Statistical Process Control, or SPC, is a collection of tools that allow a Quality Engineer to ensure that their process is in control, using statistics.

Benefit of SPC

The primary benefit of a control chart is its **unique ability to separate the normal variation within your process and the special cause variation.**

Common and Special Causes

Every process has a **normal, inherent level of variation**, often called **common cause variation** as the sources of these variation are common to the process and cannot be easily eliminated.

Special cause variation is any type of variation that can be attributed to a special cause or situation that's influencing your process. These special causes impact your process in negative ways and result in instability and unpredictability.

A control chart has 3 key elements, the **Center Line**, the **Upper Control Limit** and the **Lower Control Limit**, that **allow the control chart to distinguish between common cause variation and special cause variation.**

Typically, these control limits are set at **$\pm$ 3 standard deviations** away from the **mean** and reflect the common causes of variation with your process. These control limits are not associated with your specification limits or customer requirements.

Selection of Variable

Selecting the right variable for a control chart means understanding the difference between **discrete and continuous data**.

For **discrete data** it means understanding the difference between a **defect and a defective**.

A "**defective"** is an **entire unit** that fails to meet specifications. A "**defect"** is an **undesirable condition** within a unit. A **defective unit can have multiple defects** associated with it.

Rationale Subgrouping

For **continuous data** it means understanding the idea of **rational subgroup**.

A **rational subgroup** is usually defined as a collection of units that are all produced under the same conditions.

These samples should be as **homogenous** as possible, and any variation within these samples should only include the normal, **inherent process variation**. Not having a properly defined **rational subgroup** can hide process changes or indicate process changes where in actuality none exist.

Variable Control Charts

Common **control charts** for **variable data** include the **X-bar and R Chart**, the **X-bar and S Chart**, and the **Individual and Moving Range Chart**.

If you're **rational sub-group size is a single value (1)**, then you'll use the **I-MR (Individual and Moving Range) Chart.**

If you're **rational sub-group size is between 2 – 10**, then you'll use the **X-Bar and R Chart.** When the sample size is less than 10, the **Range** of the sample data is a better estimator of the **process variability** than the standard deviation.

If you're **rational sub-group size is greater than 10**, then you'll use the **X-Bar and S Chart.** When you've got 10 or more samples in a rational sub-group, then the **best estimator of the process variability is the standard deviation**.

X-bar & R Charts

$$\bar{\bar{X}} = \frac{\sum \bar{X}_i}{k}$$

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R} \qquad LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$$

$$Average\ Range = \bar{R} = \frac{\sum R_i}{k}$$

$$\bar{R} = \frac{Sum\ of\ Subgroup\ Ranges}{\#\ of\ Subgroups}$$

$$UCL_R = D_4\bar{R} \qquad LCL_R = D_3\bar{R}$$

$$Population\ Standard\ Deviation = \hat{\sigma} = \frac{\bar{R}}{d_2}$$

X-Bar and R Chart				
Subgroup Sample Size	X-Bar Factor	Range Factors		Variance Factor
n	A_2	D_3	D_4	d_2
2	1.880	-	3.267	1.128
3	1.023	-	2.575	1.693
4	0.729	-	2.282	2.059
5	0.577	-	2.115	2.326
6	0.483	-	2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.078
15	0.223	0.347	1.653	3.472
20	0.180	0.415	1.585	3.735
25	0.153	0.459	1.541	3.931

X-bar & S Charts

$$\bar{\bar{X}} = \frac{\sum \bar{X}_i}{k}$$

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_3\bar{s} \qquad LCL_{\bar{X}} = \bar{\bar{X}} - A_3\bar{s}$$

$$\bar{s} = \frac{\sum s_i}{k}$$

$$UCL_s = B_4\bar{s} \qquad LCL_s = B_3\bar{s}$$

$$Population\ Standard\ Deviation = \hat{\sigma} = \frac{\bar{s}}{C_4}$$

X-Bar and S Chart				
Subgroup Sample Size	X-Bar Factor	Standard Deviation Factors		Variance Factor
n	A_3	B_3	B_4	c_4
2	2.659	-	3.267	0.7979
3	1.954	-	2.568	0.8862
4	1.628	-	2.266	0.9213
5	1.427	-	2.089	0.9400
6	1.287	0.030	1.970	0.9515
7	1.182	0.118	1.882	0.9594
8	1.099	0.185	1.815	0.9650
9	1.032	0.239	1.761	0.9693
10	0.975	0.284	1.716	0.9727
15	0.789	0.428	1.572	0.9823
20	0.680	0.510	1.490	0.9869
25	0.606	0.565	1.435	0.9896

I-MR Charts

$$I\ Centerline = \bar{X} = \frac{\sum X_i}{k} = \frac{Sum\ of\ Individual\ Values}{\#\ of\ Individual\ Values}$$

$$UCL_I = \bar{X} + E_2\overline{MR} \qquad LCL_I = \bar{X} - E_2\overline{MR}$$

$$\overline{MR}\ Centerline = \frac{\sum MR_i}{k-1} = \frac{Sum\ of\ Moving\ Ranges}{\#\ of\ MR's}$$

$$UCL_{MR} = D_4\overline{MR} \qquad UCL_{MR} = D_3\overline{MR}$$

I-MR Chart				
Subgroup Sample Size	Individual Factor	Moving Range Factors		Variance Factor
n	E_2	D_3	D_4	d_2
2	2.660	-	3.267	1.128
3	1.772	-	2.575	1.693
4	1.457	-	2.282	2.059
5	1.290	-	2.115	2.326
6	1.184	-	2.004	2.534
7	1.109	0.076	1.924	2.704
8	1.054	0.136	1.864	2.847
9	1.010	0.184	1.816	2.970
10	0.975	0.223	1.777	3.078

Attribute Control Charts

Attribute control charts are normally easier to construct and execute, however they tend to be less sensitive to small changes in variation or process shifts.

		Sample Size	
		Constant	**Variable**
Type	**Defect**	c Chart	u Chart
	Defectives	np Chart	p Chart

- **p-Chart**: Trending of *Defectives* with a *Variable* Sample Size
- **np-Chart**: Trending of *Defectives* with a *Constant* Sample Size
- **u-Chart**: Trending of *Defects* with a *Variable* Sample Size
- **c-Chart**: Trending of *Defects* with a *Constant* Sample Size

np & p Charts trend the number of Defectives and the math is based on the **Binomial distribution** which operates under the assumption that every unit inspected can only be counted as "bad" one time.

u & c Charts utilize the **Poisson distribution** because they trend the number of defects where it is possible for each item inspected to contain multiple defects.

The p Chart

This chart is the most sensitive of the attribute control charts to any changes in your process. This chart trends the proportion (p) of defective items across time when the sampling size varies.

$$\bar{p} = \ Centerline = \frac{\sum np}{\sum n} = \frac{Sum\ of\ All\ Defectives}{Sum\ of\ Subgroup\ Quantity}$$

$$UCL_{\bar{p}} \ = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}} \qquad LCL_{\bar{p}} \ = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}}$$

$$\bar{n} = Average\ Sample\ Size = \frac{\sum n}{k} = \frac{\sum subgroup\ quantity}{\#\ of\ subgroups}$$

The np Chart

The NP-Chart is a variant of the P-chart where we have the luxury of a constant sample size, which makes the math easier.

In the equations below, n is the sample size, which will be constant, p-bar is the average fraction defective, and k is the number of sub-groups being analyzed.

$$n\bar{p}\ Centerline = \frac{\sum np}{k} = \frac{Sum\ of\ All\ Defectives}{\#\ of\ subgroups}$$

$$UCL_{np} = \ n\bar{p} \ + \ 3\sqrt{n\bar{p}(1-\bar{p})} \qquad LCL_{np} = \ n\bar{p} \ - \ 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$\bar{p} = \ \%\ Defective = \frac{\sum np}{\sum n} = \frac{Sum\ of\ All\ Defectives}{Sum\ of\ Subgroup\ Quantity}$$

The c Chart

The C-Chart should be utilized when trending the number of defects per unit when your sample size is constant. The c stands for "Count" as we're simply counting the number of defects per inspection.

$$\bar{c} = \ Centerline = \frac{\sum c}{k} = \frac{Sum\ of\ All\ Defects}{\#\ of\ Subgroups}$$

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}} \qquad UCL_c = \bar{c} - 3\sqrt{\bar{c}}$$

The u Chart

Similar to the c chart, the u chart controls for the **percentage of defects per subgroup** and can accommodate a variable sample size.

The u chart normalizes the number of defects by the subgroup sample size, thus trending the number defects per sub-group.

$$\boldsymbol{\bar{u}} = \ Centerline = \frac{\sum \boldsymbol{c}}{\sum \boldsymbol{n}} = \frac{Sum\ of\ All\ Defects}{Sum\ of\ units\ inspected}$$

$$\boldsymbol{\bar{n}} = Average\ \#\ of\ Samples\ per\ Subgroup = \frac{\sum \boldsymbol{n}}{\boldsymbol{k}} = \frac{\text{Sum of units inspected}}{\text{Number of subgroups}}$$

$$\boldsymbol{UCL_u = \bar{u} + 3\sqrt{\frac{\bar{u}}{\bar{n}}}} \qquad \boldsymbol{LCL_u = \bar{u} - 3\sqrt{\frac{\bar{u}}{\bar{n}}}}$$

Control Chart Analysis

Below are 8 of the most common rules used to determine if a process is in statistical control

- **Rule 1** - Any single data point outside of either the upper or lower control limits (>3σ) - outside Zone A.
- **Rule 2** - Two out of three consecutive data points in Zone A (same side) – greater than 2σ, but less than 3σ.
- **Rule 3** - Four out of Five consecutive data points in Zone B (same side) – greater than 1σ, but less than 2σ.
- **Rule 4** - Eight or more consecutive points on either side of the Centerline –a run
- **Rule 5** – Six points in a row, all increasing or decreasing – a trend
- **Rule 6** – 14 points in a row, alternating up and down – systematic variation
- **Rule 7** – 15 points in a row within Zone C (<1σ) – to little variation
- **Rule 8** – 8 points in a row outside of Zone C (>1σ) on either side

Violating any one of these rules can be a strong indication that your process is under the influence of a special cause variation and that further investigation is needed.

PRE-Control Charts

The pre-control chart monitors the center of your process to make sure it's on target. This tool is highly sensitive to the assumption of normality, and only works if your process has good process capability (Cpk).

A pre-control chart takes your specification range and create pre-control boundaries. These PC boundaries are often 25%, 15% or even as small as 7% of your overall specification range.

When a process is started, the product is compared against the PC boundaries, and a decision is made. If the product is out of specification, the process is stopped and adjusted. If the product is near the target, the process continues, if the process is nearly out of specification, the process is adjusted toward the center.

The advantage of this type of tool is that it's easy to implement and can help to keep your process on center. This tool is also a great way to make sure your process is setup correctly before starting.

The downside is that these charts can miss subtle shifts in your process and they don't reflect the natural variation in your process. Additionally, this tool losses value if your process is not already stable and capable.

Short-Run SPC

Short-run SPC can help you monitor and control processes that are infrequent or short in nature. Similar to above, there are short run SPC charts for both attribute and variable data.

One of the most common approaches in **short run SPC** is the **standardized control chart**, where are sub-group averages are normalized to find the z-value associated with each sub-group. Then the Z score is control charted.

$$Z_i = \frac{u_i - \bar{u}}{\sqrt{\bar{u}}}$$