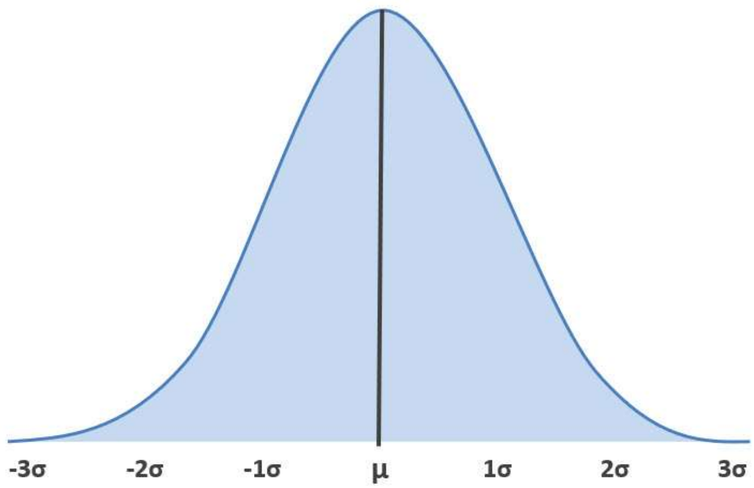
The Normal Distribution

One of the most common distributions across all of statistics, and industry, is the normal distribution, and we describe the normal distribution using 2 parameters, the **Mean** (μ) & **Standard Deviation** (σ).



The **Mean** value is a measure of the central tendency of the distribution & often exists at the peak & centerline of the distribution.

The **standard deviation** is a measure of the variation or spread associated with the distribution. The shape of the curve is governed mostly by the standard deviation.

The smaller the standard deviation the more data is centered around the mean. When the standard deviation gets bigger, the tails get longer and the data is more dispersed.

This normal distribution is going to be used throughout the CQT body of knowledge, and throughout statistics. This includes probability, control charts, six sigma and confidence intervals to name a few

The Z-Transformation of the Normal Distribution

Similar to other probability distributions, the area under the normal curve represents the probability of occurrence of X.

To more quickly calculate the area under the normal distribution curve statisticians have given us the Z-transformation, along with the Z-tables.

To perform the Z-transformation, you can use the following equation. This will transform your random variable X, into a Z-value based on the distributions mean & standard deviation.

$$Z = \frac{X - \mu}{\sigma}$$

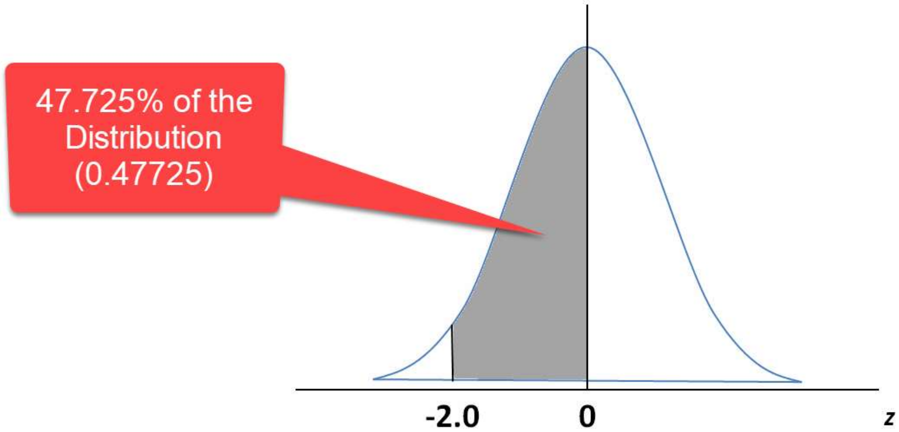
Once we have the Z-score for any random variable (X). we can translate that into a probability of occurrence, using the table below.

For example, what's shown in the image below is Z = -2.0. Now what is the probability of a Z-score of 2.0?

We can use the table for that. You can see that for a Z-score of 2.00, the area under the normal distribution curve is equal to 47.725%, and you can see that graphically in the image.

If you happen to have a Z-score of -1.0, you would capture 34.134% of the distribution.

The Z-Transformation of the Normal Distribution



Area under the Normal Curve from 0 to X

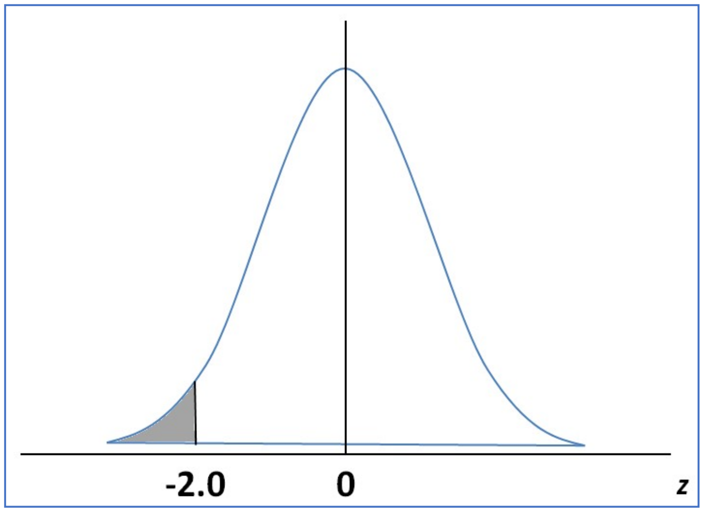
X	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00000	0.00399	0.00798	0.01197	0.01595	0.01994	0.02392	0.02790	0.03188	0.03586
0.1	0.03983	0.04380	0.04776	0.05172	0.05567	0.05962	0.06356	0.06749	0.07142	0.07535
0.2	0.07926	0.08317	0.08706	0.09095	0.09483	0.09871	0.10257	0.10642	0.11026	0.11409
0.3	0.11791	0.12172	0.12552	0.12930	0.13307	0.13683	0.14058	0.14431	0.14803	0.15173
0.4	0.15542	0.15910	0.16276	0.16640	0.17003	0.17364	0.17724	0.18082	0.18439	0.18793
0.5	0.19146	0.19497	0.19847	0.20194	0.20540	0.20884	0.21226	0.21566	0.21904	0.22240
0.6	0.22575	0.22907	0.23237	0.23565	0.23891	0.24215	0.24537	0.24857	0.25175	0.25490
0.7	0.25804	0.26115	0.26424	0.26730	0.27035	0.27337	0.27637	0.27935	0.28230	0.28524
0.8	0.28814	0.29103	0.29389	0.29673	0.29955	0.30234	0.30511	0.30785	0.31057	0.31327
0.9	0.31594	0.31859	0.32121	0.32381	0.32639	0.32894	0.33147	0.33398	0.33646	0.33891
1.0	0.34134	0.34375	0.34614	0.34849	0.35083	0.35314	0.35543	0.35769	0.35993	0.36214
1.1	0.36433	0.36650	0.36864	0.37076	0.37286	0.37493	0.37698	0.37900	0.38100	0.38298
1.2	0.38493	0.38686	0.38877	0.39065	0.39251	0.39435	0.39617	0.39796	0.39973	0.40147
1.3	0.40320	0.40490	0.40658	0.40824	0.40988	0.41149	0.41309	0.41466	0.41621	0.41774
1.4	0.41924	0.42073	0.42220	0.42364	0.42507	0.42647	0.42785	0.42922	0.43056	0.43189
1.5	0.43319	0.43448	0.43574	0.43699	0.43822	0.43943	0.44062	0.44179	0.44295	0.44408
1.6	0.44520	0.44630	0.44738	0.44845	0.44950	0.45053	0.45154	0.45254	0.45352	0.45449
1.7	0.45543	0.45637	0.45728	0.45818	0.45907	0.45994	0.46080	0.46164	0.46246	0.46327
1.8	0.46407	0.46485	0.46562	0.46638	0.46712	0.46784	0.46856	0.46926	0.46995	0.47062
1.9	0.47128	0.47193	0.47257	0.47320	0.47381	0.47441	0.47500	0.47558	0.47615	0.47670
2.0	0.47725	0.47778	0.47831	0.47882	0.47932	0.47982	0.48030	0.48077	0.48124	0.48169
2.1	0.48214	0.48257	0.48300	0.48341	0.48382	0.48422	0.48461	0.48500	0.48537	0.48574
2.2	0.48610	0.48645	0.48679	0.48713	0.48745	0.48778	0.48809	0.48840	0.48870	0.48899
2.3	0.48928	0.48956	0.48983	0.49010	0.49036	0.49061	0.49086	0.49111	0.49134	0.49158
2.4	0.49180	0.49202	0.49224	0.49245	0.49266	0.49286	0.49305	0.49324	0.49343	0.49361
2.5	0.49379	0.49396	0.49413	0.49430	0.49446	0.49461	0.49477	0.49492	0.49506	0.49520
2.6	0.49534	0.49547	0.49560	0.49573	0.49585	0.49598	0.49609	0.49621	0.49632	0.49643
2.7	0.49653	0.49664	0.49674	0.49683	0.49693	0.49702	0.49711	0.49720	0.49728	0.49736
2.8	0.49744	0.49752	0.49760	0.49767	0.49774	0.49781	0.49788	0.49795	0.49801	0.49807
2.9	0.49813	0.49819	0.49825	0.49831	0.49836	0.49841	0.49846	0.49851	0.49856	0.49861
3.0	0.49865	0.49869	0.49874	0.49878	0.49882	0.49886	0.49889	0.49893	0.49896	0.49900

The Z-Transformation of the Normal Distribution - Example

For example, let's say you've got a variable X (Grades on the CQE Exam) that follows the normal distribution with a mean value $\mu = 82$ and a standard deviation $\sigma = 6$. The Z-score for an exam grade of 70 can be calculated as:

$$Z = \frac{70 - 82}{6} = -2.0$$

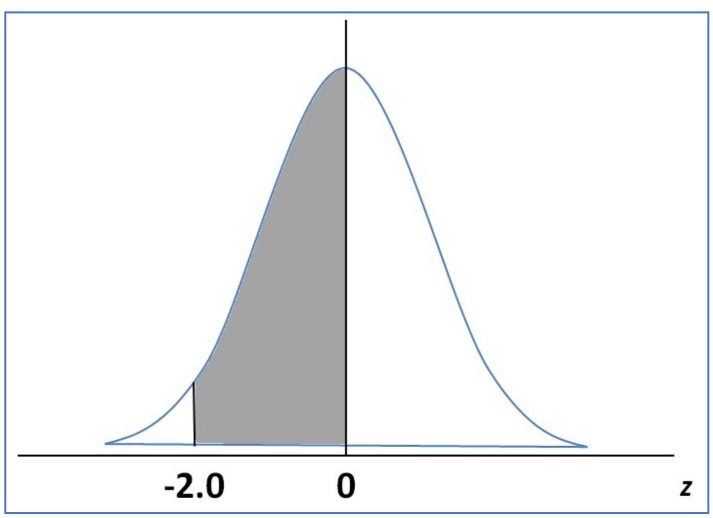
We can interpret this result by saying that the exam score of 70 is 2.0 standard deviations below the mean. If you wanted to calculate the proportion of the population which scored less than 70% on the exam, it would look like the gray shaded area below on the distribution:



Notice this distribution is not a reflection of the exam score (centered at z=0), but it's a reflection of the transformed z-score associated with the exam. We can then use the Z-score tables to answer any probability question associated with this value without having to use a calculator.

Z-Transformation Example

For example, the graph above shows all exam scores less than z = -2.0; however, you could also use the z-table to find the probability of a Z-score between -2 and 0, which graphically looks like this:



Once you've performed the Z-transformation, you can now calculate the probability associated with your Z-value using the table below. This Probability Table can be used to take your Z-value and convert it into the probability. This table is potentially different from other Z-Probability tables in that it only provides the probability of positive Z values.

Recall though that the Z-value is symmetric around the mean value, so if you were looking for the probability from -2.00 to 0, it would be the same probability as that from 0 to +2.00.