Impulse and Momentum

Key Idea

1. Impulse ($\overline{J}$): The product of the average force (F_{av}) and the time interval (Δt) during which the force acts.

$$\overline{J} = F_{av}\,\Delta t$$

2. If the exact relation between force and time is know, impulse can be expressed as

$$\overline{J} = \int \overline{F}(t)\,dt \qquad \text{Limits - } t_{initial} \text{ to } t_{final}$$

3. Impulse is also defined as the change in momentum or

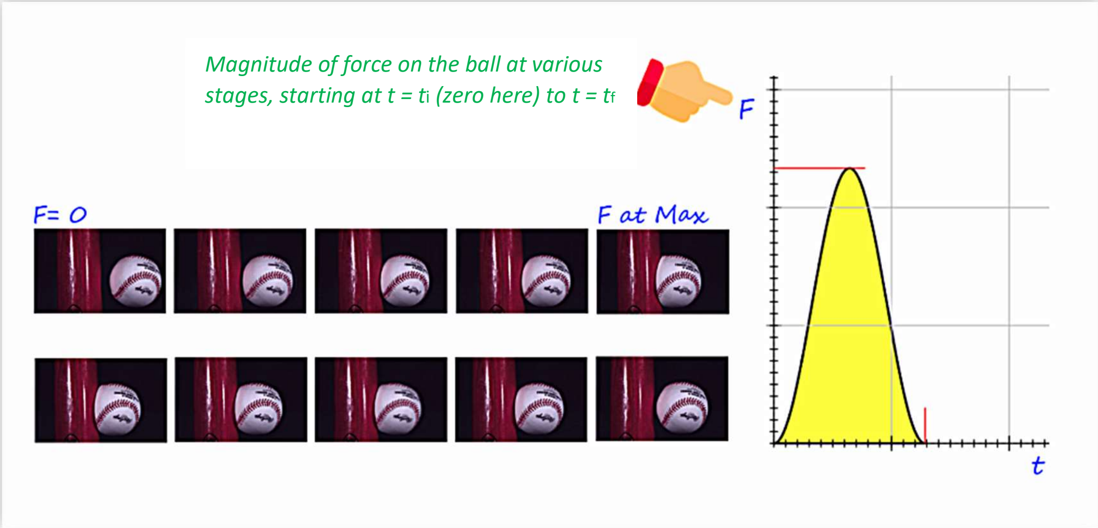
$$\Delta\overline{p} = \overline{J}$$

Understanding Impulse

When you hit a ball with a bat, a force is exerted on the ball. During the collision:

- The force starts at zero, increases to a maximum when the ball and bat are in full contact, and then decreases as they separate.
- This force varies, altering the ball's linear momentum $\overline{p}$.
- This change of momentum is termed as the impulse on the ball

Newton's Second Law in Momentum Form & Derivation of Impulse



$\bar{F} = d\bar{p}/dt$ or

$d\bar{p} = \bar{F}(t)\, dt$

integrating both sides

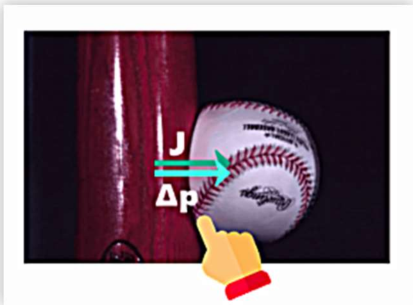
$\int d\bar{p} = \int \bar{F}(t)\, dt$ $t_{initial}$ to t_{final}

$\Delta\bar{p} = \int \bar{F}(t)\, dt = \bar{J}$ Where $\bar{J}$ is impulse

Or

$\Delta\bar{p} = \bar{J}$ *(Impulse – Momentum Theorem)*

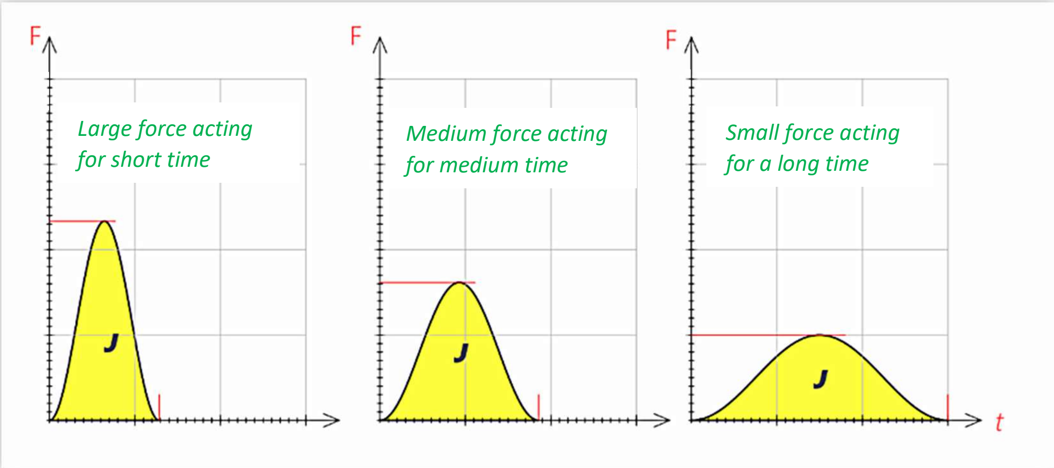
The direction of impulse is always in the direction of change of momentum



Impulse-Momentum Theorem: The impulse experienced by an object is equal to the change in its momentum

Force vs. Time Graphs

- The area under a force-time (F-t) graph represents the impulse.
- If the force varies, the area under the curve is the integral of F over time.



Area under each of the 3 graphs is equal and therefore the impulse is also equal

Average Force and Impulse

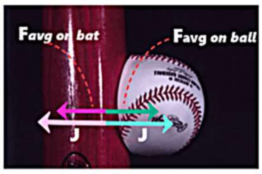
When the force is constant over a time interval Δt, the impulse can be calculated as: $J = F_{av}\,\Delta t$

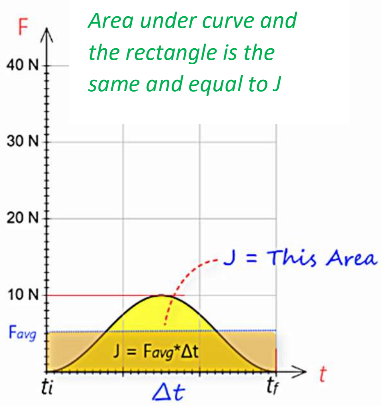
$J = \int F\,dt$

$\Delta p = J$

$J = F_{avg}\,\Delta t$ (on ball)

$J = -F_{avg}\,\Delta t$ (on bat)





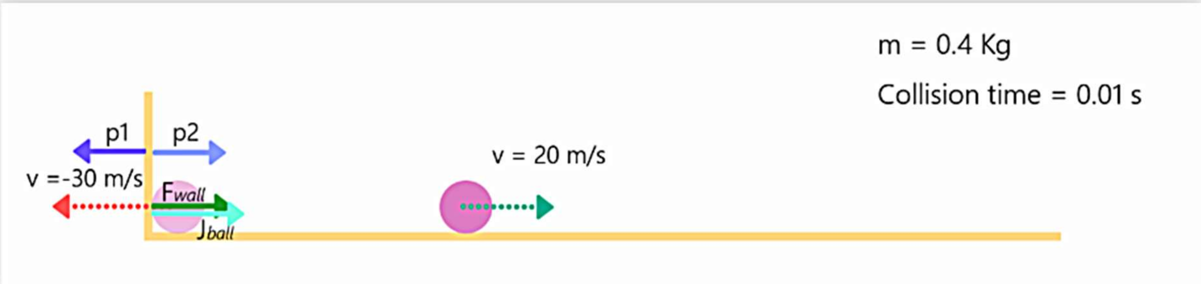
Example Problem

Given:

- A 0.40 kg ball is thrown against a wall at 30 m/s and rebounds at 20 m/s.
- Collision duration is 0.01 s.

To Find:

- Impulse on the ball.
- Average horizontal force exerted by the wall.



Solution:

1. Initial Momentum: $p_1 = mv_1 = 0.40\ kg\ (-30\ m/s) = -12\ kg \cdot m/s$
2. Final Momentum: $p_2 = mv_2 = 0.40\ kg\ (+20\ m/s) = +8\ kg \cdot m/s$
3. Impulse: $J = p_2 - p_1 = 8\ kg \cdot m/s - (-12\ kg \cdot m/s) = 20\ kg \cdot m/s$
4. Average Force: $F_{av} = J / \Delta t = 20\ N \cdot s / 0.01\ s = 2000\ N$

Comparing Momentum and Kinetic Energy

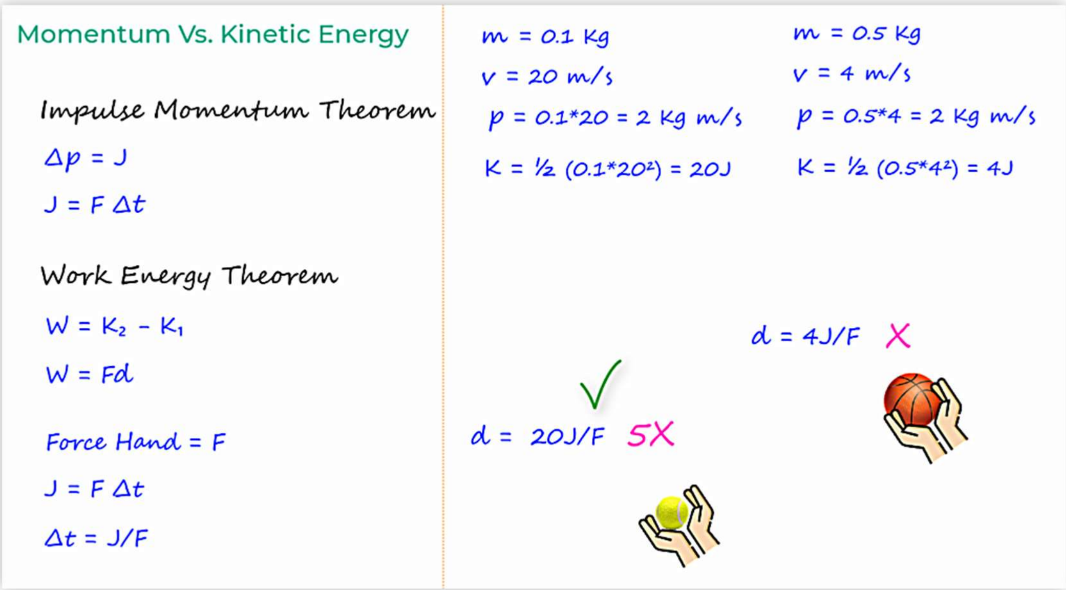
Impulse $\overline{J} = \overline{p}_2 - \overline{p}_1$ is a result of change in particle's momentum, which depends on the *time over which the net force acts* ($\overline{J} = \int \overline{F}(t)\, dt$)

Work–energy theorem $W_{tot} = K_2 - K_1$, gives us the change in kinetic energy when work is done on a particle. Total work done depends on the *distance over which the net force acts and has no dependence on time it took*. ($W_{tot} = Fd$)

Example Comparison:

Catching a 0.10-kg ball at 20 m/s vs. a 0.50-kg ball at 4 m/s. Which is easier to catch?

- Both have the same momentum (p = 2 kg·m/s).
- But, kinetic energy differs: KE of the small ball is 20 J, and the larger ball is 4 J.
- Although momentum and impulse are the same, more work is needed to stop the smaller ball due to its higher kinetic energy ($W_{tot} = K_2 - K_1$)



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Summary of Formulas & Equations

Formula	When to Use	Caution/Keep in Mind
$F = dp/dt$	When determining the relationship between force and momentum	Ensure F is the net external force
$\int dp = \int F\,dt$ from t_i to t_f $\Delta p = \int F\,dt$	To calculate total change in momentum over a time interval	Properly set the integration limits from t_i to t_f
$J = \int F\,dt$ from t_i to t_f	To find the impulse when force varies over time	Accurately integrate over the time interval
$\Delta p = J$	To relate change in momentum to impulse	Ensure consistency in the direction of vectors
$J = F_{av}\,\Delta t$	When the force is constant over the time interval Δt	Use average force F_{av}
$p = m\,v$	To calculate linear momentum of an object	Momentum p is a vector, direction is important