**QUESTIONS:**

1. **A _____ is defined as a statistical process used to make a decision between two mutually exclusive hypothesis.**
   - Hypothesis Test
   - Confidence Interval
   - Designed Experiment
   - Probability Assessment

2. **Hypothesis testing makes use of the _____ to assess the probability of the sample statistic and distinguish between the null and alternative hypothesis.**
   - Sampling Distribution
   - Probability Distribution
   - Confidence Interval
   - Theory of Constraints

3. **Identify all of the statements below regarding hypothesis testing that are True:**
   A. Rejecting the null hypothesis is considered a "strong claim" and means that the sample data was significant enough to reject the starting assumption that the null hypothesis was true.
   B. The null hypothesis, called H0, is always a statement about the value of a sample statistic.
   C. When the null hypothesis is actually true but the hypothesis test rejects it, this is known as a type II error
   D. The significance level (α) of our hypothesis test is also equal to the probability of a type I error
   - A, B
   - B, C
   - C, D
   - A, D

4. **Identify all of the statements below regarding hypothesis testing that are True:**
   A. Using the t-distribution for hypothesis testing of the population mean is required if your sample size is less than 30, or the population standard deviation is unknown.
   B. When we're comparing a sample variance against the population variance we must use the F Distribution.
   C. When testing two population variances against each other we must use the chi-squared distribution.
   D. The significance level (α) is required to determine the rejection criteria for our hypothesis test.
   - A, B
   - B, C
   - C, D
   - A, D

5. **You're performing a hypothesis test for the population mean and your critical z-score is 1.65, and you've got a 2-sided test. If your z-statistic is -1.71, what would your conclusion be?**
   - Accept the null hypothesis and thus reject the alternative hypothesis
   - Fail to reject the null hypothesis
   - Fail to reject the alternative hypothesis
   - Reject the null hypothesis in favor of the alternative hypothesis

6. **You're performing a hypothesis test for the population mean and you know the population standard deviation. You plan to sample 60 units from your population and you'd like to use a 1-sided test at a 1% significance level. What is the rejection criteria for this hypothesis test?**
   - 1.96
   - 2.33
   - 2.39
   - 2.58

7. **You're performing a hypothesis test for the population mean and the population standard deviation is unknown. You sample 20 units from your population and you'd like to use a 1-sided test at a 5% significance level. What is the rejection criteria for this hypothesis test?**
   - 1.725
   - 1.729
   - 2.086
   - 2.093

8. **What is the critical z-value associated with a 2-sided confidence interval that's associated with a 1% alpha risk?**
   - z-score = 1.96
   - z-score = 2.24
   - z-score = 2.58
   - z-score = 3.25

9. **For a sample size of 25, and a 2-sided confidence interval, identify the appropriate lower-tail chi-squared critical values associated with a 5% alpha risk.**
   - lower-tail chi-squared = 9.886
   - lower-tail chi-squared = 10.520
   - lower-tail chi-squared = 12.401
   - lower-tail chi-squared = 13.120

10. **You've taken a random sample of 15 units from the latest production lot, and measured the overall height of the part. You calculate the sample mean to be 20.0 in, and the sample standard deviation to be 2.0 in. Calculate the 95% confidence interval for the population standard deviation.**
    - $1.000 < \sigma < 3.000$
    - $1.268 < \sigma < 2.842$
    - $0.729 < \sigma < 2.645$
    - $1.464 < \sigma < 3.154$

11. **You've sampled 100 units from the last production lot and found that 3 of them are non-conforming. Find the 95% confidence interval for the true population proportion of defective products.**
    - $0.03 \pm 0.033$
    - $0.03 \pm 0.044$
    - $0.03 \pm 0.055$
    - $0.03 \pm 0.066$

12. **You're performing a hypothesis test for the population mean and you know the population standard deviation. You plan to sample 45 units from your population and you'd like to use a 2-sided test at a 5% significance level. What is the rejection criteria for this hypothesis test?**
    - 1.65
    - 1.96
    - 2.24
    - 2.58

13. **You're performing a hypothesis test for the population mean, which you believe to be 12.45". You sample 36 parts and find the sample mean to be 12.25". The population standard deviation is known to be 1.0". What is the test statistic for this hypothesis test?**
    - -0.80
    - -1.20
    - -2.35
    - -3.60

14. **You're performing a hypothesis test for the population mean and the population standard deviation is unknown. You sample 16 units from your population and you'd like to use a 2-sided test at a 20% significance level. What is the rejection criteria for this hypothesis test?**
    - 1.337
    - 2.583
    - 1.341
    - 2.602

15. **You're performing a hypothesis test for the population mean and the population standard deviation is known. You sample 100 units from your population and you'd like to use a 1-sided test at a 10% significance level. What is the rejection criteria for this hypothesis test?**
    - 1.28
    - 1.65
    - 1.96
    - 2.24

**SOLUTIONS:**

1. **A _____ is defined as a statistical process used to make a decision between two mutually exclusive hypothesis.**
   - **Hypothesis Test**
   - Confidence Interval
   - Designed Experiment
   - Probability Assessment

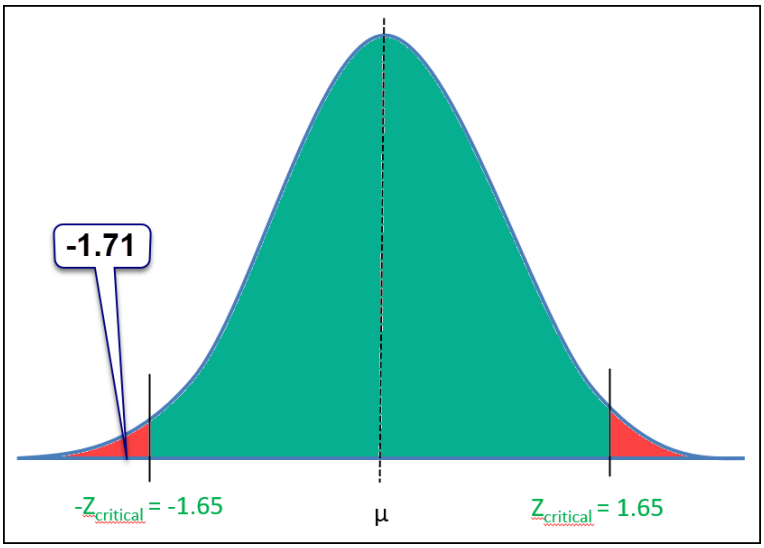
2. **Hypothesis testing makes use of the _____ to assess the probability of the sample statistic and distinguish between the null and alternative hypothesis.**
   - **Sampling Distribution**
   - Probability Distribution
   - Confidence Interval
   - Theory of Constraints

3. **Identify all of the statements below regarding hypothesis testing that are True:**
   A. Rejecting the null hypothesis is considered a "strong claim" and means that the sample data was significant enough to reject the starting assumption that the null hypothesis was true. **(true)**
   B. The null hypothesis, called $H_0$, is always a statement about the value of a ~~**sample statistic (**~~**population parameter)**. (False)
   C. When the null hypothesis is actually true but the hypothesis test rejects it, this is known as a ~~**type II error**~~ **(type 1 error)**. (False)
   D. The significance level (α) of our hypothesis test is also equal to the probability of a type I error. **(true)**
   - A, B
   - B, C
   - C, D
   - **A, D**

4. **Identify all of the statements below regarding hypothesis testing that are True:**
   A. Using the t-distribution for hypothesis testing of the population mean is required if your sample size is less than 30, or the population standard deviation is unknown. **(true)**
   B. When we're comparing a sample variance against the population variance, we must use the ~~**F Distribution**~~ **(Chi-Squared Distribution)**. (False)
   C. When testing two population variances against each other we must use the ~~**chi-squared distribution**~~ **(F Distribution)**. (False)
   D. The significance level (α) is required to determine the rejection criteria for our hypothesis test. **(true)**
   - A, B
   - B, C
   - C, D
   - **A, D**

5. **You're performing a hypothesis test for the population mean and your critical z-score is 1.65, and you've got a 2-sided test. If your z-statistic is -1.71, what would your conclusion be?**
   - Accept the null hypothesis and thus reject the alternative hypothesis
   - Fail to reject the null hypothesis
   - Fail to reject the alternative hypothesis
   - **Reject the null hypothesis in favor of the alternative hypothesis**

   Because it is a 2-sided hypothesis test, you'll be looking for a value greater than 1.65, or less than -1.65. Because our value is less than -1.65, we can **reject the null hypothesis in favor of the alternative hypothesis.**




6. **You're performing a hypothesis test for the population mean and you know the population standard deviation. You plan to sample 60 units from your population and you'd like to use a 1-sided test at a 1% significance level. What is the rejection criteria for this hypothesis test?**
   - 1.96
   - **2.33**
   - 2.39
   - 2.58

   Because we know the population standard deviation and we're sampling more than 30 units we can use the normal distribution for your hypothesis test. Based on the 1-sided test, and 1% significance level, we can look up the Z-value associated with 49.0% of the population, which is $Z_{crit}$ = ~2.33

| | Area under the Normal Curve from 0 to X | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| X | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| 1.7 | 0.45543 | 0.45637 | 0.45728 | 0.45818 | 0.45907 | 0.45994 | 0.46080 | 0.46164 | 0.46246 | 0.46327 |
| 1.8 | 0.46407 | 0.46485 | 0.46562 | 0.46638 | 0.46712 | 0.46784 | 0.46856 | 0.46926 | 0.46995 | 0.47062 |
| 1.9 | 0.47128 | 0.47193 | 0.47257 | 0.47320 | 0.47381 | 0.47441 | 0.47500 | 0.47558 | 0.47615 | 0.47670 |
| 2.0 | 0.47725 | 0.47778 | 0.47831 | 0.47882 | 0.47932 | 0.47982 | 0.48030 | 0.48077 | 0.48124 | 0.48169 |
| 2.1 | 0.48214 | 0.48257 | 0.48300 | 0.48341 | 0.48382 | 0.48422 | 0.48461 | 0.48500 | 0.48537 | 0.48574 |
| 2.2 | 0.48610 | 0.48645 | 0.48679 | 0.48713 | 0.48745 | 0.48778 | 0.48809 | 0.48840 | 0.48870 | 0.48899 |
| 2.3 | 0.48928 | 0.48956 | 0.48983 | 0.49010 | 0.49036 | 0.49061 | 0.49086 | 0.49111 | 0.49134 | 0.49158 |
| 2.4 | 0.49180 | 0.49202 | 0.49224 | 0.49245 | 0.49266 | 0.49286 | 0.49305 | 0.49324 | 0.49343 | 0.49361 |

7. **You're performing a hypothesis test for the population mean and the population standard deviation is unknown. You sample 20 units from your population and you'd like to use a 1-sided test at a 5% significance level. What is the rejection criteria for this hypothesis test?**
   - 1.725
   - **1.729**
   - 2.086
   - 2.093

   Because we do not know the population standard deviation and we're sampling less than 30 units we must use the t distribution for our hypothesis test. Based on the 1-sided test, and 5% significance level, we can look up the critical t-value associated with 5% significance and 19 degrees of freedom to be $t_{crit}$ = ~1.729

| Critical Values of the Student T's Distribution at Various Degrees of Freedom and Alpha Levels (α) | | | | | | |
|---|---|---|---|---|---|---|
| df (*v*) | α = 0.1 | α = 0.05 | α = 0.025 | α = 0.01 | α = 0.005 | α = 0.001 |
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 318.309 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 |

8. **What is the critical z-value associated with a 2-sided confidence interval that's associated with a 1% alpha risk?**
   - z-score = 1.96
   - z-score = 2.24
   - **z-score = 2.58**
   - z-score = 3.25

   Because it's a 2-sided distribution with at the 1% significance level, we're looking for the z-score that's associated with the area under the curve of 0.495. This would capture 49.5% on the left half & right half of the distribution, leaving the remaining 1% of the alpha risk in the rejection area of the tails of the distribution, which is a **Z-score = 2.58**.

| | Area under the Normal Curve from 0 to X | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| X | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| 2.2 | 0.48610 | 0.48645 | 0.48679 | 0.48713 | 0.48745 | 0.48778 | 0.48809 | 0.48840 | 0.48870 | 0.48899 |
| 2.3 | 0.48928 | 0.48956 | 0.48983 | 0.49010 | 0.49036 | 0.49061 | 0.49086 | 0.49111 | 0.49134 | 0.49158 |
| 2.4 | 0.49180 | 0.49202 | 0.49224 | 0.49245 | 0.49266 | 0.49286 | 0.49305 | 0.49324 | 0.49343 | 0.49361 |
| 2.5 | 0.49379 | 0.49396 | 0.49413 | 0.49430 | 0.49446 | 0.49461 | 0.49477 | 0.49492 | 0.49506 | 0.49520 |
| 2.6 | 0.49534 | 0.49547 | 0.49560 | 0.49573 | 0.49585 | 0.49598 | 0.49609 | 0.49621 | 0.49632 | 0.49643 |
| 2.7 | 0.49653 | 0.49664 | 0.49674 | 0.49683 | 0.49693 | 0.49702 | 0.49711 | 0.49720 | 0.49728 | 0.49736 |

**9. For a sample size of 25, and a 2-sided confidence interval, identify the appropriate lower-tail chi-squared critical values associated with a 5% alpha risk.**

- lower-tail chi-squared = 9.886
- lower-tail chi-squared = 10.520
- **lower-tail chi-squared = 12.401**
- lower-tail chi-squared = 13.120

The degrees of freedom in this sample is 24 (25 – 1).

The 2-sided confidence interval and 5% alpha risk is split in half between the upper and lower tail.

So we're looking in the 0.025 column and 24 degrees of freedom for our critical chi-squared value, which is **12.401**.

**Left-Tail Critical Value of the Chi-Squared ($X^2$) Distribution**

| df ($v$) | 0.001 | 0.005 | 0.010 | 0.025 | 0.050 | 0.100 |
|---|---|---|---|---|---|---|
| 12 | 2.214 | 3.074 | 3.571 | 4.404 | 5.226 | 6.304 |
| 13 | 2.617 | 3.565 | 4.107 | 5.009 | 5.892 | 7.042 |
| 14 | 3.041 | 4.075 | 4.660 | 5.629 | 6.571 | 7.790 |
| 15 | 3.483 | 4.601 | 5.229 | 6.262 | 7.261 | 8.547 |
| 16 | 3.942 | 5.142 | 5.812 | 6.908 | 7.962 | 9.312 |
| 17 | 4.416 | 5.697 | 6.408 | 7.564 | 8.672 | 10.085 |
| 18 | 4.905 | 6.265 | 7.015 | 8.231 | 9.390 | 10.865 |
| 19 | 5.407 | 6.844 | 7.633 | 8.907 | 10.117 | 11.651 |
| 20 | 5.921 | 7.434 | 8.260 | 9.591 | 10.851 | 12.443 |
| 21 | 6.447 | 8.034 | 8.897 | 10.283 | 11.591 | 13.240 |
| 22 | 6.983 | 8.643 | 9.542 | 10.982 | 12.338 | 14.041 |
| 23 | 7.529 | 9.260 | 10.196 | 11.689 | 13.091 | 14.848 |
| 24 | 8.085 | 9.886 | 10.856 | 12.401 | 13.848 | 15.659 |
| 25 | 8.649 | 10.520 | 11.524 | 13.120 | 14.611 | 16.473 |

**10. You've taken a random sample of 15 units from the latest production lot, and measured the overall height of the part. You calculate the sample mean to be 20.0 in, and the sample standard deviation to be 2.0 in. Calculate the 95% confidence interval for the population standard deviation.**

- 1.000 < σ < 3.000
- 1.268 < σ < 2.842
- 0.729 < σ < 2.645
- **1.464 < σ < 3.154**

Ok, let's see what we know after reading the problem statement: **n = 15, s = 2.0 in, α = 0.05 x-bar = 20.0in**

First, we must find our critical chi-squared values from the Chi-Squared Table associated with our alpha risk (5%), sample size (15), and degrees of freedom (14):

$$\textbf{\textit{Confidence Interval for Standard Deviation}}: \quad \sqrt{\frac{(n-1)s^2}{X^2_{1-\alpha/2}}} < \sigma < \sqrt{\frac{(n-1)s^2}{X^2_{\alpha/2}}}$$

$$X^2_{\alpha/2,df} = X^2_{.05/2,14} = X^2_{.025,14} = 5.629$$

$$X^2_{1-\alpha/2,df} = X^2_{1-.05/2,14} = X^2_{.975,14} = 26.119$$

$$\sqrt{\frac{(15-1)2^2}{26.119}} < \sigma < \sqrt{\frac{(15-1)2^2}{5.629}}$$

$$1.464 < \sigma < 3.154$$

11. **You've sampled 100 units from the last production lot and found that 3 of them are non-conforming. Find the 95% confidence interval for the true population proportion of defective products.**
    - **0.03 ± 0.033**
    - 0.03 ± 0.044
    - 0.03 ± 0.055
    - 0.03 ± 0.066

    First, we can calculate the **sample proportion, p** using **n = 100**, and the **number of non-conformances (3)**: $\boldsymbol{Sample\ Proportion: p = \frac{3}{100} = 0.03}$

    Then we can look up our Z-score at the 5% alpha risk: $\boldsymbol{Z_{\frac{\alpha}{2}} = Z_{\frac{0.05}{2}} = Z_{.025} = 1.96}$

    $$\boldsymbol{Confidence\ Interval\ (Proportion): p \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{p*(1-p)}{n}}}$$

    $$\boldsymbol{Confidence\ Interval: 0.03 \pm 1.96\sqrt{\frac{0.03*(1-0.03)}{100}}}$$

    $$\boldsymbol{Confidence\ Interval: 0.03 \pm 1.96\sqrt{.000291}}$$

    $$\boldsymbol{Confidence\ Interval: 0.03 \pm 0.033}$$

12. **You're performing a hypothesis test for the population mean and you know the population standard deviation. You plan to sample 45 units from your population and you'd like to use a 2-sided test at a 5% significance level. What is the rejection criteria for this hypothesis test?**
    - 1.65
    - **1.96**
    - 2.24
    - 2.58

    Because it's a 2-sided distribution with at the 5% significance level, we're looking for the z-score that's associated with the area under the curve of 0.475. This would capture 47.5% on the left half & right half of the distribution, leaving the remaining 5% of the alpha risk in the rejection area of the tails of the distribution, which is a **Z-score = 1.96**.

| | Area under the Normal Curve from 0 to X | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| X | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| 1.1 | 0.36433 | 0.36650 | 0.36864 | 0.37076 | 0.37286 | 0.37493 | 0.37698 | 0.37900 | 0.38100 | 0.38298 |
| 1.2 | 0.38493 | 0.38686 | 0.38877 | 0.39065 | 0.39251 | 0.39435 | 0.39617 | 0.39796 | 0.39973 | 0.40147 |
| 1.3 | 0.40320 | 0.40490 | 0.40658 | 0.40824 | 0.40988 | 0.41149 | 0.41309 | 0.41466 | 0.41621 | 0.41774 |
| 1.4 | 0.41924 | 0.42073 | 0.42220 | 0.42364 | 0.42507 | 0.42647 | 0.42785 | 0.42922 | 0.43056 | 0.43189 |
| 1.5 | 0.43319 | 0.43448 | 0.43574 | 0.43699 | 0.43822 | 0.43943 | 0.44062 | 0.44179 | 0.44295 | 0.44408 |
| 1.6 | 0.44520 | 0.44630 | 0.44738 | 0.44845 | 0.44950 | 0.45053 | 0.45154 | 0.45254 | 0.45352 | 0.45449 |
| 1.7 | 0.45543 | 0.45637 | 0.45728 | 0.45818 | 0.45907 | 0.45994 | 0.46080 | 0.46164 | 0.46246 | 0.46327 |
| 1.8 | 0.46407 | 0.46485 | 0.46562 | 0.46638 | 0.46712 | 0.46784 | 0.46856 | 0.46926 | 0.46995 | 0.47062 |
| 1.9 | 0.47128 | 0.47193 | 0.47257 | 0.47320 | 0.47381 | 0.47441 | 0.47500 | 0.47558 | 0.47615 | 0.47670 |

13. **You're performing a hypothesis test for the population mean, which you believe to be 12.45". You sample 36 parts and find the sample mean to be 12.25". The population standard deviation is known to be 1.0". What is the test statistic for this hypothesis test?**
    - -0.80
    - **-1.20**
    - -2.35
    - -3.60

    Because we've taken more than 30 samples, and we know the population standard deviation, we can use the Z-score to calculate our test statistic.

$$Z - Score = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{12.25 - 12.45}{\frac{1.0}{\sqrt{36}}} = -1.2$$

14. **You're performing a hypothesis test for the population mean and the population standard deviation is unknown. You sample 16 units from your population and you'd like to use a 2-sided test at a 20% significance level. What is the rejection criteria for this hypothesis test?**
    - 1.337
    - 2.583
    - **1.341**
    - 2.602

    Because we do not know the population standard deviation and we're sampling less than 30 units we must use the t distribution for our hypothesis test. Based on the 2-sided test, and 20% significance level, we can look up the critical t-value associated with 10% significance and 15 degrees of freedom to be $t_{crit}$ = ~1.341

| Critical Values of the Student T's Distribution at Various Degrees of Freedom and Alpha Levels (α) | | | | | | |
|---|---|---|---|---|---|---|
| df (*v*) | α = 0.1 | α = 0.05 | α = 0.025 | α = 0.01 | α = 0.005 | α = 0.001 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 |

15. **You're performing a hypothesis test for the population mean and the population standard deviation is known. You sample 100 units from your population and you'd like to use a 1-sided test at a 10% significance level. What is the rejection criteria for this hypothesis test?**
    - **1.28**
    - 1.65
    - 1.96
    - 2.24

    Because we know the population standard deviation and we're sampling more than 30 units we can use the z-scores and normal distribution for our hypothesis test. Based on the 1-sided test, and 10% significance level, we must look up the z-score associated with 0.40, which is a **Z-score = 1.28**.

| | Area under the Normal Curve from 0 to X | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| X | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| 0.7 | 0.25804 | 0.26115 | 0.26424 | 0.26730 | 0.27035 | 0.27337 | 0.27637 | 0.27935 | 0.28230 | 0.28524 |
| 0.8 | 0.28814 | 0.29103 | 0.29389 | 0.29673 | 0.29955 | 0.30234 | 0.30511 | 0.30785 | 0.31057 | 0.31327 |
| 0.9 | 0.31594 | 0.31859 | 0.32121 | 0.32381 | 0.32639 | 0.32894 | 0.33147 | 0.33398 | 0.33646 | 0.33891 |
| 1.0 | 0.34134 | 0.34375 | 0.34614 | 0.34849 | 0.35083 | 0.35314 | 0.35543 | 0.35769 | 0.35993 | 0.36214 |
| 1.1 | 0.36433 | 0.36650 | 0.36864 | 0.37076 | 0.37286 | 0.37493 | 0.37698 | 0.37900 | 0.38100 | 0.38298 |
| 1.2 | 0.38493 | 0.38686 | 0.38877 | 0.39065 | 0.39251 | 0.39435 | 0.39617 | 0.39796 | 0.39973 | 0.40147 |