# Proof Essential Practice

| Skill: | Proof by deduction |
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## Questions

*Attempt these questions independently showing full and clear solutions. Check each answer as you go.*

1. Prove that the sum of two consecutive integers is odd.
2. Prove that the sum of two even integers is always even.
3. Prove that the sum of two odd integers is always even.
4. Prove that the sum of the two integers on either side of any other integer is even.
5. Prove that the product of the two even integers is always a multiple of 4.
6. Prove that product of two odd integers is always odd.
7. Prove that the square of an even integer is even.
8. It is given that $n$ is an integer. Prove that $n^2 + 3n + 2$ must be a multiple of 2.
9. Prove that $(5n+3)(n-1)+n(n+2)$ is a multiple of 3 for all integer values of $n$.
10. It is given that $n$ is an integer.
    Prove that $(n-2)^2 + n(8 - n)$ is always a multiple of 4.
11. Prove that $(2n+3)^2 - (2n-3)^2$ is a multiple of 24 for all integer values of $n$.
12. Prove that the product of two consecutive odd numbers is always one less than a multiple of 4.
13. The $n^{th}$ term of the arithmetic sequence $2, 7, 12, 17, \ldots$ is $5n - 3$.
    A new sequence is formed by squaring each term of the arithmetic sequence and then adding 1 to each squared term.
    Prove algebraically that all the terms in the new sequence are multiples of 5.

14. $a, b, c$ and $d$ are consecutive integers.
Prove algebraically that $ab + cd$ is always even.

15. Prove algebraically that the sum of the squares of two consecutive integers is one greater than twice the product of the integers.

16. Two positive integers have a difference of 3.
Prove algebraically that the difference between the squares of the two integers is three times the sum of the integers.

17. Prove that twice the sum of two consecutive positive even numbers is equal to the difference between the squares of the even numbers.

18. Prove that the sum of five consecutive integers must end in a 0 or 5.

19. If $n$ is an integer, show that

$$\frac{n(n-1)}{2} + \frac{n(n+1)}{2}$$

is a square number.

20. In this problem $m, n, q$ and $r$ are integers.
(a) Expand $(x+m)(x+n)$
(b) $x^2 + qx + r \equiv (x+m)(x+n)$
Use your answer to part (a) to write $q$ and $r$ in terms of $m$ and $n$.
(c) It is given that $r$ is odd.
Use your answer to part (b) to prove that $q$ is even.

21. $y = 2n^2 + 5n + 2$
Given that $n$ is a positive integer, prove that $y$ can never have a value that is a prime number.

22. Prove that the product of two consecutive even numbers is divisible by 8.