THE SCIENCE CUBE

CUBE NOTES

Class 11/12 | AP Physics | IIT JEE | NEET

Moment of Inertia & Parallel Axis Theorem

www.thesciencecube.com

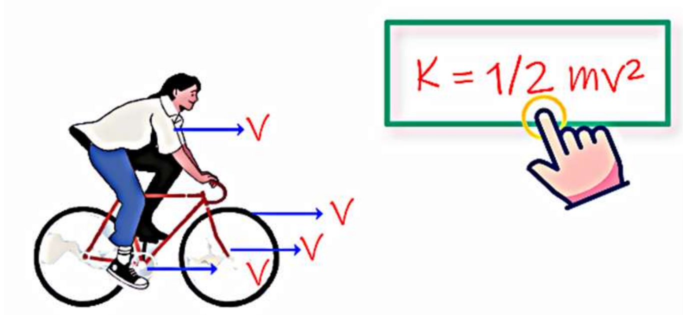
Foundational Idea

- For objects in straight-line motion,

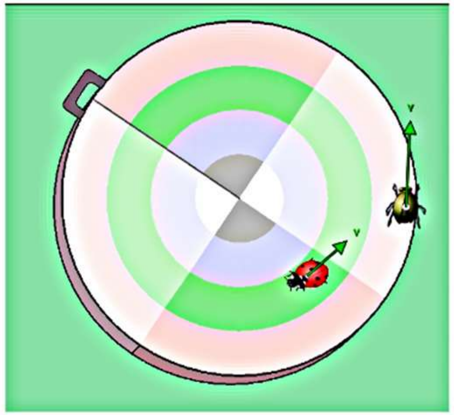
  $KE = 1/2\ mv^2$ (Here v is a constant)

- In rotating bodies, linear velocity of each particle varies with distance from the axis.
  - Need a different approach to calculate kinetic energy in rotating bodies.
- When a rigid body rotates around an axis, each particle moves in a circle with its own radius and linear velocity
  - However, the angular velocity ω is constant for each particle

Linear Motion: v is constant



Rotation: v changes, ω is constant



KE of Rotating Bodies

1. Divide the body into small masses ($m_1, m_2, \ldots, m_n$) each with its velocity ($v_1, v_2, \ldots, v_n$).

2. Find KE of each particle: $KE = 1/2\ m_i v_i^2$ where "i" represents any particle

   i changes from 1 to n, or particle number 1 to the last particle n

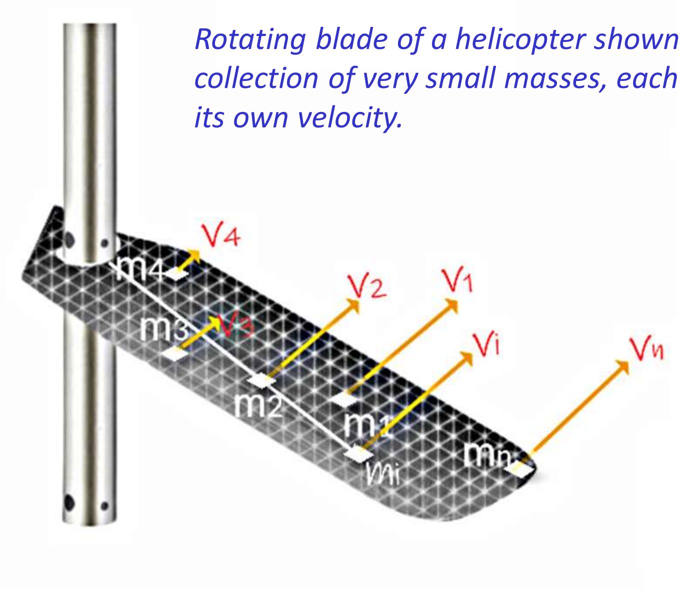
3. Sum the KEs:

   $$\text{Total KE} = \Sigma\ 1/2\ (m_i v_i^2)$$

4. Use $v = \omega r$ to find velocity of each particle

   $$\text{Total KE} = \Sigma\ 1/2\ m_i(\omega r_i)^2.$$

$$KE = 1/2\ (\Sigma\ m_i r_i^2)\ \omega^2$$



Rotating blade of a helicopter shown as a collection of very small masses, each with its own velocity.

Moment of Inertia "I" of a Body

$I = \Sigma m_i r_i^2$ (from previous slide)

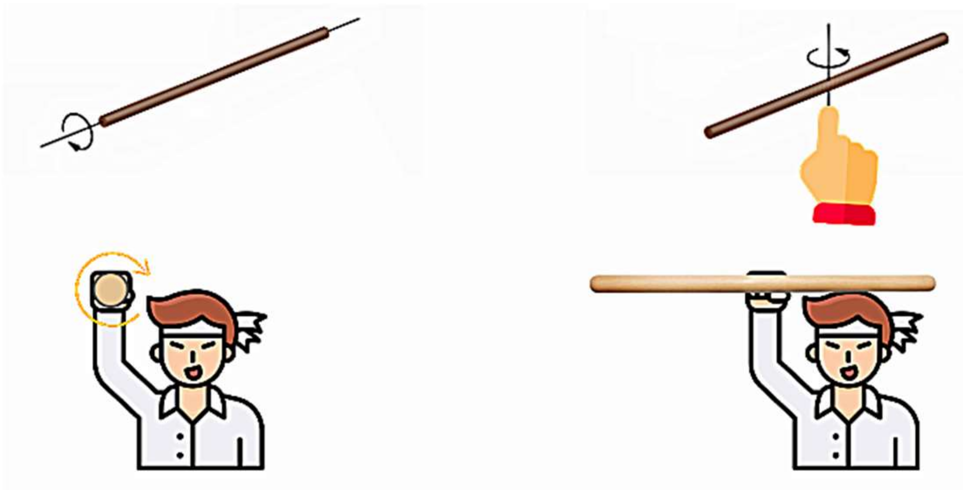
Then we can write KE in terms of I:

$$KE = 1/2\ (I\omega^2)$$

Understanding Rotational Inertia

- Depends on mass distribution relative to the axis.
- More the mass near the axis, more the I value
- Different axes yield different I values

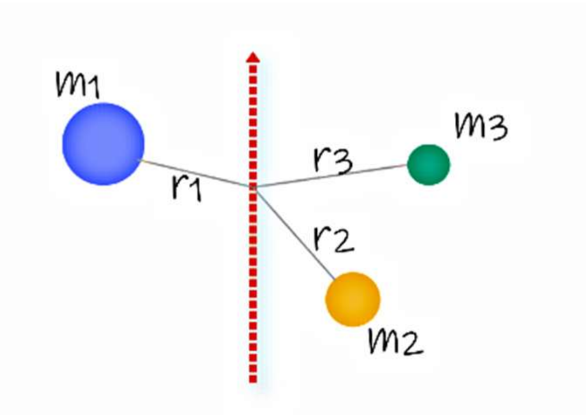
Lower I Value *Higher I Value*



Moment of Inertia

A. For discrete particles: $I = \sum m_i r_i^2$.

Example: This body is a collection of 3 discrete masses m1, m2 and m3



$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2$$

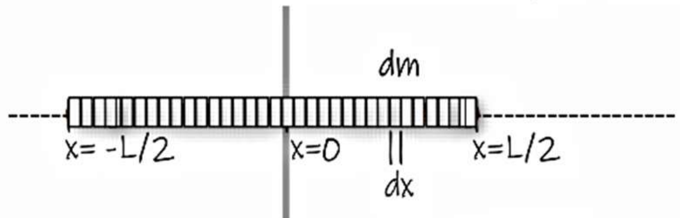
B. For continuous bodies: $I = \int r^2\, dm$.

Example: Using Integrals to find I for a rod rotating about the center

$$I = \int r^2\, dm$$

Mass = M

Mass/ unit length = M/L



$$dm = dx(M/L)$$

$$dI = r^2\, dm$$

$$I = \int x^2 (M/L)\, dx \quad (x = -L/2 \text{ to } x = +L/2)$$

$$I = (M / L) \int x^2\, dx \quad (x = -L/2 \text{ to } x = +L/2)$$

$$I = (1/12)\, ML^2$$

The Parallel Axis Theorem

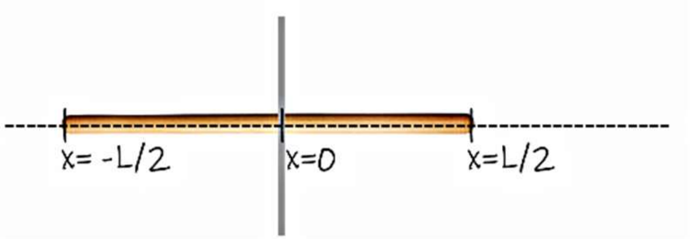
If I_0 is the rotational inertia about the axis through the center of mass, the rotational inertia I about any parallel axis is:

$$I = I_0 + Mh^2$$

h is the perpendicular distance between the axes.

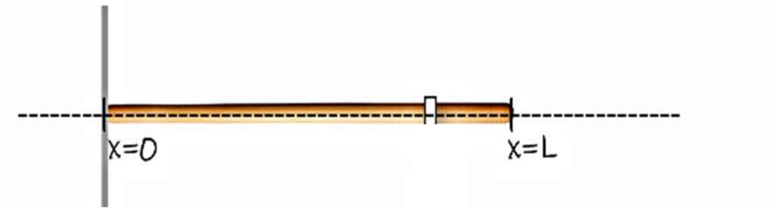
Example: Rod Rotated About 2 Different Axes

Central Axis:



$I_0 = (1/12)\ ML^2$

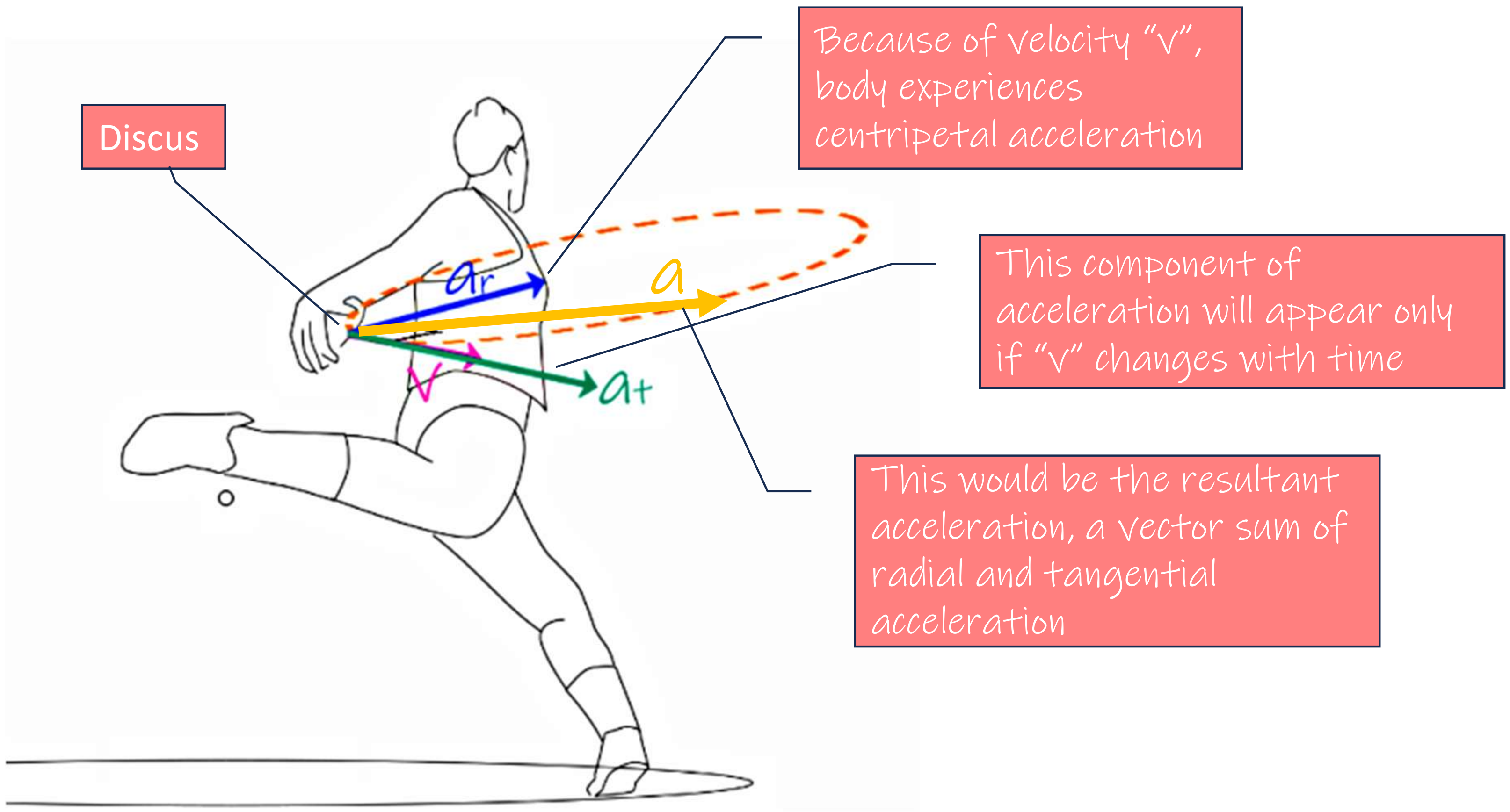
End Axis (Using Parallel Axis Theorem)



$I = (1/12)\ ML^2 + M(L/2)^2$

$I = (1/3)\ ML^2$

Discus Throw: Linear & Angular Acceleration



Equations of Motion in Rotation

SN	Angular Equation	Equivalent Linear Equation
1	$\omega = \omega_0 + \alpha t$	$v = v_0 + at$
2	$\vartheta - \vartheta_0 = \omega_0 t + 1/2\ \alpha t^2$	$x - x_0 = v_0 t + 1/2\ at^2$
3	$\omega^2 = \omega_0^2 + 2\alpha(\vartheta - \vartheta_0)$	$v^2 = v_0^2 + 2a(x - x_0)$
4	$\vartheta - \vartheta_0 = 1/2\ (\omega_0 + \omega)t$	$x - x_0 = 1/2\ (v_0 + v)t$
5	$\vartheta - \vartheta_0 = \omega t - 1/2\ \alpha t^2$	$x - x_0 = vt - 1/2\ at^2$

ω (Angular Velocity):
The rate at which an object rotates or revolves relative to another point, i.e., how fast the angle changes.

ω_0 (Initial Angular Velocity):
The angular velocity at the initial time ($t = 0$).

a (Angular Acceleration):
The rate of change of angular velocity.

ϑ (Angular Displacement):
The angle in radians through which a point or line has been rotated in a specified sense about a specified axis.

ϑ_0 (Initial Angular Position):
The initial angle at the starting time ($t = 0$).

Summary of Rotational Variables

Equation	When to Use	Notes / Caution
$s = \theta * r$	To find arc length (s) given angle (θ) and radius (r)	θ must be in radians.
$v = \omega * r$	To find linear velocity (v) given angular speed (ω) and radius (r)	The v value would be same for all points at Radius (r)
$T = (2\pi * r) / v$	To find the period of revolution (T) given radius (r) and linear velocity (v)	Ensure v is the linear speed.
$T = 2\pi / \omega$	To find the period of revolution (T) given angular speed (ω)	ω must be constant for the body.
$a_t = \alpha * r$	To find tangential acceleration (a_t) given angular acceleration (α) and radius (r)	α must be constant. Becomes zero if v is constant
$a_r = v^2 / r$	To find radial (centripetal) acceleration (a_r) given linear velocity (v) and radius (r)	If v is constant, it becomes uniform circular motion.
$a_r = \omega^2 * r$	To find radial (centripetal) acceleration (a_r) given angular speed (ω) and radius (r)	ω must be constant for the body.

Moment of Inertia of Some Shapes

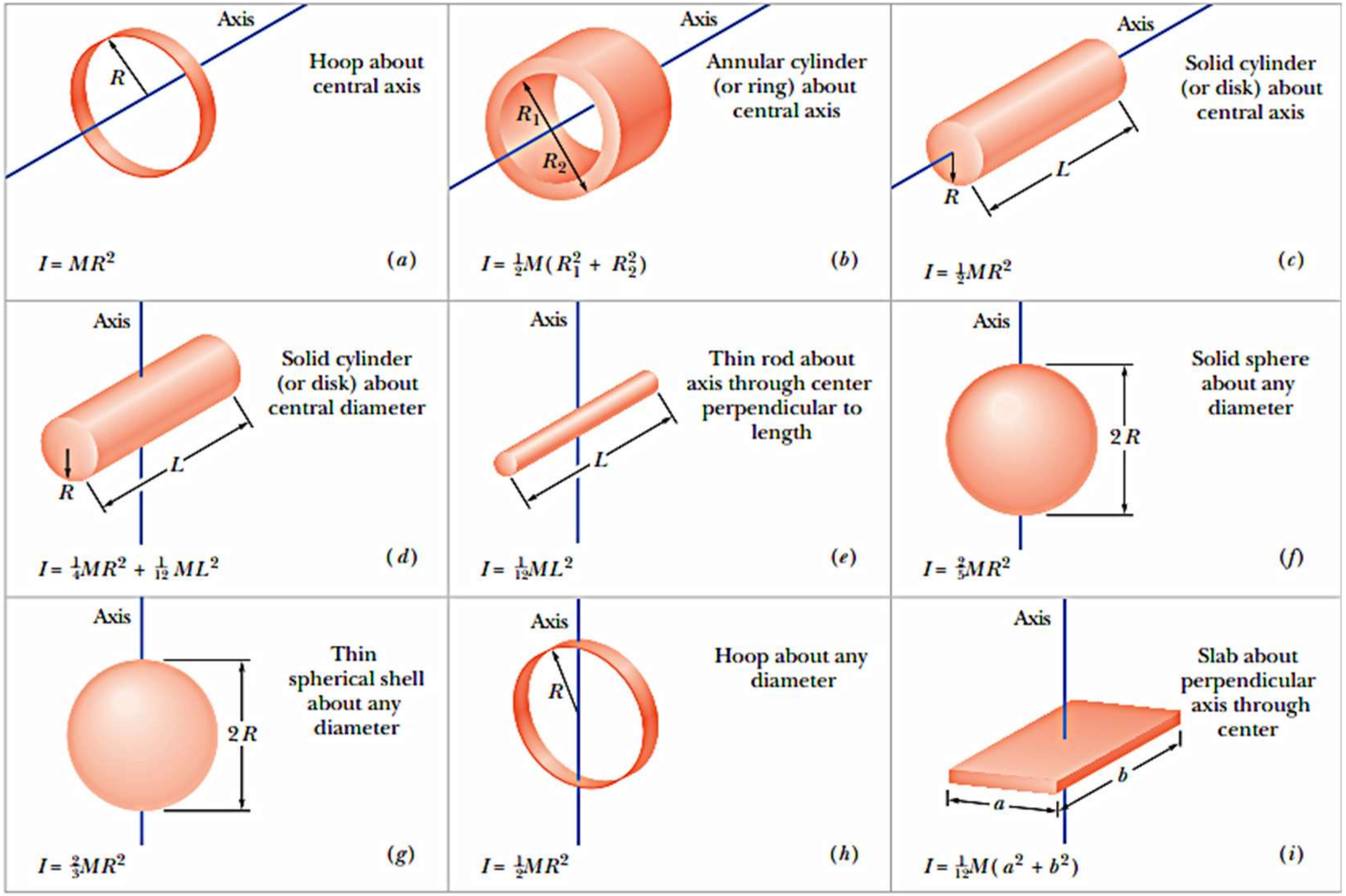


Image Source: Fundamentals of Physics Halliday & Resnick 10th edition

Angles and Radian Equivalents

Angle (Degrees)	Angle (Radians)
0°	0
30°	$\pi/6$
45°	$\pi/4$
60°	$\pi/3$
90°	$\pi/2$
120°	$2\pi/3$
135°	$3\pi/4$
150°	$5\pi/6$
180°	π

Angle (Degrees)	Angle (Radians)
210°	$7\pi/6$
225°	$5\pi/4$
240°	$4\pi/3$
270°	$3\pi/2$
300°	$5\pi/3$
315°	$7\pi/4$
330°	$11\pi/6$
360°	2π