AS CHECKING TECHNIQUES

KNOW YOURSELF — Always have your TID sheets to hand when you do maths
Do an extra check for your most common mistakes

Check a graph sketch

Sketch it on a graphical calc.
y intercept labelled?
Asymptote <u>equations</u> stated?
Does the Q ask you to find x intercepts or turning points?

Check a solution to an equation

Sub your answer back in to see if it works
Or, solve the equation on your calculator and compare answers

Check simultaneous equations eg where does $y = 3x$ cross $x^2 + y^2 = 8$

Sub the matched x and y coordinates into <u>both</u> equations to see if they work
Or, plot the graphs on a calculator and find the intersection

Check writing a number in a different form eg write $\dfrac{3}{2-\sqrt{5}}$ in the form $a + b\sqrt{5}$

Type the number into your calc then type the rearranged number in. Compare.

Check writing an expression in a different way eg write $\dfrac{2\sqrt{x}+1}{x}$ in the form $ax^n + +bx^m$

Type the original expression into your calc with any x you like subbed in. Sub the same x into your answer. Compare.

Check the equation of a line

Sub the original (known) point on the line into your equation to see if it works
Sketch your line on a graphical calc - see what its gradient is & whether it goes through the right point

Check the equation of the tangent/normal

Sketch the curve and your tangent/normal equation on your graphical calc and see if it is the tangent/normal at the right point.

Check a derivative or definite integral

Use the $\dfrac{\mathrm{d}}{\mathrm{d}x}$ button for x value and sub the same value into your derivative. Compare.

Use the $\displaystyle\int_a^b$ button on the calculator

Rules of Indices

$$a^m \times a^n = a^{m+n} \qquad \frac{a^m}{a^n} = a^{m-n} \qquad (a^m)^n = a^{mn}$$

$$a^0 = 1 \qquad a^1 = a \qquad (ab)^n = a^n b^n$$

Negative & Rational Indices

$$a^{\frac{1}{m}} = \sqrt[m]{a} \qquad a^{-m} = \frac{1}{a^m} \qquad a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

Manipulating Surds

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \qquad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Rationalising the Denominator

$$\frac{a}{b\sqrt{c}} \times \frac{\sqrt{c}}{\sqrt{c}} \qquad \frac{a}{b \pm c\sqrt{d}} \times \frac{b \mp c\sqrt{d}}{b \mp c\sqrt{d}}$$

Difference of Two Squares (DOTS) $a^2 - b^2 = (a-b)(a+b)$

Completing the Square $$x^2 + bx + c = \left(x + \frac{b}{2}\right) - \left(\frac{b}{2}\right)^2 + c$$

Turning Point of $y = A(x+B)^2 + C$ is $(-B,\ C)$

The Quadratic Formula $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solving a Quadratic Equation

Make it equal 0: $a(\dots)^2 + b(\dots) + c = 0$
Then Rearrange the Completed Square
OR Factorise & put each bracket equal to 0
OR Use the Quadratic Formula

Simultaneous Equations find Intersection of Graphs ie where $y = f(x)$ crosses $y = g(x)$

Make the simpler equation $y = \dots$ or $x = \dots$
Sub into the more complicated equation
Solve to find one of the coordinates
Use the simpler equation to get the other coordinates

The Discriminant of a Quadratic $a(\ldots)^2 + b(\ldots) + c$ is $b^2 - 4ac$

If $b^2 - 4ac$ is positive, the quadratic has 2 real distinct roots
If $b^2 - 4ac$ is **N**egative, the quadratic has **N**o real roots
If $b^2 - 4ac$ is zer**O**, the quadratic has **O**ne real repeated root

Quadratic Inequalities, $a > 0$

$ax^2 + bx + c < 0$ has solution set $\{x : \text{root } 1 < x < \text{root } 2\}$

$ax^2 + bx + c > 0$ has solution set $\{x : x < \text{root } 1\} \cup \{x : x > \text{root } 2\}$

Graphical Inequalities

$f(x) < g(x)$ is the set of values of x for which the graph of $f(x)$ is below the graph of $g(x)$

$f(x) < 0$ is the set of values of x for which the graph of $f(x)$ is below the x axis

$f(x) > 0$ is the set of values of x for which the graph of $f(x)$ is above the x axis

Sketching Inequalities

For the inequality $y < f(x)$ or $y > f(x)$ the line $y = f(x)$ is drawn as a dotted line

For the inequality $y \leq f(x)$ or $y \geq f(x)$ the line $y = f(x)$ is drawn as a solid line

The Binomial Expansion of $(a + b)^n$

The Binomial Coefficient

$^nC_x = \binom{n}{x} = \dfrac{n!}{x!(n-x)!}$ = the no. of ways x things can be chosen from a list of n things

$^nC_0 = 1,\quad {}^nC_1 = n,\quad {}^nC_2 = \dfrac{1}{2}n(n-1),\quad {}^nC_3 = \dfrac{1}{3!}n(n-1)(n-2)$

	Term 1	Term 2	Term 3
Coefficient	$^nC_0$	$^nC_1$	$^nC_2$
$a \ \downarrow$	$(a)^n$	$(a)^{n-1}$	$(a)^{n-2}$
$b \ \uparrow$	$(b)^0$	$(b)^1$	$(b)^2$

Functions have Roots. ROOTS of $f(x)$ are values of x for which $f(x) = 0$

Equations have Solutions. SOLUTIONS of $f(x) = 0$ are values of x for which $f(x) = 0$

The DOMAIN of a function is the set of possible values of x going into the function.
The domain of a function is usually given to you in the question.

The RANGE of a function is the set of possible values of y coming out of the function.
The range of a function changes when you change the domain of the function.
Find the range of a function by sketching its graph on its domain (this is an A level topic).

Graph Transformations

$f(x+a)$ is a translation of $f(x)$ by the vector $\begin{pmatrix} -a \\ 0 \end{pmatrix}$

$f(x)+a$ is a translation of $f(x)$ by the vector $\begin{pmatrix} 0 \\ a \end{pmatrix}$

$f(ax)$ is a stretch of $f(x)$, scale factor $\frac{1}{a}$, parallel to the x axis (about the y axis)

$af(x)$ is a stretch of $f(x)$, scale factor a, parallel to the y axis (about the x axis)

$f(-x)$ is a reflection of $f(x)$ over the y axis
$-f(x)$ is a reflection of $f(x)$ over the x axis

Asymptotes

The graph comes down (or goes up) towards the asymptote like a plane that is landing - but never actually touches down.

Vertical asymptotes occur when x cannot be a certain value (ie it would mean dividing by 0)
Horizontal asymptotes show long term behaviour (either for massive positive x or massive negative x or both)

$y = \frac{1}{x}$ and $y = \frac{1}{x^2}$ have 2 asymptotes, $y = 0$ and $x = 0$

$y = a^x$ and $y = e^x$ have 1 horizontal asymptote at $y = 0$

$y = \log_a(x)$ and $y = \ln(x)$ have 1 vertical asymptote at $x = 0$

$y = \frac{ax+b}{cx+d}$ has 2 asymptotes, one vertical at $x = -\frac{d}{c}$ and one horizontal at $y = \frac{a}{c}$

AS LINES & CIRCLES

Useful formulae: $\text{gradient} = m = \dfrac{dy}{dx} = \dfrac{\text{change in } y}{\text{change in } x}$

$$\text{distance } d = \sqrt{(\text{change in } x)^2 + (\text{change in } y)^2}$$

$$\text{midpoint} = \left(\text{average } x \text{ coordinate, average } y \text{ coordinate}\right)$$

Line with gradient m through (a, b):

$m(x-a) = y-b$ used to construct the line

$ax + by + c = 0$, $a, b, c \in \mathbb{Z}$ used for final ans.

$y = mx + c$ used to identify the gradient

PARALLEL lines have the same gradient.

PERPENDICULAR lines have negative reciprocal gradients: $\text{grad } 2 = \dfrac{-1}{\text{grad } 1}$

If x & y are DIRECTLY PROPORTIONAL ($y \propto x$) then $y = kx$ [a straight line through $(0, 0)$]

Circle centre (a, b), radius r

$(x-a)^2 + (y-b)^2 = r^2$

$x^2 + y^2 - fx - gy + h = 0 \rightarrow$ complete the square

Circle facts: The tangent to a circle is perpendicular to the radius

If A, B, C lie on a circle and $\angle ABC = 90°$, then AC is a diameter of the circle

The perpendicular bisectors of two chords intersect at the centre of the circle

AS VECTORS & TRIANGLES

Vectors have magnitude (length) and direction $\begin{pmatrix} a \\ b \end{pmatrix} = a\mathbf{i} + b\mathbf{j}$, **i** = $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and **j** = $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

The ANGLE between two vectors is found using the cosine rule

The MAGNITUDE (length) of $\mathbf{a} = x\mathbf{i} + y\mathbf{j}$ is found using Pythagoras: $|\mathbf{a}| = \sqrt{x^2 + y^2}$

The UNIT VECTOR (length = 1) parallel to $\mathbf{a}$ is found by dividing $\mathbf{a}$ by $|\mathbf{a}|$

If $\mathbf{a}$ and $\mathbf{b}$ are PARALLEL, then $\mathbf{a} = \lambda\mathbf{b}$, where λ is a constant.

$\mathbf{a}$ and $-\mathbf{a}$ are parallel and have the same length, but are in opposite directions.

The POSITION VECTOR of a point A is the vector from the origin O to A.

If the position vector of A is $\mathbf{a}$ and the position vector of B is $\mathbf{b}$, then $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.

If Q divides $\overrightarrow{AB}$ in the ratio $\lambda : \mu$, then $\overrightarrow{OQ} = \overrightarrow{OA} + \dfrac{\lambda}{\lambda + \mu}\overrightarrow{AB}$

(for midpoint, use $\lambda = \mu = 1$)

Cosine Rule (angle A is opposite side a) $\quad a^2 = b^2 + c^2 - 2bc\cos A$

Sine Rule (angle A is opposite side a etc) $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Area of a triangle (angle C is between sides $\mathbf{a}$ and $\mathbf{b}$) $\quad \text{Area} = \dfrac{1}{2}|\mathbf{a}||\mathbf{b}|\sin C$

AS EXPONENTIALS & LOGS

If a and b are both positive:

$$a^x = b \quad \updownarrow \quad x = \log_a(b)$$

$$e^x = b \quad \updownarrow \quad x = \ln(b)$$

If $y = a^x$, $a > 1$, then as x increases, y increases. This is EXPONENTIAL GROWTH.

If $0 < a < 1$, then as x increases, y decreases. This is EXPONENTIAL DECAY.

An EXPONENTIAL MODEL has the form $y = Ae^{kt} + B$ or $y = Ar^{kt} + B$

If $k > 0$ it models exponential growth, if $k < 0$ it models exponential decay.

As k (either positive or negative) gets closer to 0 the rate of change of y gets slower.

The further from zero k is (either positive or negative), the faster the rate of change of y.

Useful facts: $\log_x(x) = 1$ $\qquad \log_x(1) = 0$

Graphs: The graph of $y = \ln x$ is a reflection of $y = e^x$ in the line $y = x$.

The graph of $y = \ln x$ has a vertical asymptote at $x = 0$

The graph of $y = e^x$ has a horizontal asymptote at $y = 0$

Laws of Logs:

$$\log_x(a) + \log_x(b) = \log_x(ab) \qquad \log_x(a) - \log_x(b) = \log_x\left(\frac{a}{b}\right)$$

$$\log_x(a)^k = k\log_x(a) \qquad \log_x\left(\frac{1}{a}\right) = \log_x(a)^{-1} = -\log_x a$$

Differentiating e^{kx}

$$f(x) = e^{kx} \implies f'(x) = ke^{kx} \qquad y = e^{kx} \implies \frac{dy}{dx} = ke^{kx}$$

$$f'(x) = kf(x) \qquad \frac{dy}{dx} = ky$$

$$f'(x) \propto f(x) \qquad \frac{dy}{dx} \propto y$$

$y = ax^m$ can be rearranged to give $\log y = m\log x + \log a$.

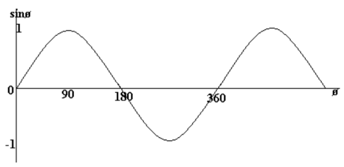
If $y = ax^n$, the graph of $\log y$ against $\log x$ is a straight line: gradient $= m$

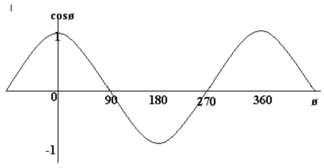
vertical intercept $= \log a$.

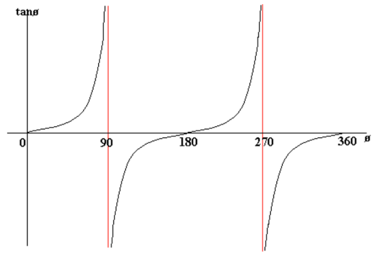
$y = ab^x$ can be rearranged to give $\log y = x\log b + \log a$.

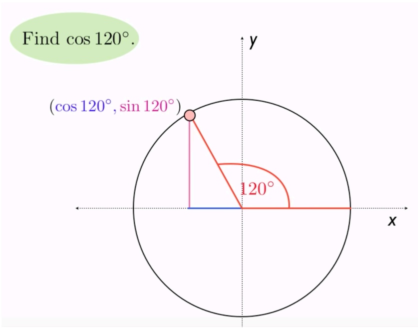
If $y = ab^x$, the graph of $\log y$ against x is a straight line: gradient $= \log b$

vertical intercept $= \log a$.









For a point (x, y) on the unit circle with angle θ from the positive x axis:

$$\cos\theta = x \text{ coordinate}$$
$$\sin\theta = y \text{ coordinate}$$
$$\tan\theta = \frac{y}{x} = \text{gradient}$$

Trig Identities: $\sin^2\theta + \cos^2\theta \equiv 1$ $\qquad$ $\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$

Solutions to $\sin\theta = a$ and $\cos\theta = x$ only exist for $-1 \leq a \leq 1$.
Solutions to $\tan\theta = a$ exist for all $a \in \mathbb{R}$.

To solve the mini-trig equation $\sin\theta = a$

(1) Use the calculator to find the first solution $\theta_1 = \sin^{-1}(a)$
(2) The second solution is $\theta_2 = 180 - \theta_1$
(3) Now ±360 as many times as you like to θ_1 and θ_2 to get more solutions.

To solve $\sin\theta = a$: solution 1 is $\theta_1 = \sin^{-1}(a)$, solution 2 is $\theta_2 = 180 - \theta_1$ then ±360

To solve $\cos\theta = a$: solution 1 is $\theta_1 = \cos^{-1}(a)$, solution 2 is $\theta_2 = -\theta_1$ then ±360

To solve $\tan\theta = a$: solution 1 is $\theta_1 = \tan^{-1}(a)$, solution 2 is $\theta_2 = 180 + \theta_1$ then ±360

To solve the mini-trig equation $\sin(\ldots) = a$ (or cos or tan)

(1) Put $y = \ldots$
(2) Solve $\sin y = a$ as above and ±360 to get many y solutions
(3) Rearrange $y = \ldots$ to get the θ solutions in the required range

AS DIFFERENTIATION

$$y = ax^n \quad \updownarrow \quad \frac{dy}{dx} = anx^{n-1}$$

$$f(x) = ax^n \quad \updownarrow \quad f'(x) = anx^{n-1}$$

The GRADIENT OF A CURVE at any given point x is actually the gradient of the tangent to the that curve, at that point. The DERIVATIVE of the function $y = f(x)$, written as $f'(x)$ or $\frac{dy}{dx}$, tells you the rate of change of the original function, ie the gradient of the tangent to the original function.

Differentiating the derivative of a function gives you the 2nd derivative, written $f''(x)$ or $\frac{d^2y}{dx^2}$.

Differentiation from FIRST PRINCIPLES: $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The TANGENT to the curve $y = f(x)$ at the point $(\alpha, f(\alpha))$: $$f'(\alpha)(x - \alpha) = y - f(\alpha)$$

The NORMAL to the curve $y = f(x)$ at the point $(\alpha, f(\alpha))$: $$-\frac{1}{f'(\alpha)}(x - \alpha) = y - f(\alpha)$$

$f'(a) > 0$ the function $f(x)$ is INCREASING at $x = a$ (going up/tangent gradient is positive)

$f'(a) < 0$ the function $f(x)$ is DECREASING at $x = a$ (going down/tangent grad is negative)

$f'(a) = 0$ $x = a$ is a STATIONARY POINT. The gradient of the tangent to the function at $x = a$ is zero. It could be a maximum, a minimum or point of inflection.

At a MINIMUM point: y is going down, then stops, then goes up
$\frac{dy}{dx}$ is negative, then $\frac{dy}{dx} = 0$, then $\frac{dy}{dx}$ is positive
$\frac{dy}{dx}$ is going up, so $\frac{d^2y}{dx^2}$ is positive

At a MAXIMUM point: y is going up, then stops, then goes down
$\frac{dy}{dx}$ is positive, then $\frac{dy}{dx} = 0$, then $\frac{dy}{dx}$ is negative
$\frac{dy}{dx}$ is going down, so $\frac{d^2y}{dx^2}$ is negative

At a POINT OF INFLECTION, the graph changes from CONVEX to CONCAVE (or vice versa)

ie) $\frac{d^2y}{dx^2}$ just before the point is positive, $\frac{d^2y}{dx^2}$ just after the point is negative (or vice versa)

A point of inflection may or may not be a stationary point.

AS INTEGRATION

$$\frac{dy}{dx} = x^n \quad (n \neq -1) \qquad\qquad f'(x) = x^n \quad (n \neq -1)$$

$$\updownarrow \qquad\qquad\qquad\qquad \updownarrow$$

$$y = \frac{1}{n+1}x^{n+1} + c \qquad\qquad f(x) = \frac{1}{n+1}x^{n+1} + c$$

$$\int \frac{dy}{dx}\ dx = y + c \qquad\qquad \int f'(x)\ dx = f(x) + c$$

You can integrate terms individually: $\int f(x) + g(x)\ dx = \int f(x)\ dx + \int g(x)\ dx$

To find c: Substitute both x and y values (coordinates of a point on the curve or the value of the function at a given point) into the integrated function

Solve the equation to find c

The area between the positive section of the curve $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$ is given by

$$\int_a^b f'(x)\ dx = [f(x)]_a^b = f(b) - f(a).$$

If the curve $y = f(x)$ is below the x-axis between $x = a$ and $x = b$, the integral $\int_a^b y\ dx$ will be negative. Remember though, the area is positive.

If the graph of $y = f(x)$ is above the graph of $y = g(x)$ then the area between the graphs of $y = f(x)$ and $y = g(x)$ and the lines $x = a$ and $x = b$ is given by $\int_a^b f(x) - g(x)\ dx$.

AS PROOF

$x \in \mathbb{N}$ x is a NATURAL number 0, 1, 2, 3, 4, … All natural numbers are integers: $\mathbb{N} \subset \mathbb{Z}$

$x \in \mathbb{Z}$ x is an INTEGER 0, ±1, ±2, ±3, ±4, … All integers are rational: $\mathbb{Z} \subset \mathbb{Q}$

$x \in \mathbb{Q}$ x is a RATIONAL number: $x = \dfrac{m}{n}$, $m, n \in \mathbb{Z}$ All rational numbers are real: $\mathbb{Q} \subset \mathbb{R}$

$x \in \mathbb{R}$ x is a REAL number (all numbers not involving $\sqrt{-1}$)

$\text{Thing } 1 \Rightarrow \text{Thing } 2$ If thing 1 is true, then thing 2 is definitely true
$\text{Thing } 1 \Leftarrow \text{Thing } 2$ If thing 2 is true, then thing 1 is definitely true

$\text{Thing } 1 \Leftrightarrow \text{Thing } 2$ Thing 1 and thing 2 are EQUIVALENT
If thing 1 is true then thing 2 is definitely true
and also if thing 2 is true then thing 1 is definitely t

$\text{Thing } 1 \equiv \text{Thing } 2$ Thing 1 and thing 2 are the same for all values of the unknown
eg $x^2 + 3x \equiv x(x+3)$ because it's always true
… but $x^2 + 3x = x(x+1)$ because it's not always true

Interval Notation $x \in [a, b]$ means $a \le x \le b$ $x \in [a, b)$ means $a \le x < b$

$x \in (a, b)$ means $a < x < b$ $x \in (a, b]$ means $a < x \le b$

Set Notation $\{x \in \mathbb{R} : x < a\} \cup \{x \in \mathbb{R} : x > b\}$ means $x < a$ OR $x > b$

$\{x \in \mathbb{R} : x < a\} \cap \{x \in \mathbb{R} : x > b\}$ means $x < a$ AND ALSO $x > b$

A mathematical proof
States any assumptions made
Shows every step clearly
Every step follows on logically from the previous step
Covers all possible cases
Has a statement of proof at the end

To prove an identity you should:
Begin with one side of the identity
Use algebra to manipulate it until it matches the other side
Show every step of your working

You can <u>prove</u> a mathematical statement by EXHAUSTION: breaking the statement into smaller cases and proving each case separately (eg prove for odds then evens)

You can <u>disprove</u> a mathematical statement by COUNTER-EXAMPLE: give one example that does not work for the given statement

AS: LDS & SAMPLING

POPULATION:	whole set of items of interest.
CENSUS:	measures every individual in a population.
SAMPLE:	a selection of observations from a subset of the population, which is extrapolated to estimate information about the whole population.
SIMPLE RANDOM SAMPLE:	every individual in a population is equally likely to be selected.
SYSTEMATIC SAMPLING:	individuals are chosen at regular intervals from an ordered list.
STRATIFIED SAMPLING:	population is divided into strata (groups) and random samples are taken.
QUOTA SAMPLING:	sample that reflects the characteristics of the population is chosen.
OPPORTUNITY SAMPLING:	sample is chosen from suitable individuals available at the time.

LDS Key Features

AS DATA REPRESENTATION

The MIDPOINT is the average of the upper and lower class boundaries.
The CLASS WIDTH is the difference between the upper and lower class boundaries.

RANGE	measures spread. It is the difference between the largest and smallest values.
IQR	measures spread. It is the difference between the upper and lower quartiles.
VARIANCE	measures spread. It is the average squared distance from each data point from the mean

$$\frac{\sum (x-\bar{x})^2}{n} = \frac{\sum x^2}{n} - \bar{x}^2 \qquad \text{Standard Deviation} = \sqrt{\text{Variance}}$$

The MEAN can be calculated using the formula $\bar{x} = \frac{\sum x}{n}$

The MEDIAN is the middle value when the data values are put in ascending order.

For ungrouped data: For the lower quartile, calculate $\frac{n}{4}$. For the 7th decile, calculate $\frac{7n}{10}$ etc

If this is a whole number, the data point you need is halfway between this point and the point above. If not, round up.

For grouped data, use LINEAR INTERPOLATION to find quantiles

An OUTLIER is any value greater than $Q_3 + \frac{3}{2}(\text{IQR})$ or less than $Q_1 - \frac{3}{2}(\text{IQR})$

Removing these values 'CLEANS' the data.

On a histogram: $\text{frequency density} = \text{height of bar} = \frac{\text{frequency}}{\text{width}} \times k$

Joining the middle of the top of each bar on the histogram forms a FREQUENCY POLYGON.

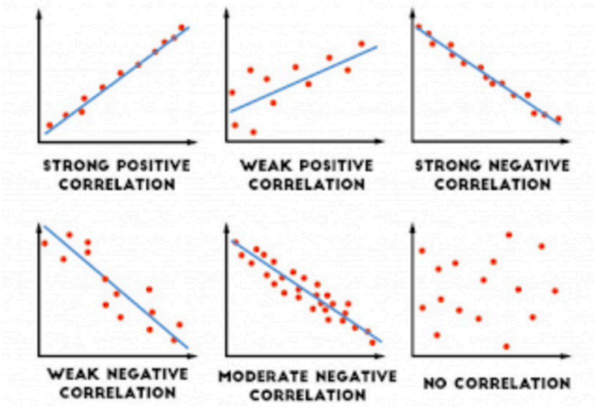
To find the dimensions of a bar use

$$\frac{\text{cm width}}{\text{maths width}} = \frac{\text{cm width}}{\text{maths width}} \quad \text{and} \quad \frac{\text{cm height}}{\text{freq . density}} = \frac{\text{cm height}}{\text{freq . density}}$$

for the bar you know about and the bar you don't know about.

BIVARIATE DATA has pairs of two variables, allowing scatter graphs to be drawn.

CORRELATION (and correlation coefficients) describe the relationship between two variables.



A REGRESSION LINE, in the form $y = a + bx$, is the line of best fit of a scatter graph.
a and b can be found using your calculator

The value a is interpreted as

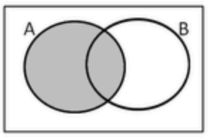
> "a is the value of y (use the context and units from the question) corresponding to zero x (use the context and units from the question)"

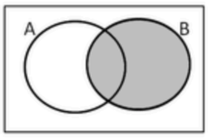
The value b is interpreted as

> "for every increase of one x (use the context and units from the question), y (use the context from the question) increases/decreases by b (use the units from the question)"

Watch out for changed units eg P is cost in thousands of pounds, then if the cost is £12,000 you need to sub in $P = 12$

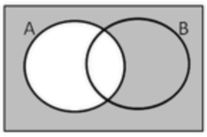
AS PROBABILITY

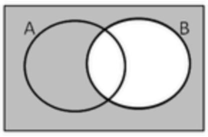

$P(A)$


$P(B)$

Useful formula:

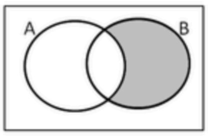
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

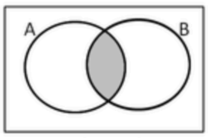

$P(A')$


$P(B')$

If A and B are MUTUALLY EXCLUSIVE, $P(A \cap B) = 0$

Therefore, $P(A \cup B) = P(A) + P(B)$


$P(A' \cap B)$


$P(A \cap B)$

If A and B are INDEPENDENT $P(A \cap B) = P(A) \times P(B)$

Therefore, $P(A \cup B) = P(A) + P(B) - P(A)P(B)$

A PROBABILITY DISTRIBUTION is a table or formula showing all of the possible outcomes and their associated probabilities.

The sum of all probabilities is 1: $\sum P(X = x) = 1$

The CUMULATIVE PROBABILITY is the probability of obtaining up to and including the outcome.

A TREE DIAGRAM can be used to show the outcomes of two or more events occurring in succession.

A VENN DIAGRAM is a graphic representation of two or more events.

AS BINOMIAL DISTRIBUTION

X = The number of …. out of n

$X \sim \mathrm{B}(n, p)$

Conditions for a binomial distribution

- two possible outcomes
- a fixed number of trials, n
- fixed probability of success, p
- trials are independent

The probability of an individual outcome: $P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$

The BINOMIAL COEFFICIENT

$$\binom{n}{x} = \frac{n!}{x!(n-r)!}$$

= the number of ways x things can be chosen from a list of n things

$P(X \leq x)$ is found using your calculator

$$P(X < x) = P\big(X \leq (x - 1)\big)$$

$$P(X \geq x) = 1 - P(X < x)$$

$$P(X > x) = 1 - P(X \leq x)$$

AS HYPOTHESIS TESTING

A hypothesis test determines whether there is sufficient evidence that a population parameter has changed. In the case of the Binomial Distribution $X \sim B(n,\ p)$, the population parameter is p

The NULL HYPOTHESIS is what we currently think is true $\mathrm{H}_0 : p = \dots$
The ALTERNATIVE HYPOTHESIS is what someone is claiming (and we are testing). $\mathrm{H}_1 : p \dots$

The idea: The chance of correctly guessing the suit of a playing card is 0.25. Someone claims they are psychic so the chance of them getting it right will be higher than 0.25. They guess the suit of 20 cards and get 8 correct. Does this mean they are psychic, or got lucky?

The probability of guessing 8 <u>or more</u> correctly - $P(X \geq 8)$ - is 0.10 so 10% of people could get 8/20 or more just by being lucky…. so they are psychic, or got lucky guessing?

The significance level α, set for the test, is how 'weird' or 'unlikely' we require the result to be before we say the result is 'significant'. Usually it's 5% (or even 1%) meaning that only 5% of people could get the result by guessing & getting lucky. So in this example, the result 8/10 is not big enough to be significant because 10% is not less than 5% . They could just be guessing.

A one-tailed test:

$\mathrm{H}_1 : p < \dots$ find $P(X \leq a)$ and compare it to the significance level set for the test.

$\mathrm{H}_1 : p > \dots$ find $P(X \geq a)$ and compare it to the significance level set for the test.

A two-tailed test:

$\mathrm{H}_1 : p \neq \dots$ find either $P(X \leq a)$ if a is 'weirdly small' or find $P(X \geq a)$ if a is 'weirdly big'
Compare this to <u>half</u> the significance level set for the test.

If the probability is <u>less</u> than the significance level then the result is <u>significant</u> and H_0 is <u>rejected</u>

The CRITICAL VALUES are the first values of X to fall inside the critical region.

The CRITICAL REGION is the set of values of X for which H_0 <u>would be</u> rejected

The ACCEPTANCE REGION is the set of values of X for which H_0 <u>would not be</u> rejected

The SIGNIFICANCE LEVEL OF A TEST is the probability of incorrectly rejecting H_0
= the full probability of the critical region

AS CONSTANT ACCELERATION

DISPLACEMENT is a vector. The magnitude of displacement is called DISTANCE.

VELOCITY is the rate of change of displacement.
On a displacement-time graph, velocity is represented by the gradient.
If a displacement-time graph is a straight line, velocity is constant.
Velocity is a vector. The magnitude of the velocity is called SPEED.

$$\text{Average } \textbf{velocity} = \frac{\textbf{displacement}}{\text{time}} \qquad \text{Average } \textbf{speed} = \frac{\textbf{distance}}{\text{time}}$$

ACCELERATION is the rate of change of velocity.
On a velocity-time graph, acceleration is represented by the gradient.
If a velocity-time graph is a straight line, acceleration is constant.
Acceleration is a vector. The magnitude of the acceleration is also called acceleration.

Most important facts: Gradient of $v - t$ graph is acceleration
Area between $v - t$ graph and t axis is displacement

The suvat equations can be used to solve problems about particles with constant acceleration.

(1) Decide which way is positive (up or down, left or right)
(2) Complete the table (use a horizontal table, not vertical). Be careful with **s**, **u**, **v**, **a** ± signs

s	u	v	a	t

(3) Choose the equation without the thing you're not interested in

$$v = u + at \qquad s = \left(\frac{u+v}{2}\right)t \qquad v^2 = u^2 + 2as \qquad s = ut + \frac{1}{2}at^2 \qquad s = vt - \frac{1}{2}at^2$$

(4) Sub in clearly then solve

GRAVITY causes all objects to accelerate towards the centre of the earth at a constant rate (ignoring air resistance) of 9.81ms^{-2} so for vertical motion under gravity, $a = 9.81 \text{ ms}^{-2}$ downwards

AS VARIABLE ACCELERATION

Differentiate

s or x (displacement)

v (velocity)

a (acceleration)

Integrate + c

The constant of integration, c, can be found by substituting in a known displacement/velocity.

The change in displacement from time t_1 to time t_2 is $\int_{t_1}^{t_2} v \; \mathrm{d}t$.

If the particle doesn't change direction from time t_1 to time t_2, then the change in displacement is the same as the distance travelled.

The change in velocity from time t_1 to time t_2 is $\int_{t_1}^{t_2} a \; \mathrm{d}t$.

Don't forget: displacement, velocity and acceleration are vectors, so can be positive or negative.

AS FORCES & ACCELERATION

NEWTON'S FIRST LAW: an object at rest will stay at rest and an object with constant velocity will move at that velocity unless unbalanced forces act upon it.

NEWTON'S SECOND LAW: $F = ma$, where F is force, measured in Newtons (N), m is mass, measured in kg and a is acceleration, measured in ms^{-2}

NEWTON'S THIRD LAW: For every action there is an equal and opposite reaction.

The RESULTANT FORCE is the sum of all forces acting on an object. If there is a resultant force acting on an object, it will accelerate in the direction of the resultant force.

$\sum F = ma$ The sum of the forces in a particular direction is equal to the mass times the acceleration in that direction

If 3 forces act on an object in equilibrium, you can form a TRIANGLE OF FORCES and use the cosine rule to find missing information.

WEIGHT is a force which acts downwards.

$W = mg$ Weight is equal to the mass times the acceleration due to gravity, g

TENSION occurs when a string or rod is in tension

THRUST occurs when a rod is in compression

FRICTION occurs between an object and a non-smooth surface. It opposes motion and acts parallel to the surface

NORMAL REACTION occurs between an object & a surface, it acts perpendicular to the surface

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If particles are connected by an INEXTENSIBLE STRING, their accelerations are equal

If particles are connected over a SMOOTH PULLEY, the tension in the string is equal on each particle

If a string is LIGHT you do not need to consider its weight