directly proportional to velocity. Object B has twice the velocity of object A so it has twice the momentum.
   $p = mv$

4. **Answer: B, C**
   Momentum is a vector which has the same direction as the velocity. The blocks have the same mass and same speed so the magnitude of their momentum is the same. The velocities of the blocks are in opposite directions so they have opposite signs, one block has positive momentum and the other has negative momentum.

5. **Answer: A, D**
   Momentum is a vector which depends on the direction of the velocity, but kinetic energy is a scalar which does not have a direction. The ball has the same speed at the start and end of the motion but its velocity is in opposite directions, so its kinetic energy is the same but its momentum is different.

6. **Answer: B**
   The equation for momentum is given below. Object B has three times the speed of object A so it has three times the momentum of object A, and object A has 1/3 the momentum of object B.

7. **Answer: D**
   Using kinematics, we can find the speed of the ball after it accelerates at g for Δy, and then find the momentum.
   $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$ $\quad v_{yf}^2 = (0\text{ m/s})^2 + 2(g)(\Delta y)$ $\quad v_{yf} = \sqrt{2g\Delta y}$
   $p = mv = m\sqrt{2g\Delta y}$
   Using conservation of energy for the ball-earth system, gravitational potential energy is converted into kinetic energy, which we can use to find the speed and then the momentum.
   $E_i = E_f$ $\quad U_{g\,i} = K_f + U_{g\,f}$ $\quad mg(\Delta y) = \frac{1}{2}mv_f^2 + mg(0\text{ m})$ $\quad v_f = \sqrt{2g\Delta y}$
   $p = mv = m\sqrt{2g\Delta y}$

8. **Answer: A, B, D**
   An object has angular momentum when it is rotating.

9. **Answer: D**
   The SI unit for angular momentum is kg·m²/s.

10. **Answer: A, B, C**
    The equation for angular momentum is given below. Angular momentum depends on the angular velocity and the rotational inertia, which depends on the mass and radius of the cylinder.
    $L = I\omega \qquad I = \frac{1}{2}mR^2$

11. **Answer: A, D**
    The wheels have the same rotational inertia because they have the same size, shape and mass. The wheels have the same radius and linear speed so we know they have the same angular speed, which is given by $v = r\omega$. Angular momentum is a vector which depends on the direction of the angular velocity, and the wheels are rotating in opposite directions so their angular momentums have opposite signs (positive and negative).

12. **Answer: C**
    We are comparing the magnitudes of the momentum which does not depend on the direction.
    $p_A = m_A v_A = (5\text{ kg})(3\text{ m/s}) = 15\text{ kg·m/s}$
    $p_B = m_B v_B = (2\text{ kg})(4\text{ m/s}) = 8\text{ kg·m/s}$
    $p_C = m_C v_C = (3\text{ kg})(2\text{ m/s}) = 6\text{ kg·m/s}$
    $p_D = m_D v_D = (4\text{ kg})(2\text{ m/s}) = 8\text{ kg·m/s}$

13. **Answer: 36,000 kg·m/s**
    $\frac{72\text{ km}}{\text{h}} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ h}}{60\text{ min}} \times \frac{1\text{ min}}{60\text{ s}} = 20\text{ m/s}$
    $p = mv = (1800\text{ kg})(20\text{ m/s}) = 36{,}000\text{ kg·m/s}$

14. **Answer: 6 m/s**
    $p_1 = p_2 \qquad m_1v_1 = m_2v_2 \qquad (60\text{ kg})v_1 = (90\text{ kg})(4\text{ m/s}) \qquad v_1 = 6\text{ m/s}$

15. **Answer: 31.3 kg·m/s**
    We can use conservation of energy of the block-earth system to find the speed of the block at the bottom of the incline. The gravitational potential energy is converted into kinetic energy.
    $E_i = E_f \qquad U_{g\,i} = K_f + U_{g\,f} \qquad mg(2\text{ m}) = \frac{1}{2}mv_f^2 + mg(0\text{ m}) \qquad v_f = 6.26\text{ m/s}$
    $p = mv = (5\text{ kg})(6.26\text{ m/s}) = 31.3\text{ kg·m/s}$

16. **Answer: 25.5 kg**
    $\omega = \frac{150\text{ rev}}{\text{min}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} \times \frac{1\text{ min}}{60\text{ s}} = 15.71\text{ rad/s}$
    $L = I\omega \qquad L = \frac{1}{2}mR^2\omega \qquad (8\text{ kg·m²/s}) = \frac{1}{2}m(0.2\text{ m})^2(15.71\text{ rad/s}) \qquad m = 25.5\text{ kg}$

17. **Answer: $p_x = 24$ kg·m/s, $p_y = -41.6$ kg·m/s**
    $p_x = mv_x = (8\text{ kg})(6\text{ m/s})\cos(60°) = 24\text{ kg·m/s}$
    $p_y = mv_y = (8\text{ kg})(6\text{ m/s})\sin(60°) = -41.6\text{ kg·m/s}$ (negative because it's in the negative y direction)

18. **Answer: 180 kg·m²/s**
The rotational inertia can be found by treating the sphere as a point mass of 10 kg at a distance of 3 m from the axis of rotation.
$L = I\omega = mr^2\omega = (10\text{ kg})(3\text{ m})^2(2\text{ rad/s}) = 180\text{ kg·m}^2\text{/s}$

Answers - Impulse

19. **Answer: C, D**
Impulse is defined as the change in momentum of an object. An impulse is caused by a force and can be calculated as the average force applied multiplied by the period of time the force is applied.

20. **Answer: C**
The SI unit for impulse is kg·m/s which is the same unit as momentum.

21. **Answer: A**
The direction of an impulse is the same as the direction of the force exerted on the object. This is also the direction of the change in momentum, in the same way that the direction of the acceleration vector is the same as the direction of the change in velocity.

22. **Answer: A**
As the block slides down the incline it accelerates and its velocity increases so its momentum also increases.

23. **Answer: B**
We use the average force multiplied by the period of time to calculate impulse.

24. **Answer: C**
The equation for impulse is given below. If the force is divided by 2 and the time is multiplied by 2 then the change in momentum is the same. The change in velocity is also the same because it's the same object with the same mass.
$F_{avg}\Delta t = \Delta p = m\Delta v$

25. **Answer: D**
The impulse on each astronaut is equal to the force exerted on them multiplied by the period of time the force is applied. According to Newton's 3rd law of motion, the force exerted by astronaut A on astronaut B is equal in magnitude and opposite in direction to the force exerted by astronaut B on astronaut A (this is a force pair). The period of time the force is applied is the same for both astronauts, so the impulse on each astronaut is the same.

26. **Answer: B, D**
A net torque applied to an object for a period of time will change the object's angular momentum, which we might call a "rotational impulse". Unless given a reason to assume otherwise, we assume an object's rotational inertia (it's mass and shape) does not change so the object's angular speed must change.
$\tau\Delta t = \Delta L = I\Delta\omega$

27. **Answer: B**
The equation for rotational impulse is given below. The change in angular momentum is the same in each case, so if the torque is multiplied by 3 then the period of time must be divided by 3.
$\Delta L = \tau\Delta t$

28. **Answer: A**
The object starts with a counterclockwise angular velocity so it has a counterclockwise angular momentum. A counterclockwise torque would increase its angular velocity and angular momentum. A clockwise torque would decrease its angular velocity and angular momentum.

29. **Answer: C**
The impulse is equal to the change in the momentum of the block. Impulse and momentum are vectors so the direction (positive or negative sign) must be included. The initial velocity is to the right so it's positive and the final velocity is to the left so it's negative.
$J = \Delta p = p_f - p_i = mv_f - mv_i = m(-v) - m(v) = -2mv$

30. **Answer: A**
The spring force exerted on each block is the same (the force at each end of the spring is the same) and the spring is in contact with each block for the same amount of time, so the impulse on each block is the same. That means the change in momentum of each block is the same. If m_2 is greater than m_1 then Δv_2 must be less than Δv_1.
$\Delta p_1 = \Delta p_2 \qquad m_1 \Delta v_1 = m_2 \Delta v_2$

31. **Answer: 19.6 m/s**
When the ball reaches the maximum height its velocity is zero. The change in the ball's momentum (the impulse) is equal to the gravitational force multiplied by the period of time. If we say up is the positive direction:
$\Delta p = F\Delta t \qquad mv_f - mv_i = -mg\Delta t \qquad m(0\text{ m/s}) - mv_i = -mg(2\text{ s}) \qquad v_i = 19.6\text{ m/s}$

32. **Answer: 1.67 m/s**
$\Delta p = F\Delta t \qquad mv_f - mv_i = F\Delta t \qquad (18\text{ kg})v_f - (18\text{ kg})(0\text{ m/s}) = (10\text{ N})(3\text{ s}) \qquad v_f = 1.67\text{ m/s}$

33. **Answer: 50,000 N**
$\Delta p = F_{avg}\Delta t \quad mv_f - mv_i = F_{avg}\Delta t \quad (1{,}600\text{ kg})(0\text{ m/s}) - (1{,}600\text{ kg})(25\text{ m/s}) = F_{avg}(0.8\text{ s}) \quad F_{avg} = 50{,}000\text{ N}$

34. **Answer: 0.49 s**
We can use impulse and momentum to find the period of time that the average spring force is applied to change the momentum of the block to zero. But we first need to find the average force, which is half of the maximum force, which we can find using conservation of energy and Hooke's law (leaving things as variables).

$$E_i = E_f \qquad K_i = U_{sp\,f} \qquad \frac{1}{2}mv_i^2 = \frac{1}{2}k\Delta x_f^2 \qquad \Delta x_f = \sqrt{\frac{mv_i^2}{k}}$$

$$F_{sp\,max} = k\Delta x = k\sqrt{\frac{mv_i^2}{k}} = \sqrt{kmv_i^2}$$

$$\Delta p = F_{avg}\Delta t \qquad mv_f - mv_i = \frac{1}{2}\sqrt{kmv_i^2}\,\Delta t \qquad (6\text{ kg})(0\text{ m/s}) - (6\text{ kg})(3\text{ m/s}) = \frac{1}{2}\sqrt{(100\text{ N/m})(6\text{ kg})(3\text{ m/s})^2}\,\Delta t$$

$\Delta t = 0.49\text{ s}$

35. **Answer: 2 m/s**
The ground exerts an impulse (a force for a period of time) on the ball in the upwards direction which changes the ball's momentum. Impulse is the area under the curve of the force vs time graph, which in this case is a triangle.

$$\Delta p = \text{area} = \frac{1}{2}bh = \frac{1}{2}(3\text{ s})(8\text{ N}) = 12\text{ kg·m/s or N·s}$$

$$\Delta p = mv_f - mv_i \qquad (12\text{ kg·m/s}) = (2\text{ kg})v_f - (2\text{ kg})(-4\text{ m/s}) \qquad v_f = 2\text{ m/s}$$