

Probability

General concepts of probability :

1- Probability that event A happens = $P(A) = \frac{\text{Number of outcomes satisfying A}}{\text{Total number of possibilities}}$

2- Probability must be between 0 and 1.

3- Probability that event A doesn't happen = $P(A') = 1 - P(A)$

4- Relative frequency of A = $\frac{\text{Frequency of A}}{\text{Total frequency}}$

5- Single event : The experiment is done in one phase.

Double event : The experiment is done in two phases.

6- Conditional probability : The probability that an event happens given that another event has already happened.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

↳ given that we are selecting from B, what's the probability that A happens.

Ex: $P(\text{female} | \text{Math}) = 0.2$ means that, if we select a student from the math class, there's a probability of 0.2 of selecting a female.

7- Expected number of A = probability of A $\times$ total number.

Ex: expected number of girls = prob. of selecting a girl $\times$ total.

8- Probability of at least 1 $\otimes$ = $1 - \text{all } \otimes'$.

2 Types of probability diagrams:

I. Venn diagram:

Worked example:

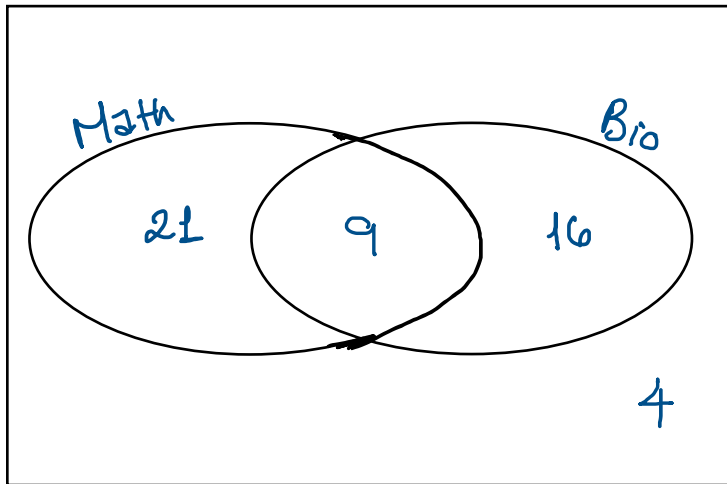
A school has 30 students studying Maths and 25 students studying Biology. 4 students study neither Maths nor Biology.

The total number of students in the school is 50.

- (i) Draw a Venn diagram to represent the above.
- (ii) One student is selected at random. Find the probability that the student studies only one subject.
- (iii) Two students are selected at random. Find the probability that they both study Maths.
- (iv) Two students are selected at random. Find the probability that they both do not study Biology.
- (v) Two students are selected at random. Find the probability that one of them studies Maths only & the other studies Biology only.
- (vi) A student from those who study Biology is selected, find the probability that he studies Maths.

Solution:

(i)



Total students = 50

Study neither = 4

Total of Maths and Bio
= $50 - 4 = 46$

$30 + 25 = 55$

$55 - 46 = 9$



This means that

9 students were
counted twice.

(ii) $\frac{21 + 16}{50} = \frac{37}{50}$

(iii) Possible Scenarios:

Math & Math
 $\frac{30}{50} \times \frac{29}{49} = \frac{87}{245}$

(iv) Possible Scenarios:

Bio' & Bio'
 $\frac{25}{50} \times \frac{24}{49} = \frac{12}{49}$

(v) Possible Scenarios:

(Math only & Bio only) or (Bio only & Math only)

$$\left(\frac{21}{50} \times \frac{16}{49} \right) + \left(\frac{16}{50} \times \frac{21}{49} \right) = \frac{48}{175}$$

(vi) $P(\text{Math} | \text{Bio}) = \frac{\text{Math and Bio}}{\text{Bio}} = \frac{9}{25}$